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ES[illegible] SCHOOL MATHEMATICS

TEACHERS' EDITION

STIGATION DISCUSSION UTILIZATION



Accompanying
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CURRICULUM

	Kindergarten	Book One	Book Two
	Units P and R	Units a, b, c, and d	Units e, f, g, and h
A Sets, Logical Reasoning, and Patterns	Concept of sets: P13–14 Comparisons of sizes: P1–8, P11–12, P15 Similarities and differences: P17–32 Patterns: P9–10, P16 Maze: P32	Comparing sets: a1–9 Equivalent sets: a5–8 Cardinal number of a set: a13–14 Empty set: a19 Union of sets and addition: b1–13, b25–32, b41 Sets and subtraction: b15–23, b33–37, b45 Sets of ten: c1–4 Skip counting: d54, d62 Logic: a59–60, d42 Patterns: c14	Cardinal number of a set: e1–7 Maze: e40, g22 Sets and addition: e43 Sets and subtraction: e47 Counting sequences: e15, 62; f36; g6, 11, 13; h38 Informal logic: e52, 57; f55; g12, 34, 37; h20, 50 Patterns: f10 Attribute pieces: g23 Puzzle problems: g34, g62, h8 Multiplication and sets: g50–52, g55, g57
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Teachers' Edition to accompany

Investigating School Mathematics

ROBERT E. EICHOLZ

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Collaborator, Teachers' Edition
THERESA BURKE

Collaborators, Reference Material and Metrication
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Foreword

The *Investigating School Mathematics* series co-ordinates the precise concepts of modern mathematics with an approach that stimulates the child to actively participate in his own learning experiences. The series provides for the necessary mastery of basic number skills, and presents the material in a way that emphasizes the exciting, creative nature of mathematics. As the child becomes involved in exciting explorations and investigations, the structure and beauty of mathematics unfolds. The children are encouraged to investigate and discover ideas for themselves, to look for interesting patterns and relationships, and to develop their own generalizations. New and fascinating topics are explored not solely for their mathematical value, but also because they stimulate interest and motivate children to put forth their best efforts.

In our view, the development of a sound mathematical structure need not be hindered by an exciting, activity-oriented approach. Rather, the activity approach can and should reinforce the child's experiences as he investigates mathematical topics in an orderly, structured fashion. The same, sound mathematical structure that was called "modern" in the 1960's is present in *Investigating School Mathematics*. The important difference in this new series lies in its approach. The child learns through continual active participation in activities and investigations that lead to the discovery of each new idea.

As each new concept unfolds, the child is given an opportunity to investigate the ideas by using a wide variety of manipulative materials and activities. Then, through guided discussion, he is led to a deeper understanding of the ideas and their relation to the overall structure of mathematics. Following the investigation and discussion, he is provided with sufficient problem-solving practice to develop speed and accuracy.

The *Investigating School Mathematics* series is unprecedented in its careful provision for individual differences. Throughout each text, the child is challenged to do what he *can* do, not what someone else *thinks* he can do. Each child has the opportunity to experience individual success in an environment that stresses co-operation and communication rather than competition. This careful provision for individual differences makes the *Investigating School Mathematics* series unusually adaptable to such diverse teaching situations as ungraded schools, individual or small-group instruction, or whole-class instruction.

The essence of the *Investigating School Mathematics* series is reflected in the beliefs to which we are committed: that there are fundamental mathematical concepts which can be isolated and set forth with sharpness and clarity; that these concepts, when truly understood, provide powerful tools for extending knowledge; that children of every level should be encouraged to actively participate, to think, to question, and to seek understanding; that, although a certain body of knowledge must be passed on to each generation from preceding generations, the individual creativity of each new generation must not be stifled by pedagogy which forces upon its pupils patterns of thought which have served us well in the past but which may be inadequate for the future.

Mathematics can be successfully taught in this spirit. At every stage in the learning of mathematics the discovery of new relationships can be a delight. It is in this spirit that *Investigating School Mathematics* has been written.

The authors wish to express their appreciation to Ball State University and to the Educational Research Council of Greater Cleveland, where many of the ideas were generated and tested for the *Elementary School Mathematics* series, which served as a forerunner of *Investigating School Mathematics;* to Edith Biggs and the Nuffield Project in England, for their leadership in bringing the activity-oriented laboratory approach into prominence; to Mrs. Nancy Hildebrand, whose contributions to the teachers' manuals for *Elementary School Mathematics* are still reflected in this manual; to Theresa Burke, who assisted in the preparation of this manual by bringing, from a wealth of classroom experience, many of the activities and teaching suggestions found in each lesson; and finally, to the many teachers and children who have proved that studying mathematics can be an exciting and stimulating experience in the elementary school.

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Mathematics of the Book 4 Program

To gain a better appreciation of the philosophy and teaching strategy that underlies the mathematics for Book 4 of the *Investigating School Mathematics* series, you should take the time to study *Investigating Mathematics Learning: A Text for Teachers*, which is found at the back of this Teachers' Edition.

All of the important mathematical concepts contained in Book 3 of this program are reviewed, reinforced, and extended in Book 4. In writing Book 4, we have assumed that the children know all of the addition and subtraction facts through 18 and that they have been exposed to some of the multiplication facts.

After the introductory chapter on measurement, the children review place value and numeration in Chapter 2. Chapter 3 reviews and extends addition and subtraction with regrouping. Chapter 5 deals with the concepts of multiplication and division. Skills in multiplication and division are the main concern in Chapters 6, 7, 8, and 10. Considerable attention is given to the multiplication and division algorithms in these chapters. Complete mastery of the long-division algorithm is not expected in the Book 4 program inasmuch as this topic will be reviewed and extended in Book 5.

Chapter 11, Number Theory, provides background for certain fractional number concepts such as multiples, factors, greatest common factor, and least common multiple. Fractions and fractional numbers are the topics of Chapters 12 and 14. Little in the way of computational mastery with fractional numbers is expected in the Book 4 program. However, the groundwork laid in fractional number concepts in this book will be invaluable in Books 5 and 6, where computation with fractional numbers will be studied in detail.

There are four chapters covering basic geometry concepts. Chapter 1 is devoted to linear measure, area, and volume. Chapter 4 reviews points, lines, segments, and angles, as well as simple polygons and space figures such as the cube and tetrahedron. Additional space figures and the concepts of congruence and symmetry are introduced in Chapter 9. Chapter 13, the final geometry chapter, deals with co-ordinate geometry. This chapter reviews the idea of locating a point in the plane by means of a number pair and uses co-ordinate geometry to illuminate such concepts as symmetry, similarity, and translations. A brief introduction to negative integers by means of co-ordinate geometry is also included.

A wide variety of story problems, challenge problems (labelled *Think*), and problems involving logical reasoning are integrated throughout the text to help the children improve their skills in reasoning, analysis, and problem solving.

General Bibliography

To gain a broader understanding of the overall mathematical development in the *Investigating School Mathematics* series, the list of books that follows should prove helpful.

Forbes, Jack, and Robert Eicholz, *Mathematics for Elementary Teachers* (Menlo Park, Cal.: Addison-Wesley, 1971).

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Elliott, H. A., James MacLean, and Janet Jorden, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Ontario: Holt, Rinehart and Winston of Canada, Ltd., 1968).

National Council of Teachers of Mathematics, *Insights into Modern Mathematics* (23rd Yearbook, 1957); *The Growth of Mathematical Ideas, Grades K-12* (24th Yearbook, 1959); *Enrichment Mathematics for the Grades* (27th Yearbook, 1963); *Topics in Mathematics for Elementary School Teachers* (29th Yearbook, 1964); *More Topics in Mathematics for Elementary School Teachers* (30th Yearbook, 1969), (Washington, D.C.: NCTM).

Newman, J. R., *The World of Mathematics* (New York: Simon and Schuster, 1956).

School Mathematics Study Group, *Studies in Mathematics, Volume IX, A Brief Course in Mathematics for Elementary School Teachers*, Revised Edition (Stanford University, 1963).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (New York: Cambridge University Press, 1967).

Williams, E. M., and Hilary Shuard, *Elementary Mathematics Today, Grades 1-8* (Menlo Park, Cal.: Addison-Wesley, 1972).

Teaching the Book 4 Program

Design Features of Book 4

Each lesson is titled with either a question or a provocative phrase inviting the child to explore a given idea. The core lessons of Book 4 are designed in one of two ways: those lessons which include an investigation activity begin on the left-hand page with the investigation, followed by a set of discussion exercises and then by a set of exercises on the right-hand page for the children to work independently. The titles of these sections are *Investigating the Ideas*, *Discussing the Ideas*, and *Using the Ideas*, respectively; other lessons are designed around a set of discussion exercises, titled *Discussing the Ideas*, and a set of independent exercises, titled *Using the Ideas*.

Many lessons throughout the book contain starred exercises for enrichment and a *Think* problem for the more able or interested children. Each chapter contains a chapter review (*Reviewing the Chapter*) and every chapter, except the first, contains at least one cumulative review (*Keeping in Touch*).

Though each lesson in Book 4 is presented on facing pages, this format may be treated with considerable flexibility. You may find that for some lessons you will want to spend your entire allotment of time for mathematics on the Investigation and Discussion, saving the exercises for another day. In other lessons, however, you may find that you can cover as many as two facing-page lessons in one day. Keep in mind, though, that, in general, the core lessons are designed to be used for a single mathematics lesson in one day.

Color is used functionally throughout the text whenever it is felt that color-coding of numerals and symbols will facilitate learning and understanding of key concepts.

Teaching Strategies for Book 4

While specific teaching strategies will be made clear through your teachers' edition notes, there is a broad general plan for the teaching strategy throughout the book. The organization of the teachers' manual, as well as the material in the child's book, continually suggests this strategy. It is intended that each day's lesson in which the child is presented with a new concept be divided into four parts: Preparation, Investigation, Discussion, and Using the Exercises. The preparation usually should be kept fairly short, and care should be taken to see that this work does not preempt either the Investigation or the Discussion. Generally, the Preparation should do nothing more than provide the children with that readiness which they need before they begin the Investigation. The Investigation presents the rudiment of the concept treated in the lesson and should be the "main event" in terms of pupil activity and involvement in the unfolding of the concept.

In general, it is expected that the Investigation be done by the children either independently or in small groups. Think of the Investigation as student-centred activity. It is fully anticipated that the students will grope, question, search, and explore. Most Investigations are designed to provide for individual differences; that is, the child is frequently asked to perform a certain task as many ways as *he* can, or to find how many ways he can do a certain thing. By presenting the child with this type of challenge, at least some degree of success is assured. That is, your slowest student will find that he can do something more than one way, while your more able children will find many ways to do a given task. Thus, as you guide the children through an investigation, it is important for you to recognize that they will achieve in widely differing ways, and that you should give recognition for all levels of achievement. Perhaps the most important thing to remember in working with the children during the Investigation is to encourage them to do the thinking and exploring. It is their section. Do not help them too much.

Following the Investigation, the children are given an opportunity in the Discussion section to talk about what they did and to summarize the mathematical ideas of the lesson in preparation for working independently in the Using the Ideas section. Generally, the beginning discussion exercises are designed to stimulate the children to talk about what they did in the Investigation. You should encourage them to discuss the various methods that they used to investigate and explore the concepts. Also, you should follow your teacher's guide carefully to be sure that whatever mathematical ideas are to be developed in the section are actually summarized and understood by the children.

The section titled *Using the Ideas* (called *Using the Exercises* in the teacher's manual) does exactly what the title implies. The children, having come through the first three parts of the day's lesson — the Preparation, the Investigation, and the Discussion — should be ready to work on their own to use the ideas of the lesson. Again, you should provide for individual differences in assigning work in this section; in other words, base your assignments on the needs and abilities of the children. The exercise sets in this section are generally graded, beginning with the easier exercises and ending with the more challenging (starred) problems. Often, there will be a challenging puzzle-type problem for a given lesson. From time to time, a fairly easy challenge problem is provided to encourage less able children to attempt it. Also, quite often, all of the children will benefit from a discussion of the challenge problem.

At the top of each left-hand page of the teachers' edition, under the heading *Objective(s)*, the goal of the lesson is stated in terms of what the child should be able to do as a result of the lesson. This objective summarizes the key idea of the lesson in terms of the child's performance. Throughout the teaching suggestions for the Investigation and Discussion, the most important ideas of each lesson are re-emphasized

and clarified. It is important that you carefully consider the objective of each lesson so that you know how to direct the children through the development of ideas to the desired goal.

Following the statement of the objective for each lesson, the teaching suggestions follow the format of the general teaching strategy mentioned above. The Preparation lists any materials the child will need for the Investigation, and, occasionally, materials recommended for use by the teacher during the Discussion. The preparation section is designed to review ideas, motivate children, or give them necessary background information regarding the Investigation. Depending on your classroom organization, you may choose to use only those Preparations which are essential to the Investigation. At times it is recommended that you have children begin immediately with the Investigation.

The next three sections of the teaching suggestions correspond to the sections of the child's text:

Investigating the Ideas — Investigation

Discussing the Ideas — Discussion

Using the Ideas — Using the Exercises

Each section contains teaching suggestions related specifically to the corresponding section in the child's text. When lessons in the child's text deviate from the standard format, the teaching suggestions fall under the Discussion, and/or the Using the Exercises heading.

A section titled *Mathematics* is included in certain lessons to provide background or to clarify for the teacher the principal mathematical concepts treated in the lesson. This section is strictly for the teacher; terms used here need not be used with the children other than as indicated in the teaching suggestions or in the child's text.

The *Follow-up* suggests various activities, games, or worksheets to help reinforce the concepts developed in the lesson. These are included only as suggestions to be used as your schedule allows. However, some of these activities and many of the materials recommended in the *Resources for Active Learning* may be used by the children independently at various activity or free periods throughout the day.

Since effective use of the investigative approach requires knowledge of the materials available for activity-oriented classrooms, suggestions are provided in the section titled *Resources for Active Learning*. The lists of resources in the chapter introductions and in most of the lessons offer an ample start. Provided some of these resources are available, you should be able to use or adapt the ideas contained in them to various situations.

The "Manipulative Devices" can be used to support the lessons. The "Commercial Games" and "General Activities" in the Chapter Introduction should be useful as continuous activities throughout the chapter, for review or practice of basic skills and concepts. The Resources listed for specific lessons are directly related to those lessons' objectives. Choose one or two, and try them out in a variety of situations.

At the time of this writing, the authors cited those resources which are of high quality and which directly complement the active-learning approach. Familiarize yourself with those materials which have been marketed subsequently. Check with your principal or supervisor for more recent materials to support this type of learning approach. Such materials can often be obtained from your local supplier. Additional sources are listed on page xi.

Provision for Individual Differences

Minimum, average, and maximum assignments are provided for each lesson other than review lessons. These assignments are given to assist you in providing for the individual needs of the children. It is *not* intended that you give the minimum assignment to the slower children, the average assignment to the average children, and the maximum assignment to the more able children. Rather, these designations are given to assist you in making individual assignments according to needs, abilities, and time available for each individual child. For example, if time is short and you need to move rapidly through a particular lesson, you may choose to use the minimum assignment for all children. The minimum assignment will, in general, provide the children with sufficient practice and mastery of skills to move ahead to the next lessons. On the other hand, you may sometimes choose to use the maximum assignment with slower children over a period of two or three days. Also, it is highly likely that you will not want to assign the maximum assignment to the more able children, since quite often they need less practice than some average and below average children. For example, when your more able children demonstrate the ability to perform a particular skill with great efficiency, they should not be made to drill excessively in that skill. In some cases, an asterisk is placed beside an assignment to indicate that the lesson could be omitted without loss of continuity.

Long-Range Planning Chart

The long-range planning chart is designed to provide the teacher with some guidelines for planning basic, average, maximum course coverage of *Investigating School Mathematics.*

The basic course outline covers all the essential parts of the program, but provides little in-depth or extension material. The average course covers all the material of the basic course plus considerable extension material. The maximum course provides for nearly total coverage of all topics presented in the text. Optional material for each of the three courses is shown by shaded page numbers.

The suggested time schedule, covering 36 weeks, should be viewed only as an *aid* in helping you plan a time schedule that allows for the individual differences within your own class. You should not view it as a rigid schedule. For example, the basic course may be used with children who are achieving below the average grade level. This same course might be used in other ways, such as with a brighter child who may move rapidly through these same pages to

LONG-RANGE PLANNING CHART						
Chapter	Basic Course		Average Course		Maximum Course	
1	4-17, 20-21, 26-29, 32-33 22-23	3 weeks	4-23, 26-29, 32-33 24-25	3 weeks	4-33	3 weeks
2	34-39, 42-45, 56-57 40-41	3 weeks	34-39, 54-57 50-51	$2\frac{1}{2}$ weeks	34-49, 54-57 50-53	$2\frac{1}{2}$ weeks
3	58-63, 66-69, 72-73, 76-77, 80-84, 88-90, 94-97 64-65, 70-71	5 weeks	58-69, 72-77, 80-91, 94-97 70-71	$4\frac{1}{2}$ weeks	58-77, 80-91, 94-97 78-79, 92-93	4 weeks
4	98-99, 102-105, 110-111, 114-115	2 weeks	98-107, 110-111, 114-115 106-109	2 weeks	98-111, 114-115 112-113	2 weeks
5	116-129, 132-143, 152-154 130-131, 144-147	$5\frac{1}{2}$ weeks	116-147, 152-154 148-149	5 weeks	116-149, 152-155 150-151	$4\frac{1}{2}$ weeks
6	156-165, 168-169	$1\frac{1}{2}$ weeks	156-165, 168-169 166-167	$1\frac{1}{2}$ weeks	156-169	$1\frac{1}{2}$ weeks
7	170-177, 180-181	1 week	170-177, 180-181 178-179	1 week	170-181	$\frac{1}{2}$ week
8	182-193, 202-205 194-197	$3\frac{1}{2}$ weeks	182-197, 202-205 198-199	3 weeks	182-197, 200-225 198-199	3 weeks
9	206-211, 218-219 214-215	1 week	206-211, 214-219 212-213	1 week	206-219	$1\frac{1}{2}$ weeks
10	220-231, 234-236, 246-249 232-233	4 weeks	220-236, 246-249 240-243	$3\frac{1}{2}$ weeks	220-236, 244-249 237-243	$3\frac{1}{2}$ weeks
11	250-253, 260-261 254-255	1 week	250-255, 260-261 256-257	1 week	250-255, 260-261 256-259	1 week
12	262-285, 292-295 288-289	4 weeks	262-285, 292-295 286-289	$3\frac{1}{2}$ weeks	261-289, 292-295 290-291	4 weeks
13	296-307, 314-315 308-309	$1\frac{1}{2}$ weeks	296-307, 314-315 310-311	2 weeks	296-311, 314-315 312-313	2 weeks
14			316-329, 342-345 330-337	$2\frac{1}{2}$ weeks	316-339, 342-345 340-341	3 weeks

make up time lost through absence.

General Suggestions

We offer two general suggestions regarding use of the chapter and page lesson notes:

(1) Read and consider each point as it applies to the immediate objectives for the lesson and the overall objectives of the unit.

(2) Do not allow the teachers' manual notes to deter you from using your own effective teaching methods or to stifle creative efforts.

Your manual does not attempt to dictate all of the activities in the day-to-day handling of your class and the individuals in it. You should use your manual as a guide to be co-ordinated with those methods which you have found to be most effective in teaching mathematics in the past. One of the key techniques in presenting a structured system of mathematics consists of developing a topic and pursuing it until you have reached a desired level of understanding with regard to that topic. This is one of the guiding philosophies behind the development of *Investigating School Mathematics*. You will notice this philosophy particularly in the development of some of the more intense sections of work. That is, when the topic is being introduced, it is explored in great detail; however, we

have tried to provide interesting activities to relieve the intensity of these longer sections. Treat the relief materials with a light touch and in the spirit of having fun and playing games with mathematics.

Classroom Organization

The Book 4 program can be used in situations where the entire class works together on the same lesson, where small groups work together on the same lesson, or where individual students are allowed to proceed at their own rate of speed. You will want to employ the type of classroom organization that best suits the physical facilities of your particular situation. Teachers often find it stimulating to vary group sizes for different lessons and units of work. Smaller-size groups can often work more effectively together and allow greater opportunity to participate in the Investigation and the Discussion. Whatever class organization you choose, keep in mind that the key to the *Investigating School Mathematics* program is active student participation.

Evaluation of Progress

A child's attitude toward mathematics is often influenced by the methods used for evaluating his progress. All too often, evaluation procedures focus attention on what the child *did not* understand or master, rather than on what the child *did* accomplish. In evaluating a child's progress, try to maintain a positive view, one which capitalizes on successes and develops confidence.

Achievement and diagnostic tests for Book 4 may be obtained from the publisher. Chapter reviews as well as cumulative reviews are provided in the text, to aid in evaluation of the child's progress. However, you may find that a day-to-day evaluation of the children, often involving personal interviews, will help you determine how well a child grasps the concepts and how well he is able to apply them.

Children's Bibliography

Books listed in the Books to Explore section of the pupils' text (pages A31–A33) were selected to appeal to the interests of children, as well as to provide historical background and reference material.

Additional books which may be of interest to you and the children are listed below.

More Books to Explore

Adler, Irving, *Logic for Beginners Through Games, Jokes and Puzzles* (New York: John Day, 1964). This book contains 285 challenging brain teasers.

Adler, Irving and Ruth, *Numbers Old and New* (New York: John Day, 1960). This book explains the counting methods of Australian natives and Mayan Indians, and the fractions used by Egyptians and Greeks.

Carona, Philip, *Things That Measure* (Englewood Cliffs, N.J.: Prentice-Hall, 1962). This book contains many pictures and diagrams to help explain units called cubits, how tobacco is used for money, and what is measured on a jeweller's stick.

Froman, Robert, *Faster and Faster: A Book About Speed*, (New York: Viking Press, 1965). The reader can compare the speed of snails, turtles, people, fish, cheetahs, boats, airplanes, the wind, the earth, spacecraft, light waves, and radio waves.

Hine, Al, *Money Round the World* (New York: Harcourt Brace Jovanovich, 1963). This book tells about trading and money.

Kaplan, Philip, *Posers* (New York: Harper and Row, 1963). You will find 80 hurdles for reasonably agile minds in this book.

Kettlekamp, Larry, *Puzzle Patterns* (New York: Morrow, 1963). This book neatly blends a brief history of puzzles with the everyday uses of puzzle patterns in gear and keys.

Luce, Marnie, *The Math Concept Books*(Minneapolis, Minn.: Lerner Publications, 1969). This series is designed to give increased understanding of the way numbers work.

Shimek, William, *Patterns: What Are They?* (Minneapolis, Minn.: Lerner Publications, 1969). A Math Concept Book. This book discusses the mathematical importance of patterns and examines the natural patterns around us.

Smith, David Eugene, *Number Stories of Long Ago* (Washington, D.C.: NCTM, 1969). This book consists of stories about numbers, counting, numeral writing, and computing done by boys and girls of long ago.

Smith, David Eugene, and Ginsberg, Jekuthiel, *Numbers and Numerals* (Washington, D.C.: NCTM, 1958). Unusual things about measures and curious number superstitions are a few of the subjects of this book.

Smith, George O., *Mathematics: The Language of Science* (New York: G.P. Putnam, 1961). The stories in this book tell of how the first men kept track of their possessions, how Roman businessmen struggled with Roman numerals, and how the Babylonians discovered place value.

Whitney, David, *The Easy Book of Fractions* (New York: Franklin Watts, 1970). The reader is shown the many ways in which people use fractions.

Other Books of Interest

Barr, Donald, *Arithmetic for Billy Goats* (New York: Harcourt Brace Jovanovich, 1966). Base-two arithmetic.

Bendick, Jeanne, *Names, Sets, and Numbers* (New York: Franklin Watts, 1971). Sets, numbers, and classification.

Carroll, Lewis, *Through the Looking Glass* (New York: Grosset & Dunlap, 1963). Mirrors and mathematics.

Charosh, Mannis, *Straight Lines, Parallel Lines, Perpendicular Lines* (New York: Thomas Crowell, 1970). A Young Math Book. Relating the ellipse to other curves.

Frédérique and Papy, *Graph Games* (New York: Thomas Crowell, 1971). A Young Math Book. Sets and relations.

Hertzberg, Hendrik, *One Million* (New York: Simon & Schuster, 1970). Number.

Linn, Charles, *Estimation* (New York: Thomas Crowell, 1970). A Young Math Book. Making calculated guesses.

Luce, Marnie, and A. B. Lerner, *Infinity: What Is It?* (Minneapolis, Minn.: Lerner Publications, 1969). A Math Concept Book.

McCloskey, Robert, *Time of Wonder* (New York: Viking Press, 1957). Measurement and comparison.

Sitomer, Harry and Mindel, *Circles* (New York: Thomas Crowell, 1971). A Young Math Book. The nature of a circle.

Srivastava, Jane, *Weighing & Balancing* (New York: Thomas Crowell, 1970). A Young Math Book. Measurement and comparison.

Thurber, James, *The Great Quillow* (New York: Harcourt Brace Jovanovich, 1944). Number, measurement, comparison.

Bibliography of Resources for Active Learning

Abott, Janet, et. al., Franklin Mathematics Series: *From Fingers to Computers; Learning About Measurement; Learn to Fold—Fold to Learn; Making and Using Graphs and Nomographs; Mirror Magic; Patterns and Puzzles in Mathematics; Pencil and Paper Geometry* (Chicago: Lyons and Carnahan, 1970).

Bates, John, Donald Irwin, and Garry Hamilton, *Developmental Math Cards* (Don Mills, Ont.: Addison-Wesley, 1970).

Biggs, Edith, and James MacLean, *Freedom to Learn* (Don Mills, Ont.: Addison-Wesley, 1969).

Clarkson, Dave, *Math Activity Cards* (New York: Macmillan Co., 1969).

Cohen, Donald, *Inquiry in Mathematics via the Geo-board* (New York: Walker, 1967).

Cohen, Donald, *Mathematics Using String* (Danbury, Conn.: Math Media Inc., 1968).

Davis, Robert, Madison Project: *Discovery in Mathematics: A Text for Teachers* and *Student Discussion Guide* (Menlo Park, Cal.: Addison-Wesley, 1964).

Elementary Science Study: *Attribute Games and Problems; Mirror Cards; Peas and Particles* (St. Louis: Webster Division, McGraw-Hill Book Co., 1968).

Fletcher, Harold, Ruth Walker, and Arnold Howell, *Mathematics in Modules* (Sydney, Australia, Addison-Wesley, 1973).

Galton, Grace, Arlene Fair, and Patricia Davidson, *Chip Trading Activities—SET 1* (Fort Collins, Colo.: Sigma, Scott Scientific, 1970).

Goddard, T.R., and A.W. Grattidge, *Applied Mathematics Cards* (Huddersfield, England: Schofield & Sims Ltd., 1965) (available from Math Media Inc., Danbury, Conn.).

Huff, M. Elizabeth, and Donald Irwin, *Activities in Geometry for Primary Pupils* (Don Mills, Ont., Addison-Wesley, 1973).

Mathex: Matching and Graphing No. 1; Numeration No. 2; Operations No. 3; Geometry No. 4; Graphing and Probability No. 6; Numeration No. 7; Operations and Problem Solving No. 8; Geometry No. 9; Measurement No. 10 (Teacher's Resource Books and pupil pages) (Toronto, Ontario: Encyclopaedia Britannica Publications Ltd., 1970).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (Cambridge, England: Cambridge University Press, 1967).

Nuffield Mathematics Project: *Environmental Geometry; Problems—Green Set; Computation and Structure 2, 3, 4; Beginning 1; Shape and size 2, 3; Pictorial Representation 1; Graphs Leading to Algebra 2* (New York: John Wiley, 1967-1969).

Pearcy, J.F.F., and K. Lewis, *Experiments in Mathematics*, Stages 1 and 2 (Boston: Houghton Mifflin, 1966).

Sealey, Leonard, *Early Number Multigroup Lab* (Englewood Cliffs, New Jersey: Responsive Environments Corp., 1969).

Swartz, Clifford, *Measure and Find Out*, Books 1 and 2 (Glenview, Ill.: Scott, Foresman & Co., 1969).

Turner, Ethel, *Teaching Aids for Elementary Mathematics* (New York: Holt, Rinehart and Winston, 1966).

Wirtz, Robert, et al. *Math Workshop:* Games and Enrichment Activities (Chicago: Encyclopaedia Britannica Educational Corp., 1964).

Wirtz, Robert, Morton Botel, and B. G. Nunley, *Discovery in Elementary School Mathematics* (Chicago: Encyclopaedia Britannica Educational Corp., 1963).

Suppliers of Resources for Active Learning

Addison-Wesley (Canada) Ltd., Don Mills, Ontario

Cadaco, Inc., Chicago, Illinois

CCM School Materials, Inc., Chicago

Creative Publications, Palo Alto, California

Cuisenaire Company of America, Inc., New Rochelle, New York

Edmund Scientific Co., Barrington, New Jersey

Educational Teaching Aids, Chicago

Encyclopaedia Britannica Publications Ltd., Toronto, Ontario (Canada)

Geyer Instructional Aids, Fort Wayne, Indiana

J. L. Hammett Co., Braintree, Massachusetts

D. C. Heath Canada Ltd., Toronto, Ontario

Herder and Herder, Inc., New York

Holt, Rinehart and Winston, of Canada, Ltd., Toronto, Ont.

Kohner Bros., Inc., East Paterson, New Jersey

LaPine Scientific Co., Chicago

E. S. Lowe Co., Inc., New York

The Macmillan Co., (Collier-Macmillan, Don Mills, Ont.)

Math Media Inc., Danbury, Connecticut

Midwest Publications Co., Inc., Birmingham, Michigan

Milton Bradley, Springfield, Massachusetts

Responsive Environments Corp., Englewood Cliffs, New Jersey

Selective Educational Equipment, Newton, Massachusetts

Scott, Foresman and Co., (Gage Educational Publishing Ltd., Agincourt, Ont.)

Sigma Division, Scott Scientific Inc., Fort Collins, Colorado

TUF, Rowayton, Connecticut

Walker Educational Book Corp., New York

Webster Division, McGraw-Hill Book Co. (McGraw-Hill Ryerson Ltd., Scarborough, Ont.)

PAGES 2–3

Objective

Given the Investigating School Mathematics *text, the child will understand the purpose of the three sections of each regular lesson.*

Preparation

This first step of the four-part teaching plan for each lesson in the text is the briefest. However, it will often be the key to inspiring children with an enthusiastic approach to the lesson. In some lessons, it is essential to the investigation; in others, it simply serves as a review. It should be kept brisk and lively, and seldom should it extend beyond five minutes.

As preparation for this introductory lesson, you might simply encourage the children to flip through the pages of the book, pausing to look at whatever interests them. After one or two minutes, refer them to the title and author page and the table of contents, asking them what chapters they think they will enjoy the most. Finally, refer them to the first introductory lesson.

Investigation

As stated in the introductory material, the investigation presents the child with an opportunity to explore, often with minimum teacher direction, the ideas presented. Sometimes this exploration consists of an investigation with concrete objects; at other times, the child simply uses paper and pencil to explore ideas about number. In any case, this section should be totally child-centred. It is the child's opportunity to explore new ideas or new ways of looking at familiar ideas. It is the kick-off for the lesson. The child's efforts in this section should not be assessed in terms of "right" and "wrong." Rather, he should be encouraged to explore things on his own, with the understanding that he often learns as much from his "incorrect" methods as from his correct methods.

In this introductory lesson, you will probably want to read with the children the paragraph at the top of the page and explain that you want them to read and answer questions 1, 2, and 3 by themselves. Move around the room encouraging children who find the required sections and suggesting that they look for others. Stress the importance of recording their answers even when the text does not explicitly request it.

Discussion

The discussion section provides an opportunity for children to discuss the ideas they investigated and for you to guide the discussion in order to stress the main point of the lesson. It is in this section that you will sometimes want to demonstrate various computational skills or use visual materials to emphasize a particular concept. The questions in the text help the children to focus on the most important concepts, but your skill in guiding discussion and presenting ideas will be invaluable in this section.

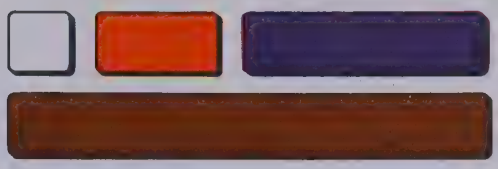
Notice that the first question, 1A, gives children an opportunity to talk about their answers from the investigation. Then, questions 1B and 2 use these results as a springboard to other related ideas. Have children note the color bands for each section and point out that every investigation section is followed by a discussion section. However, in some lessons no investigation precedes the discussion section.



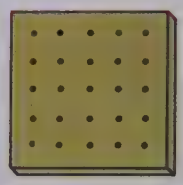
Investigating the Ideas

This is a sample lesson. It will help you understand how to use your book. In this part of a lesson there are things for you to **investigate** and discover.

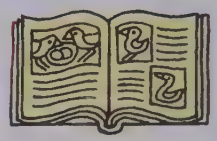
1. Can you find an "Investigating the Ideas" section where you use colored strips like these? Sample answer: page 4



2. Can you find an "Investigating the Ideas" section where you use dot paper or a board with nails in it? Sample answer: page 266



3. Can you find an "Investigating the Ideas" section where you look in the newspaper, a magazine, or another book? Sample answer: page 42





Discussing the Ideas

In this part of a lesson you will **discuss the ideas** you investigated. You will be sharing your ideas with others. You will be getting ready to **use the ideas.**

1. A Which of the "Investigating the Ideas" sections you looked at seemed interesting? Why? Answers will vary.
 B Does a "Discussing the Ideas" section follow that "Investigating the Ideas" section? Does it ask you to explain something or to give your ideas? Yes Yes
 Read one of the questions.

2. Does every "Discussing the Ideas" section follow an "Investigating the Ideas" section? No; see Discussion.

2

Using the Ideas

In this part of the lesson you will **use the ideas.** You will work problems to improve your understanding of the ideas you have discussed. Try these.

1. How many "Investigating the Ideas" sections are in Chapter 4? 8

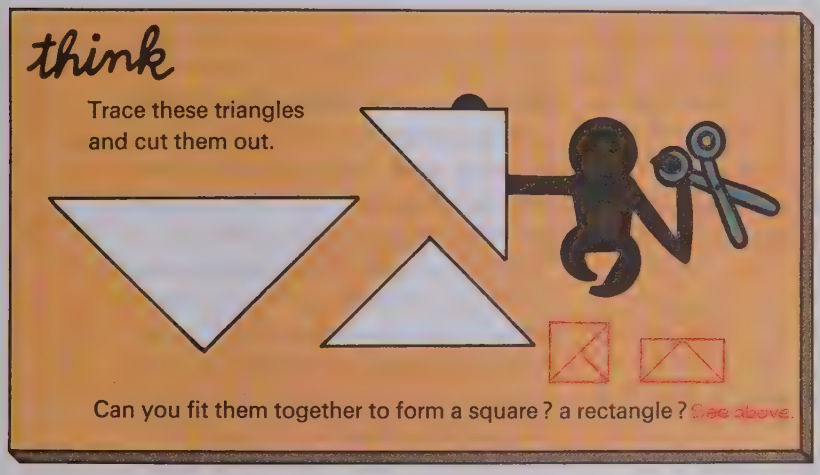
2. Find the number of "Discussing the Ideas" sections in Chapter 6. 5

3. How many "Using the Ideas" sections are in Chapter 8? 6

4. A Give the page number for a lesson called "Keeping in Touch."
 B Find the **Activity Card** for this lesson. What page number is it on? Sample answer: page 347
 Sample answer: page 57

5. Look up *segment* in your **index.** What page numbers are given?
 Pages 102-103, 114

Problems in these boxes are a **special challenge** for you. Be sure to try some of them. See if you can do this one.



3



Using the Exercises

The section in the child's text entitled *Using the Ideas* is referred to in the teacher's manual as *Using the Exercises*. As indicated by this title, this section is intended to provide the children with an opportunity to use the ideas they have investigated and discussed. Ordinarily children should be expected to work by themselves for this section. Occasionally, however, you might want to use a few of the exercises as a basis for introduction, directions, and discussion. Exercises on these pages may be assigned selectively; that is, it is rarely essential that each child do all the exercises. However, on many pages the exercises progress developmentally, so children should do at least some parts of each exercise. Starred exercises refer to exercises which extend the concept of the lesson or give more difficult applications of it. They are not intended for all children.

Explain to the children the purpose of this section and assign the exercises. In some of these sections the amount of reading involved may make it desirable for you to read the exercises aloud with some children. The *Think* problems constitute a special challenge. Encourage children to try them, even though they are intended primarily for enrichment.

General Objectives

To provide experience in working with length, area, and volume
To introduce the idea of selecting appropriate units
To provide practice in estimation
To introduce the concept of perimeter
To provide experience in working with units for liquid measure

This chapter explores three basic types of measurement—length, area, and volume—and introduces the concept of perimeter for the first time.

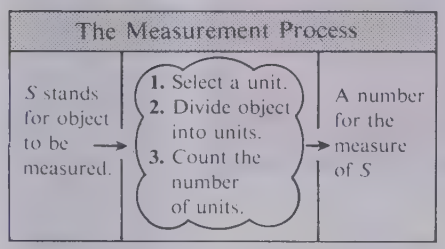
Length, area, and volume are concepts extended somewhat from Book 3, but they are primarily review topics. Hence, the early work on these three topics moves quite rapidly.

In this chapter, measurement can be divided into two parts: (1) counting distinct objects in a set and (2) approximation or estimation. When we can obtain measurements by counting distinct objects, we have no need for approximation or estimation. However, the concept of approximation or estimation is used when we cannot count distinct objects. In this chapter, the children learn to make some approximations to determine the lengths of given objects. Fractional numbers in measurement are developed as an outgrowth of approximations, and indirectly provide the child with a background for fractional numbers. The concept of perimeter as the distance around a figure is carefully developed as an application of measurement of length.

Carefully planned sequences of exercises lead the children to discover some basic formulas for finding area of plane regions and volume of space figures. However, no attempt is made to formalize these discoveries. The children are also given some experience in working with volumes of liquids. Although the concepts are essentially the same, the children will find that measuring liquids is quite different from counting cubes to find volume. In both kinds of measurement, it should be noted that we find volume with respect to a given unit. By working the exercises, the children see the relationship between finding volume with units of liquid measure and finding volume with cubes.

Mathematics

Measurement is a process of associating numbers with certain objects. Before an object in the real world can be measured, a unit must be chosen. The number 1 is assigned to this unit. The measuring process then consists of comparing the object to be measured with this unit, usually by counting the number of units it takes to "cover" or "fill up" the object. Thus the choice of the unit determines which numbers are assigned to other objects. This process is illustrated below.



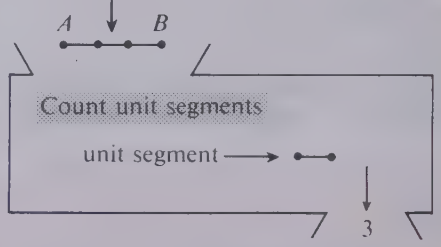
In this chapter we are concerned with measurement of length, area, and volume. Notice, however, that a broad interpretation of the measurement process also allows the measurements of angles, weight, and time to be included.

The following illustration shows the kind of unit used in the types of measurement discussed in this chapter.

Idea	Unit
Length	Segment
Area	Square
Volume	Cube

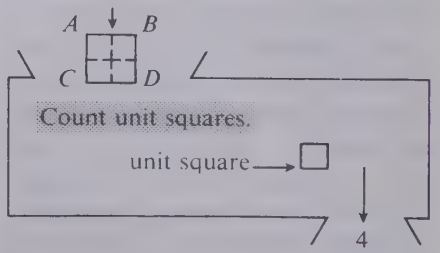
We could specify particular examples of these units—a metre, a square metre, and a cubic centimetre—but we want to discuss these units in general terms.

The length of segment *AB* is a number that compares the size of the segment to be measured with the size of a unit segment. Perhaps the following illustration will help clarify the concept of measurement of length.



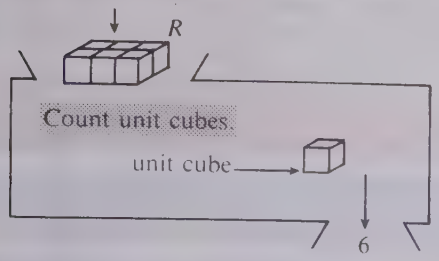
The input is line segment *AB*. The machine performs the operation of counting unit segments, and the output, 3 in this example, is the number of unit segments in the input.

To find the *area* of a given region, a unit must be chosen and counted just as in determining length. Usually a unit of length is chosen and a square region that has sides of that unit length is used as the area unit. This unit is then counted to determine how many of these *unit-square* regions it takes to cover some region. Consider region *ABCD*.



The number of unit squares counted, 4, is the area of region *ABCD*. To find the area of geometric figures such as rectangles, parallelograms, triangles, and circles, special formulas which provide shortcuts for the counting process may be used. The basic concept of area measurement is, however, one of counting unit squares.

To find area, a region of a plane is measured. To find *volume*, a region of space is measured. Just as with area, a unit must be chosen and counted. A common unit for volume is that region of space in a cube that has edges of length 1, a *unit cube*.



The number of unit cubes it takes to fill the region, R, is the volume of R, 6 unit cubes. In measuring volume, a number is associated with a region of space. Again, when it is impractical to divide the space into unit cubes and count, it is necessary to develop shortcut formulas; but the formulas are based on the counting process.

In each type of measurement discussed, a more accurate measure could be obtained by choosing a smaller unit, such as a part of the unit shown. For example, for distances that cannot be measured accurately enough by using the metre as a unit, the centimetre, which is $\frac{1}{100}$ as long as a metre, is often used. This suggests that the output number in practical measurement is often a fractional number.

Teaching the Chapter

Materials

Blocks, sugar cubes
Centimetre ruler
Colored strips
Construction paper
Containers for liquid measure: 2 litre, 1 litre, 500 ml, 250 ml, 100 ml
Demonstration squares, marked in fractional regions
Empty boxes to be filled by cubes in measuring volume
Flannelboard
Graph paper
Metre stick
Notebook paper or newsprint
Overhead projector
Sand or water for measuring volume
String (1-metre piece per 2 children)
Transparent ruler for use on overhead projector

Vocabulary

area	metre
centimetre	millilitre
cube	minute
day	perimeter
estimate	scale
fractional part	second
hour	segment
kilometre	tonne
length	unit
litre	volume
measurement	week

Because measurement concepts can be explored best through physical activities, it is imperative that you give the children an opportunity to make considerable use of various materials in class activities. The investigations provide adequate manipulative experiences, but you may choose to extend use of materials in other physical activities, depending on the children's need for such experiences.

The vocabulary list contains all the words pertinent to the discussions in this chapter. Many of the words are already familiar to the children, but you should be alert to those words which are new to the class and give them particular emphasis.

Lesson Schedule

Because many varied activities can be conducted to illustrate the lessons in this chapter, there is some danger that these activities may take too much time. This does not mean that you should greatly limit the activities for these lessons but only that you should exercise caution to see that this chapter does not take an undue amount of time. Plan to spend no more than three weeks on the chapter.

Evaluation of Progress

The chapter review on pages 32 and 33 can be used for evaluation, or you can construct a test similar to this review. In judging the children's achievement, keep in mind that exploration and recognition of the ideas is of primary importance and that complete mastery of all material is not expected.

Treat the chapter with a light touch, since it is not a necessary background for the rest of the year's work. Nevertheless, these ideas are important to the full development of mathematics. Since one of the primary aims of this program is to stimulate the children's interest in arithmetic, try to see that the children have a successful and enjoyable experience with this chapter.

Resources for Active Learning

General Activities

Developmental Math Cards, G$\frac{2}{}$6, Addison-Wesley
Freedom to Learn, "Length," "Perimeter and Area," "Volume and Capacity," pp. 121–134, Addison-Wesley
Nuffield Project: *Computation and Structure* 2, "Length," pp. 6–23, Wiley. [Includes many suggestions for practical work.]

Manipulative Devices

Area Discovery Blocks (Educational Teaching Aids)
Caliper (Edmund Scientific; Selective Educational Equipment)
Capacity and Weight Measures (Educational Teaching Aids; school supplier)
Cubical Counting Blocks (Milton Bradley, school supplier)
Geoboards (Addison-Wesley)
Lake and Island Board (Math Media)
Map Measurer (Edmund Scientific)
Metre Stick (Geyer Instructional Aids)
Surveyor's Land Chain (Selective Educational Equipment)
Trundle Wheel — Metre (Math Media; Selective Educational Equipment)

Commercial Games

The Weighting Game—A Game of Strategy and Balance (Cadaco)
Which Is More? (Selective Educational Equipment)

PAGES 4-5

Objective

Given an appropriate pair of the colored strips, the child will be able to give the measurement of one strip by using the other as a unit.

Preparation

Materials

colored strips

The only preparation essential to this lesson is to make sure each child has a set of colored strips. The strips can be made easily, but are also available from Addison-Wesley. It is suggested that you supply each child with an envelope or other suitable container; direct him to put his name on the envelope; and, after each use of the strips, collect them to be used again when needed.

If the majority of the children are familiar with the strips, having used them previously with books of this series, you might let them spend a few minutes recalling ways in which the strips were used. If the strips are new to the children, let them speculate about how the strips might be used.

Investigation

Because this is the first regular lesson in the book, you might want to give particular guidance to the class in working through it. Direct the children to arrange their strips as shown on page 4. Read the directions with the class, and let the children study the strips. Then write the colors of the strips on the chalkboard and suggest that, as they work on the first question, they record the number with the color to which they would assign it. The color names in order of length, *when the white strip is the unit*, are (1) white, (2) red, (3) light green, (4) purple, (5) yellow, (6) dark green, (7) black, (8) brown, (9) blue, and (10) orange. Children should use the unit (1) strip to count how many times it fits on another strip, to prove to themselves which number should be assigned to each strip. By choosing the red strip as the unit and fitting it onto another strip, the child can find the length of that strip in terms of the red strip. Of course only the red, purple, dark green, brown, and orange strips will have whole-number measures (1, 2, 3, 4, and 5 respectively). Children might give the lengths of the other strips, but lead them to concentrate on the strips whose measure can be given in whole numbers.

1 Numbers and Measurement

How do we use numbers in measurement?

Investigating the Ideas

Think of the number 1 when you see the white strip. What numbers do you think of when you see each of the other strips?



White-½; lt. green-1½; purple-2; yellow-2½; dark green-3; black-3½; brown-4; blue-4½; orange-5.

? Think of the number 1 when you see the red strip. Can you give numbers for some of the other strips?
See Investigation.

Discussing the Ideas

Now think of the number 1 when you see the light green strip. Give the lengths of some other strips. (The length of any strip is the number of times the chosen unit fits on that strip.)



4

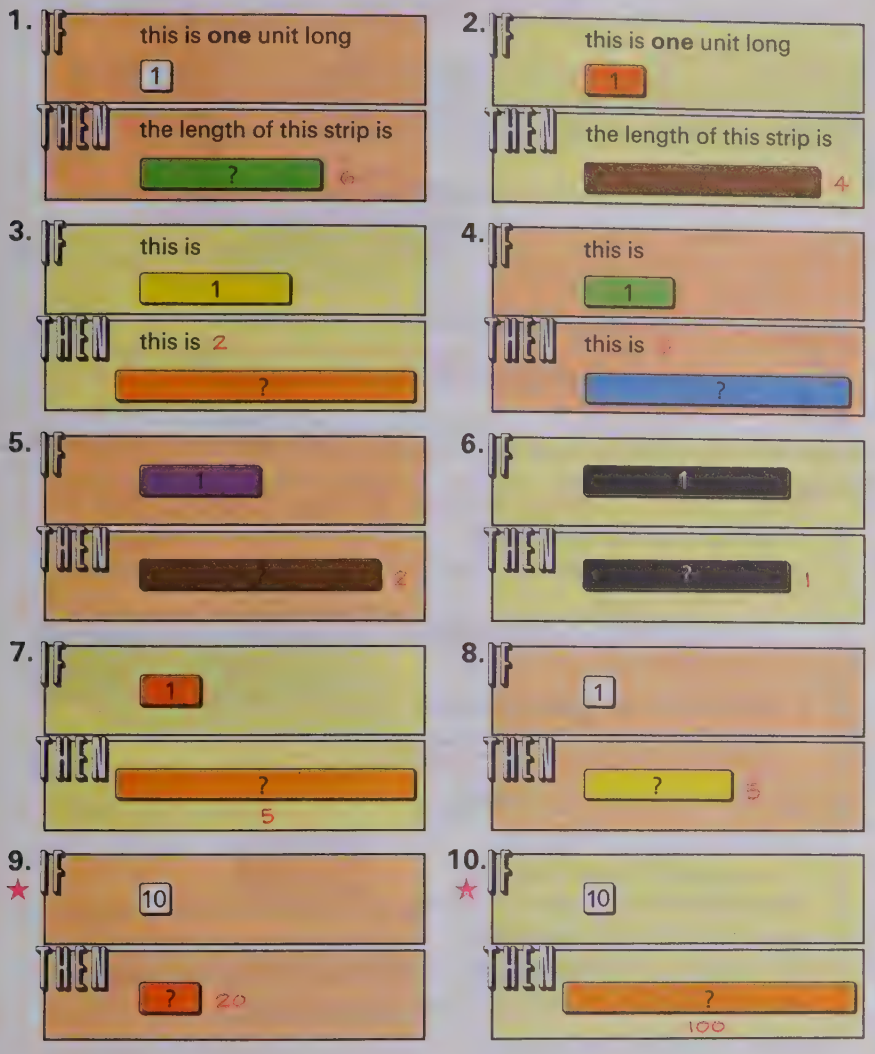
White-⅓; red-⅔; purple-1⅓; yellow-1⅔; dark green-2; black-2⅓; brown-2⅔; blue-3; orange-3⅓

Discussion

One of the most important concepts treated in this lesson is that of *unit*. In the investigation, the children intuitively explore the idea that the number assigned as the length of an object depends upon the unit used to measure it. Encourage the children to discuss the results of the investigation. Help them realize that their unit strip was first the white strip and then the red strip; and for each different strip the number they thought of, or the length, was also different. For example, when the white strip is the unit the purple strip should be assigned the number 4; but when the red strip is the unit,

Using the Ideas

Give the missing number in each exercise below.
You may want to use your strips to find the answers.





the purple strip should make one think of the number 2. Then allow the children time to find the measure for other strips when the light green strip is the unit, as suggested in the text, and let them compare and discuss their findings. Again, as in the investigation, children should concentrate on the strips whose measure can be given in whole numbers.

Using the Exercises

If the children have understood how to use the unit strip to measure other strips, assign the exercises on page 5 as independent work. If necessary, work through a few exercises with the children to ensure that they know what they are to do. The starred exercises are more challenging, but many children would benefit from an explanation of how to use the white strip as 10 and count the red strip and the orange strip by tens. When the children have finished, help them check their work together.

Assignments (page 5)

Minimum: 1–8, oral. Average 1–8. Maximum: 1–10.

Mathematics

Once a unit has been chosen for the measurement of a segment, the length of the segment is determined. The measurement of the segment is found by counting the number of times the unit is contained in the segment being measured. Length, then, is a number associated with the segment in relation to the chosen unit. By considering lengths of their strips in terms of various chosen units, the children explore these basic concepts.

Follow-up

To provide the children with more experience in using different units, prepare a worksheet similar to the one below.

Give the length for each line according to the unit given for each exercise.

Line A
6 cm

Unit	Line Length
1. white strip	1.
2. red strip	2.
3. light green strip	3.

Line B
8 cm

4. white strip	4.
5. red strip	5.
6. purple strip	6.
7. brown strip	7.

Line C
9 cm

8. white strip	8.
9. light green strip	9.
10. blue strip	10.

Resources for Active Learning

Franklin Series: *Learning About Measurement*, pp. 5–7, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Workshop: Games and Enrichment Activities, "Three Boxes," pp. 10-11, Encyclopaedia Britannica Educational Corp. [A game of logical reasoning through If-Then situations]

Mathex: Measurement No. 10, "Measurement with Different Units," pupil page 3, Encyclopaedia Britannica Publications Ltd.

Workbook, page 1

PAGES 6–7

Objective

Given a centimetre ruler, the child will be able to measure the length of appropriate objects.

Preparation

Materials

colored strips; centimetre ruler

To prepare for this lesson, you might tell the children that although the investigation directs them to use the white strip, they should have their other strips available. They may find a use for them in the investigation or in other parts of the lesson. (It is hoped that some children *will* use strips other than the white strip, but they should be given the opportunity to think of this themselves.)

Investigation

In this investigation, children are given a chance to relate the idea of unit to the convenience of a ruler. Most children at this level will realize that the procedure of using the white strips to measure the length of the top of the desk is very tedious. Without giving specific suggestions, encourage the children to look for easier ways of finding the length. The most difficult way would be to use one single white strip. The possibility of using two or more white strips might occur to several children, and some might realize that these white strips could be replaced by a strip of another color and length. Some may see that the orange strip is the best replacement since it would require the fewest number of moves. Some capable children may recognize that the unit is one centimetre and use the centimetre ruler. Note that the uncolored side of the orange strip, with its markings, may be considered to be a centimetre ruler.

Allow children ample time to work out their procedure by themselves. If necessary, you might give a few suggestions to those having trouble finding an easy way, but do so in the form of questions to individual children as you move around the room.

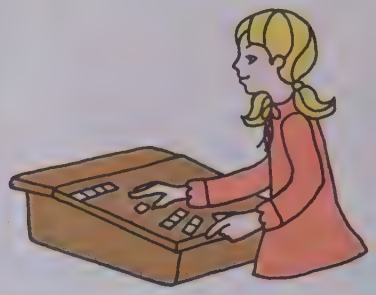


How can a ruler help you find length?

Investigating the Ideas

If the white strip is one **unit** long, what is the length of the top of your desk?

Answers will vary.



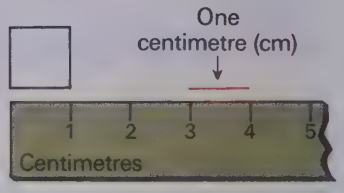
Can you find an easy way to find this length?

See Investigation.

Discussing the Ideas

1. What easy way did you find to measure your desk?

See Investigation.



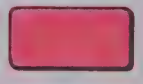
2. A What does the picture show?

Centimeter ruler

B How could you find the length of the top of your desk with a centimetre ruler?

Sample answer: Count how many times ruler will fit across length of desk.

3. To find length, we may choose any unit, but it is easier if we use special units, like the **centimetre**. Do you think the length of the top of your desk found using red strips is more or less than the length found using the centimetre ruler? Check your answer.



Less when measured with red strips, because 1 red strip is greater than 1 centimetre.

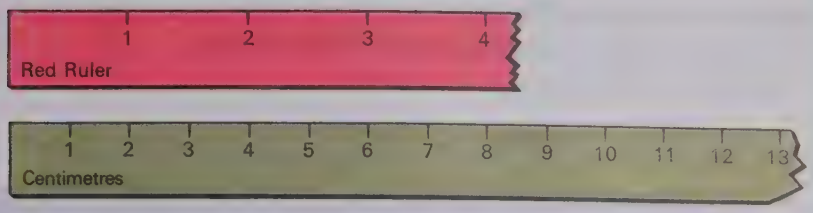
6

Discussion

When most of the children have decided upon an "easy way," use the discussion exercises as a basis for letting the children share the results of the investigation. In exercise 3, emphasize that the white strip is the centimetre unit. Also, if centimetre rulers are not available for each child, now would be a good time to have the children make them. (This might be done by distributing strips of stiff paper about 15 cm long. Then guide the children in lining up their strip of paper with the picture in exercise 1 on page 7. Be sure that they put the end of the strip exactly at the zero mark of the illustrated ruler.)

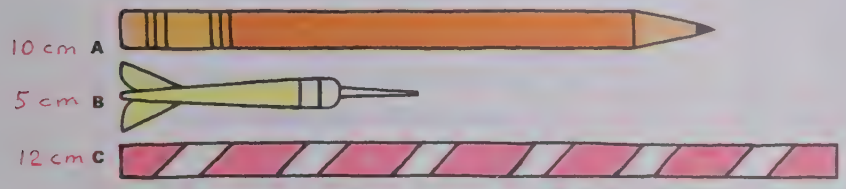
During the discussion, develop the idea that different units assign a different number as the length of an object. Thus, the children's measures of their desk tops will differ according to the different units they chose. Exercise 3 leads into the idea of standard units of measure and enables children to compare two units, the red strip and the centimetre. Help them realize that, since the length given in centimetres is greater than that given in red strips, the centimetre unit must be smaller than the red strip.

Using the Ideas

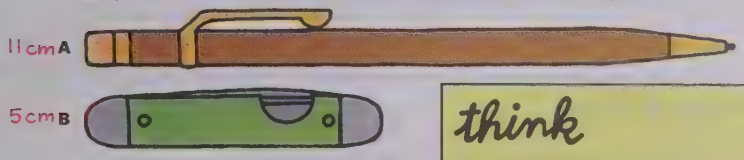
1. 

 A Which is longer, a red strip or a centimetre ? red strip
 B Give the length, in centimetres, of each of your strips.
 C Which strip is about 4 centimetres long ? 4 red strips long ? purple strip brown strip

2. Use your centimetre ruler to measure these objects.



3. Use your centimetre ruler to measure these objects.



4. Draw pictures of objects that are this tall.
 A 12 cm B 20 cm

5. A boy is 6 times as tall as this page. Will you get a number greater or less than 150 when you measure him with a centimetre ruler ? Less

think

How many paper clips would you guess it would take to reach across your classroom ? Make a ruler, using a paper clip as the unit, and check your guess.
Answers will vary. See Using the Exercises.



Answers, exercise 1B, page 7

1B white—1	dark green—6
red—2	black—7
light green—3	brown—8
purple—4	blue—9
yellow—5	orange—10

Follow-up

To give the children further experience in using different units to measure objects, write a list of available objects on the chalkboard. Then have the children work in groups of three or four, first choosing a unit of measure and then measuring the objects in terms of this nonstandard unit. You might have them display on a bulletin board the list of objects and the various lengths assigned them according to the units used by the different groups.

Possible Objects to Be Measured

window frame
chalk tray
teacher's desk
bookshelf
height of door
length of corridor
length of classroom

Resources for Active Learning

Applied Mathematics Cards, Group 1/2, 3, Schofield and Sims. [Measuring length] (Available from Mafex Associates, Willowdale)

Franklin Series: *Learning About Measurement*, pp. 7–8, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Using the Exercises

For exercise 1, the children need one of each of the colored strips and a centimetre ruler.

Assign the exercises as independent work, requesting that the children record the results of their measurements. When they have finished, check their results and discuss any difficulties.

Allow those who try the *Think* problem freedom to check their guess. However, the point in having them make a string of paper clips is not just for them to see how many they would need, but for them to understand the role of unit as they make the ruler and use it to measure the width of the room.

Assignments (page 7)

Minimum: 1–3. Average 1–5.
Maximum: 1–5.

PAGES 8–9

Objective

Given familiar objects, and using the centimetre unit, the child will be able to estimate the lengths of the objects and check his estimates by measuring.

Preparation

Materials

centimetre ruler (1 per child)
metre stick

To prepare for this lesson, write the word "estimate" on the chalkboard. Ask the children if anyone knows what the word is and what it means. Let them give their opinions and then direct them to the text, where they will do some estimating.

Investigation

As you read through the directions with the class, it would be helpful to extend the list of items given in part 2. For example, write on the chalkboard the additional suggestions:

the height of the door
the length of a bookshelf
the width of the teacher's desk
the length of a school corridor
the height of a bulletin board

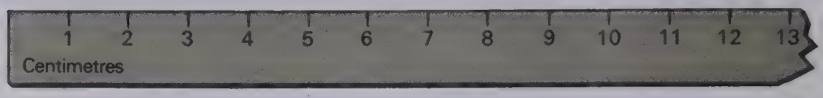
Having given these additional suggestions, you might direct the children to work in groups of two or three. Encourage them to let each member make a guess and then work as a team to check their estimates by measuring. It would be helpful for as many groups as possible to estimate and measure three or four items. Remind them that before each guess they must choose a unit to use. Some may use the same unit for each object they estimate and measure; others may choose a different unit for each object.

Can you estimate length?

Investigating the Ideas

How good are you at **guessing** lengths of objects?
Try this Investigation with a classmate.

Use centimetre units.



Write your guesses for:
Guesses will vary.

the length of your desk top
the height of a window
the length of your shoe
the length of the teacher's desk
the width of a closet door

? Can you check your guesses by measuring?
See Investigation.



Discussing the Ideas

Answers will vary throughout. See Discussion.

1. Which of your guesses is closest to the length of the object you measured?
2. Do you have any special ways to make good guesses of objects' lengths?
3. Guess the height of a classmate, using the centimetre as the unit. Check your guess.
4. Guess your own height in centimetres. Then check your guess.

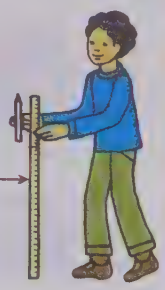
8

Discussion

As children talk about the estimates they made in the investigation, use the discussion exercises to stress one of the main ideas in the lesson, namely, that estimates can be made more easily when a standard tool of measure is kept in mind. For example, imagining a centimetre ruler and mentally comparing the length of an object with it, would help someone make a fairly accurate estimate. In exercise 3 someone may imagine a centimetre ruler. Thinking of a metre stick as well as a centimetre ruler, would help many give accurate estimates of the lengths of the objects suggested in the investigation. Discuss this with the children, exhibiting a metre stick to help them recall that a metre stick is equal in length to 100 centimetres.

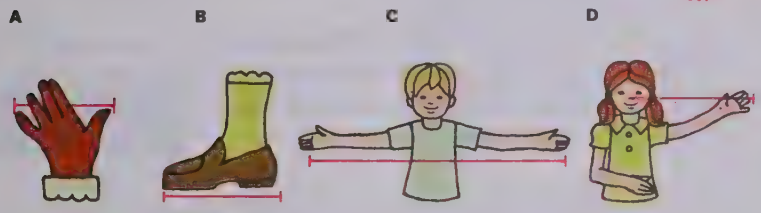
Using the Ideas

Comparing Metric Units	
1 centimetre ⟵	1 cm
A decimetre is 10 centimetres long.	1 dm = 10 cm
A metre is 10 decimetres long.	1 m = 10 dm
A kilometre is 1000 metres long.	1 km = 1000 m



1. Give the word [centimetre(s), metre(s), kilometre(s)] that best completes each sentence.
 - A pencil is about 15 ___ ? ___ long. centimetres
 - B A man is about 200 ___ ? ___ tall. centimetres
 - C A basketball hoop is 3 ___ ? ___ high. metres
 - D A jet plane might fly 7 ___ ? ___ high. Kilometres
 - E A football field is 96 ___ ? ___ long. metres
 - F A book is about 20 ___ ? ___ wide. centimetres
 - G A man might be about 2 ___ ? ___ tall. metres
 - H You could walk a distance of one ___ ? ___ in about 15 minutes. Kilometre
 - I A basketball court is about 27 ___ ? ___ long. metres
 - J Your hand is about 7 ___ ? ___ wide. centimetres
 - K A postage stamp is about 2 ___ ? ___ high. centimetres
 - L Niagara Falls is about 50 ___ ? ___ high. metres

2. First estimate and then find (to the nearest centimetre) the distances on your body suggested by these pictures. Answers will vary.



Using the Exercises

With the class, read the various units for length and then read the directions for exercise 1. Give the children an opportunity to make estimates for exercise 1 independently. Suggest that they help each other when they make the measurements to check their estimates in exercise 2. You may want to assign partners for this purpose. Also, you may need to review for the children that when we say, for example, that the measure of an object is 4 cm *to the nearest centimetre* what we mean is that its measure is closer to 4 cm than to 3 cm or 5 cm.

When children have had sufficient time to complete this page, give them the correct estimates for exercise 1 so that they can check their work. Encourage discussion of any item which seems more difficult to estimate.

Assignments (page 9)

Minimum: 1, 3. Average: 1–3. Maximum: 1–3.

Follow-up

Several children might enjoy drawing and labelling the lengths of the parts of the body they measured in exercise 2. These drawings may then be displayed for all to compare.

Others may be interested in research work on the history of measurement. For example, they might report on terms such as the following: cubit, digit, fathom, nail, span.

Resources for Active Learning

Developmental Math Cards, G²1, Addison-Wesley.

Applied Mathematics Cards, Group 1/8–10, Schofield and Sims. [Estimating] (Available from Mafex Associates, Willowdale.)

Franklin Series: *Learning About Measurement*, pp. 11-19, Lyons and Carnahan. (Available from McGraw-Hill Ryerson.)

Math Activity Cards, "Cubits," B23; "Fathoms," C22, Macmillan.

Mathematics in Modules, M7, Addison-Wesley.

Mathex: Measurement No. 10, "A Ball Diamond," "Making a Measuring Tape," pp. 2–3; "Make a Metre Stick," p. 6; "Metric Units," pupil page 22, Encyclopaedia Britannica Publications Ltd.

Workbook, page 2

PAGES 10–11

Objective

Given an object whose length is less than the unit length, the child will be able to give the length of the object as a fractional part of the unit.

Preparation

Materials

colored strips

No specific preparation is recommended for this lesson. The term *fractional number* is presented in the investigation and in the discussion, so it need not be discussed before these sections.

Investigation

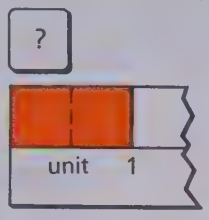
Each child needs at least one of each of the colored strips. You might want to discuss illustrations A and B at the top of page 10 in order to clarify for the children what they are intended to do in this investigation. The ruler in the illustration is an imaginary ruler showing which strip is to be regarded as the unit. When the children proceed to consider the other strips as the unit, they should only imagine such a ruler; they should not actually use a ruler, because they would need a different ruler for each strip. You might find it necessary to explain the *fractional number* $\frac{1}{2}$ and the *fractional number* $\frac{1}{3}$, to help the children understand the directions. When you are sure the children know how to proceed, encourage them to work independently, recording the length of the white strip when each of the other strips is the unit.



Investigating the Ideas

A

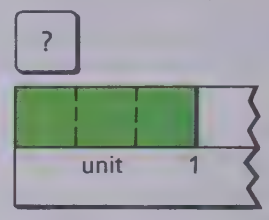
If the red strip is the unit, what is the length of the white strip? ½



The answer to question A is the **fractional number** $\frac{1}{2}$ (**"one half"**).

B

If the light green strip is the unit, what is the length of the white strip? ⅓



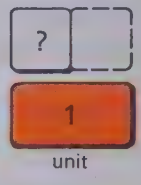
The answer to question B is the **fractional number** $\frac{1}{3}$ (**"one third"**).

Can you give the length of the white strip when each of the other strips is the unit?

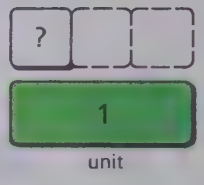
Red-½; lt. green-⅓; purple-¼; yellow-⅕; dk. green-⅙; black-⅐; brown-⅛; blue-⅑; orange-⅒

Discussing the Ideas

1. In A the white strip is 1 of 2 parts needed to match the unit. How are these numbers used to write the fractional number for the length of the white strip? ½



2. In B the white strip is 1 of 3 parts needed to match the unit. How are these numbers used to write the fractional number for the length of the white strip? ⅓



3. Explain what you found in the Investigation.

See Discussion.

10

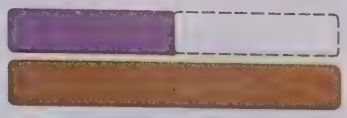
Discussion

Exercises 1 and 2 on page 10 further develop the idea treated in the illustrations of the investigation. Write the fractional number on the chalkboard or use felt numerals on the flannelboard to show $\frac{1}{2}$ and $\frac{1}{3}$ as you speak of 1 of 2 parts and of 1 of 3 parts. Exercise 3 leads back to a discussion of the investigation. Use the chalkboard or strips on the flannelboard to demonstrate each situation. Ask for volunteers to explain, for instance, what the length of the white strip is if the purple strip is the unit, and have a child write the fractional number on the chalkboard as you show the white strip beneath the purple strip on the flannelboard. Continue similarly with each of the strips. Depending on the understanding shown by the children, you might use two or three white strips and show $\frac{2}{3}$'s or $\frac{3}{4}$'s or $\frac{2}{5}$'s and so on. In each case, relate the phrase "1 of 2 parts" or "2 of 3 parts" to the fractional number $\frac{1}{2}$ or $\frac{2}{3}$, to help the children see how the expressions are associated.

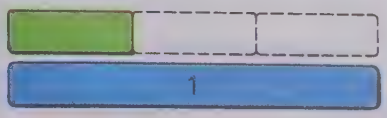
Using the Ideas

1. The unit strip has a "1" on it in each exercise. Give the missing numbers.

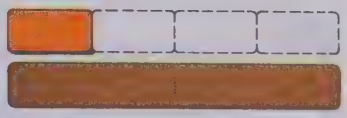
 A The purple strip is __? 1 __ of __? 2 __ parts needed to match the unit. Its length is __? ½ __.

 B The light green strip is __? 1 __ of __? 3 __ parts needed to match the unit. Its length is __? ⅓ __.

 C The red strip is __? 1 __ of __? 4 __ parts needed to match the unit. Its length is __? ¼ __.

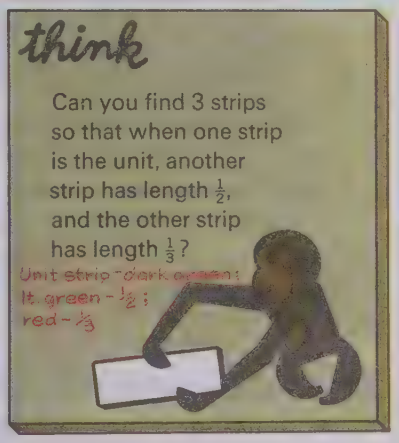
 

 D The white strip is __? 1 __ of __? 10 __ parts needed to match the unit. Its length is __? 1/10 __.

2. If the dark green strip is the unit, what is the length of the
 A white strip? 1/6
 B red strip? ⅓
 C light green strip? ½
 ★ D yellow strip? 5/6

3. If the orange strip is the unit, what is the length of the
 A yellow strip? ½
 B red strip? ⅕
 C orange strip? 1
 ★ D light green strip? 3/10





11

Using the Exercises

You might like to work through these exercises together. However, if you think the children are capable, assign the page as independent work.

Exercises 2 and 3 may require special discussion; help the children think, "What number of each strip is needed to match the unit?" If the yellow strip is thought of as 5 white strips, and the dark green as 6 white strips, starred exercise 2D will be simplified. Similarly, in exercise 3D capable children may see that the light green strip can be thought of as 3 white strips, and the orange strip as 10 white strips, so the length of the light green strip is $\frac{3}{10}$ when the orange strip is the unit that is being considered.

Assignments (page 11)

Minimum: 1–2C. Average: 1–3C. Maximum: 1–3.

Follow-up

To provide children with practice in using fractional parts of a metre, suggest that they find objects in the classroom which they think are less than one metre in length. Then give the measure as a fractional part of a metre. They might try to measure such items as a softball using string to measure around it.

PAGES 12–13

Objective

Given a centimetre ruler, the child will be able to measure appropriate objects to the nearest centimetre or to the nearest half centimetre.

Preparation

Materials

centimetre ruler

Depending on how well the children understood the previous lesson, you might choose to review halves and fourths, particularly as treated with the strips. For example, show the brown strip as the unit, the purple strip as one half, and the red strip as one fourth. Spend only a few minutes on such a review. Also, be sure that each child has a centimetre ruler.

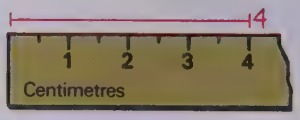
Investigation

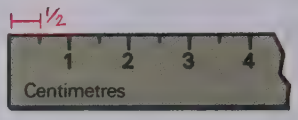
Since the child's main activity in this investigation is drawing line segments using fractional parts of the unit of measure, it would be helpful to read the initial question together. Ask volunteers to give the lengths of the illustrated segments. When the half and unit of the centimetre are identified, direct the children to proceed to the challenge question and draw the line segments on their own. If some finish quickly, suggest that they draw line segments of other lengths that you write on the chalkboard. Ask the children to be sure to label the length of each line segment. As they work, move around the room and help any child who has difficulty reading and using the ruler.

● *Let's use fractional numbers in measurement.*

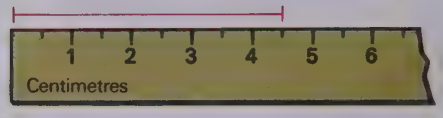
Investigating the Ideas

If the centimetre is the unit, how long is each segment ?





This segment is $4\frac{1}{2}$ centimetres long. (Read "four and one half.")



? Can you use your ruler to draw segments that have these lengths ?

A $3\frac{1}{2}$ cm B $5\frac{1}{2}$ cm C $1\frac{1}{2}$ cm

See Investigation.

Discussing the Ideas

1.



A Is the length of the segment closer to 8 centimetres or to $8\frac{1}{2}$ centimetres ? How can you tell ? 8cm because the end of the segment is closer to the end mark than to the $8\frac{1}{2}$ cm mark.

B The length of the segment (to the nearest of the marks that are one-half centimetre apart) is 8cm.

2. What is the length of this segment to the nearest centimetre ?

$6\frac{1}{2}$ cm

3. Draw a segment that is longer than $4\frac{1}{2}$ centimetres, but whose length **to the nearest half centimetre** would be $4\frac{1}{2}$ cm.

(Segment should be longer than $4\frac{1}{2}$ cm but shorter than 4.75 cm.)

12

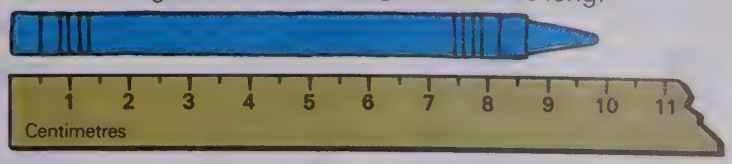
Discussion

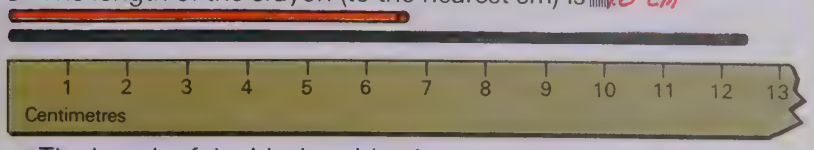
If possible, let the children use a plastic transparent ruler and an overhead projector to point out the marks on the ruler which they used for each segment in the investigation, and continue this activity for discussion exercise 1.

As you discuss exercise 2, instruct the children to line up their ruler with the left end of the line at the zero mark of the ruler. (Explain that this zero mark is sometimes at the end of the ruler and sometimes a short distance away from the end.) Suggest that they use the half-centimetre mark on the ruler to help them decide whether the segment is closer to 5 or to 6. When they decide that it is closer to 5, explain that we call it 5 cm *to the nearest centimetre.*

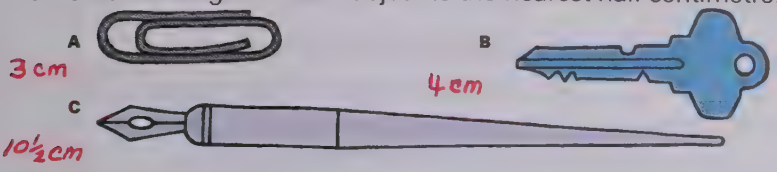
As you discuss exercise 3, explain what it means to measure to the nearest half centimetre. If an object appears to fall exactly halfway between two units, when measuring to the nearest unit, some agreement must be made to call this either the next whole number of units or the previous whole number of units. By convention, we call it the next whole number of units.

Using the Ideas

1. Draw a square that has each side $6\frac{1}{2}$ centimetres long.
2. Draw a triangle with one side $4\frac{1}{2}$ centimetres long.
3. 

 A Is the length of the crayon closer to 9 or to 10? 10
 B The length of the crayon (to the nearest cm) is 10 cm
4. 

 A The length of the black rod (to the nearest half cm) is 12½ cm
 B The length of the red rod (to the nearest half cm) is 6½ cm
5. Give the length of each object to the nearest half centimetre.



6. Give the length of each object in exercise 5 to the nearest cm.
 A: 3 cm B: 4 cm C: 10 cm
7. Give the length of each segment to the nearest centimetre.

A 6 cm (6 cm) B
C 6 cm (5½ cm) D
E 13 cm (12½ cm) F
G 9 cm (9 cm) H
I 8 cm (7½ cm) J

8. Give the length of each segment in exercise 7 to the nearest half centimetre. Answers in parentheses above.

More practice, page A-1, Set 1



Using the Exercises

On page 13, you might find it necessary to guide the children through exercises 1 and 2. Help them recall that a square has all right angles and 4 equal sides. Suggest that they use a corner of a tablet or crayon box to form the right angles. To draw the triangle, they should draw the $4\frac{1}{2}$-centimetre side first and then extend the other two sides from it.

Assign the rest of the exercises as independent work. The centimetre is not divided into halves, fourths, eighths; it is divided into tenths, so the half-centimetre mark will be distinguishable as five tenths, but there is no "quarter-centimetre" mark.

Assignments (page 13)
Minimum: 1–5. Average: 1–7.
Maximum: 1–9.

Mathematics

The fact that all measurement of physical objects is subject to error leads us to an understanding of the need for giving measures to the nearest unit or fraction of a unit. For example, if the length of an object, to the nearest unit, is 5, this simply means that the true length is between $4\frac{1}{2}$ and $5\frac{1}{2}$ units. If the length of an object is reported as $2\frac{1}{2}$ cm to the nearest half centimetre, than the true length of the object must be between $2\frac{1}{4}$ cm and $2\frac{3}{4}$ cm. In other words, the maximum error is always one half of the smallest unit used in the measurement process.

Follow-up

The children would benefit from further opportunity to make measurements to the nearest unit and the nearest half unit. Have two or three children work together and measure objects around the classroom, recording their measurements for each item. You might supply other material for them to measure, such as pieces of colored yarn or string, shoelaces, strips of paper, and pencils. The list of items and their measurements might then be displayed around the classroom.

Resources for Active Learning

Applied Mathematics Cards, Group 1/4–6, Schofield and Sims. [Measuring to the nearest unit] (Available from Mafex Associates, Willowdale)

Developmental Math Cards, "Fractions and Weight," H²2, Addison-Wesley.

Franklin Series: *Learning About Measurement*, pp. 8–10, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Workbook, page 3

PAGES 14–15

Objective

Given a closed figure, the child will be able to estimate or measure its perimeter.

Preparation

Materials

centimetre ruler; string (at least 1 metre for every 2 children)

No specific preparation is necessary for this lesson. However, you might spend just a few minutes speaking about investigations in general. For example, remind the children that in an investigation they should always keep in mind the basic question they are being asked and they should always record their results in some manner.

Investigation

In this investigation, the children are to use simple tools (string and ruler) to find perimeters of some objects in their classroom. Before they begin, stress the importance of keeping a record of the results. You might write the names of suitable objects on the chalkboard so that they can list the items they measure and record the distances they find for each. Have the children work as partners or in groups of three. Let them discover for themselves that they can wrap the string around the object and then measure the length of the string used. You might ask them to find the distance around at least three objects. You might make a rule that only one group can measure the same object at the same time. It would also be helpful to have as many suitable objects available as possible, such as a large juice can, boxes of various sizes, a large milk carton, a dictionary, an encyclopedia, etc.

Let's explore the idea of perimeter.

Investigating the Ideas

The red paths below suggest the **distance around**:



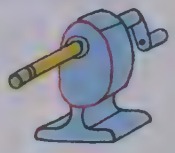
your waist



a book



a desk



the pencil sharpener

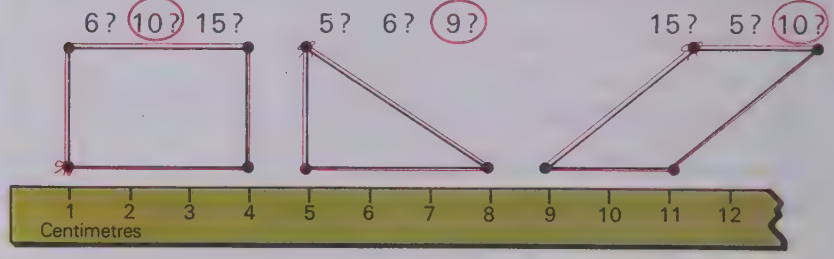
?	Can you use a piece of string and your ruler to find the distance around some objects in your classroom? Answers will vary. See Investigation.	Record your findings.

Discussing the Ideas

The distance around a figure is called the **perimeter**. See Discussion.

1. A How can you make a string rectangle on your bulletin board that has a perimeter of 72 centimetres?
 B Can you make string rectangles of different shapes, each having a perimeter of 72 cm?
 C What other figures can you make with perimeter 72 cm?

2. Think about the string around each figure. Choose the best estimate for the perimeter of each figure. Explain your choice.





Discussion

As the children are measuring objects, write the word *perimeter* on the chalkboard. When several have measured the perimeter of three or four objects, direct attention to the text. Teach the word *perimeter*. Then, as you suggest that the children compare the lengths of the perimeters that they measured, encourage them to use the word *perimeter* in their discussion.

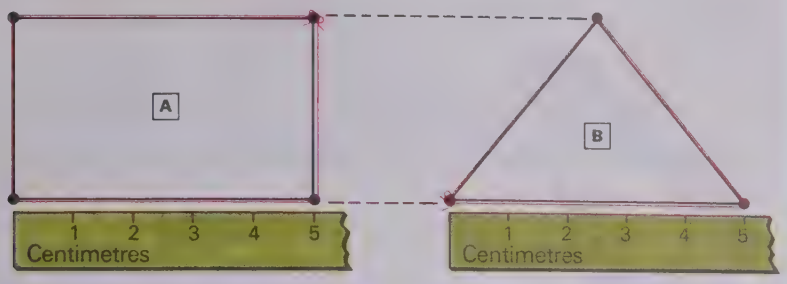
As you discuss exercise 1, have children make several string rectangles of perimeter 72 cm on the bulletin board. For each rectangle, have someone pin the string in position so that it can be compared to another rectangle. The children can use the 72 cm length to make several other figures for exercise 1C (triangles, parallelograms, etc.).

In exercise 2, although the children might check their estimates by measuring the sides with a centimetre ruler, they should first choose an estimate for each perimeter by making a careful guess for each figure.

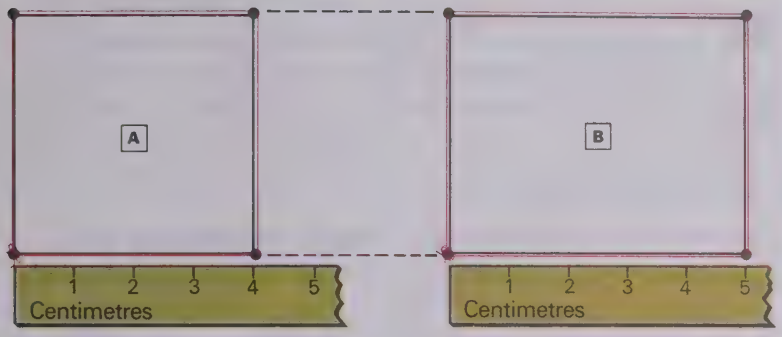
Using the Ideas

Think about measuring the red string that is wrapped around each figure. Without measuring, tell which figure, A or B, has the greater perimeter. Can you explain why?

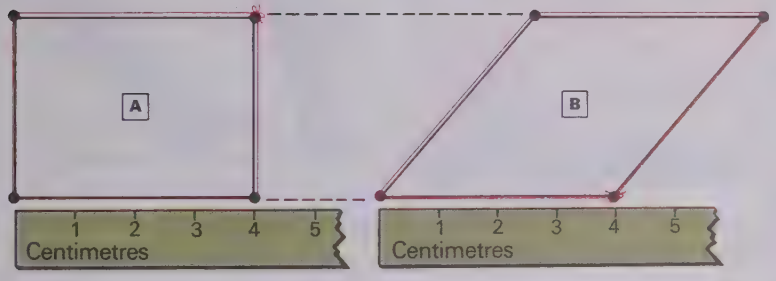
1. A



2. B



3. B



15



Resources for Active Learning

Developmental Math Cards, H²20, Addison-Wesley.

Nuffield Project: *Beginnings* 1, pp. 74–81, Wiley. [Introductory activities for studying perimeter]

Mathex: Measurement No. 10, "Perimeter of Irregular Shapes," p. 7 (pupil page 23), Encyclopaedia Britannica Publications Ltd.

Using the Exercises

It would be appropriate to treat the exercises on page 15 as a basis for discussion. Notice that an estimate for each perimeter is to be given; that is, the children are to make a judgment without measuring. If some doubt the decision of the majority, have them use centimetre rulers to determine for themselves which figures have the greater perimeter.

Assignments (page 15)

Minimum: 1–3, oral.

Average: 1–3, oral. Maximum 1–3.

PAGES 16–17

Objective

Given a polygon, the child will be able to find its perimeter by measuring the length of its sides.

Preparation

Materials

centimetre ruler

To prepare for this lesson, write the word *perimeter* on the chalkboard and have some children explain the meaning of the word and how they found perimeters of various objects in the previous lesson. However, keep such a review short and move quickly to the investigation.

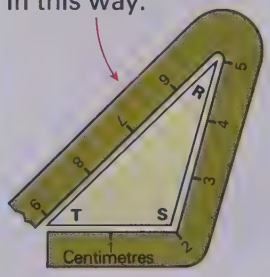
Investigation

In the previous investigation, children studied the concept of perimeter. In this investigation, children explore how to measure the perimeter of a figure. After you read the directions with them, direct the children to work independently. Remind them to record their results, giving the measure of the perimeter of each figure. If a child finishes measuring quickly, ask him to draw other figures and measure their perimeters. You might even challenge capable children to seek a shortcut when measuring the perimeter of squares and rectangles.

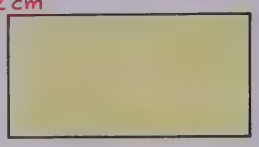
How can we find perimeter by measuring?

Investigating the Ideas

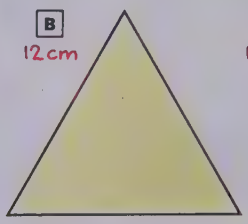
We could measure the perimeter of triangle RST easily **if** we could bend our ruler, like string, in this way.



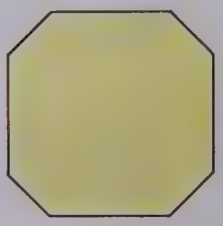
A 12 cm



B 12 cm



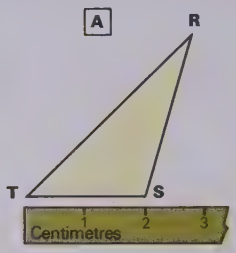
C 12 cm

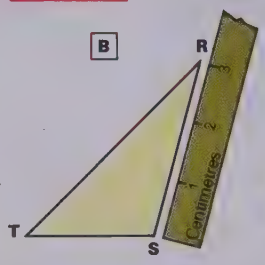


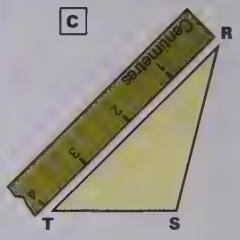
? Can you figure out a way to use only your centimetre ruler, **without bending it,** to measure the perimeter of each figure above? See Investigation and Discussion.

Discussing the Ideas

1. These pictures show how to measure the perimeter of triangle *RST*. Can you explain them? What is the perimeter? 9 cm
See Discussion.







2. How did you find the perimeters of figures B and C in the Investigation?
Sample answer: By measuring each side and then adding

16



Discussion

In discussion exercise 1, children are given an opportunity to check the method they used in the investigation. Observe with them that each of the three sides of the triangle is measured and then the three lengths are added to obtain the perimeter. Encourage children to explain how they found the perimeters of figures B and C in the investigation. It would be helpful to use the overhead projector or chalkboard to demonstrate finding the perimeters of other figures.

If you think the children would benefit from further discussion, have them measure the perimeters of the figures on page 15 of the previous lesson.

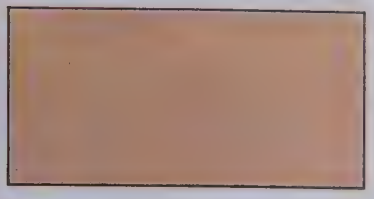
Using the Ideas

1. Use your centimetre ruler to find the perimeter of each rectangle.

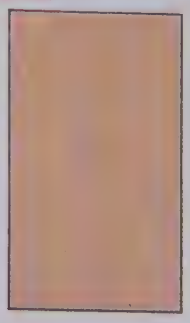
A 8 cm



B 18 cm

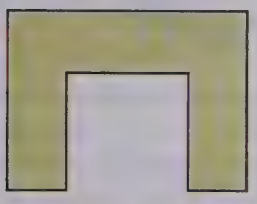


C 16 cm



2. Use a centimetre ruler to find the perimeter of each figure.

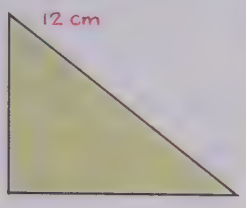
A 18 cm



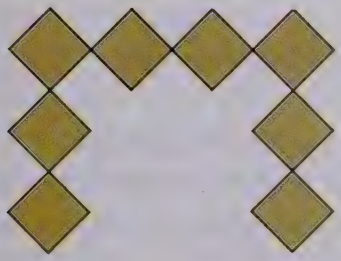
B 10 cm



C 12 cm



3. A What is the perimeter of this figure with 8 centimetre squares? 32 cm

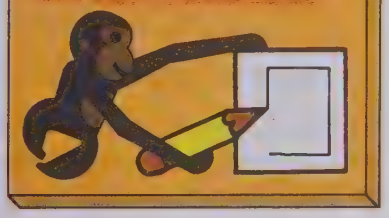


B What is the smallest perimeter you can get by rearranging these 8 squares? Draw a figure to show your answer. 12 cm



think

Using just whole numbers, how many rectangles can you draw that have a perimeter of 36?



More practice, page A-2, Set 2



Using the Exercises

Have the children work independently through the exercises on page 17. When they have finished, guide them in checking their answers and allow time for discussion. You might wish to call the children's attention to interesting comparisons of the perimeters of the various figures. For example, the "small" star in 2B has a perimeter more than half of that of the "large" rectangle in 1B. It is also noteworthy that the irregularly shaped figure in 3A has the greatest perimeter of all the illustrated figures, yet the shaded area is obviously less than that of the large rectangles.

If time permits, you might have volunteers display on the chalkboard some of the rectangles with perimeter of 36 that they found for the *Think* problem. Possible dimensions are 1×17, 2×16, 3×15, 4×14, 5×13, 6×12, 7×11, 8×10, and 9×9.

Assignments (page 17)

Minimum: 1. Average: 1–2.
Maximum: 1–3.

Follow-up

To provide the children with interesting practice in finding perimeters, have them trace outlines of various objects on large pieces of paper and then measure the perimeter of each. You might limit the objects to those which will be straight-lined figures measurable with a ruler, or you might let children include objects with curved outlines as well and challenge them to find the perimeter of these also. (For these, they might use string, as they did for the investigation on page 14.) To create such figures, they might trace their hands, feet, or shoes as well as items of classroom equipment.

Resources for Active Learning

Applied Mathematics Cards, "Shapes —Perimeters," Group 1/28, 29, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Mathex: Measurement No. 10, "Perimeter of Regular Figures," p. 7 (pupil page 23); "Shapes and Pencils," pupil page 9, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 1
Workbook, page 4

PAGES 18–19

Objective

Given a scale drawing, the child will be able to find the distances represented on the map.

Preparation

Materials

centimetre ruler

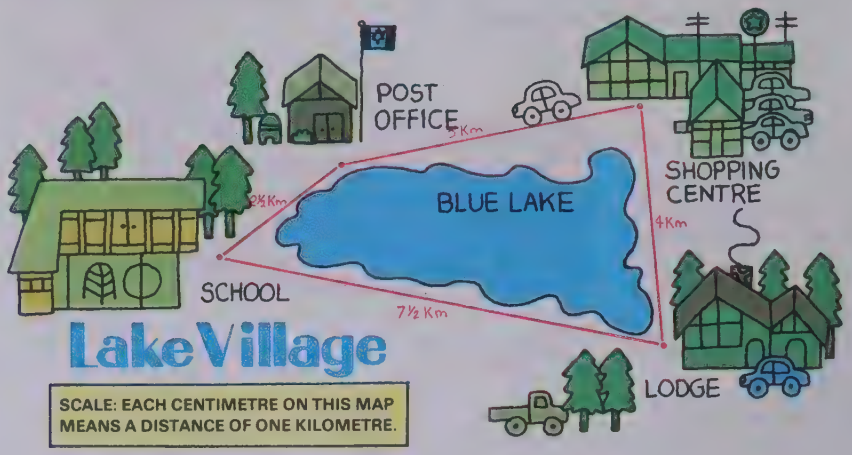
To prepare for this lesson, you might write the word *scale* on the chalkboard and ask children to show what the word means by using it in a sentence. The *scale* most familiar to the children will probably be the weight scale, which they may have used either for purchasing groceries or for weighing themselves. If children use the word *scale* in reference to *scale drawing*, direct them to the investigation; otherwise, first explain that they will find a new use for the word *scale* in the investigation for this lesson.

Investigation

If children understand the information given in the scale, they need only to measure the centimetres and substitute kilometres for centimetres for each distance. If the concept of scale drawing seems difficult for them, encourage them to study more carefully the chart which explains what each centimetre represents. For those who finish quickly, write another scale on the chalkboard and see if they can answer the same question for it. For example, write: What are the actual distances if each centimetre on the map represents a distance of 2 km? of 5 km?

How can we find distances on maps by measuring?

Investigating the Ideas



?	Can you find the number of kilometres between any two places shown on this map ?	Record your findings.

Discussing the Ideas

1. Explain how you used the **scale** to help you find the distances on the map.
2. If you went from the school to the post office and then to the shopping centre, how many kilometres would you go ? 7½ Km
3. How many kilometres, to the nearest half kilometre, is it from the lodge to the shopping centre ? 4 km
4. About how far is it around the lake ? 19 Km
- ★ 5. If you went across the lake in a boat from the lodge to the post office, about how many kilometres (to the nearest half kilometre) would you go ? 6 Km

18

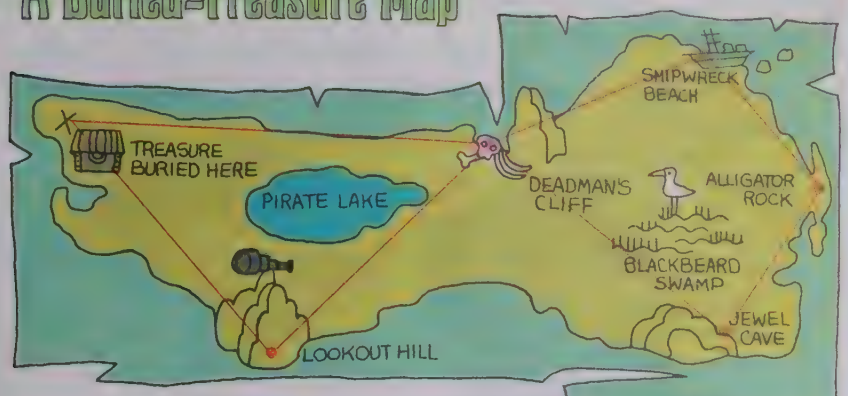


Discussion

Since the scale matches each centimetre with one kilometre, children need only to substitute the unit kilometre for every measurement they made in centimetres. Point out that this means that every fractional part of a centimetre corresponds to that same fractional part of a kilometre. Thus in discussion exercise 3, since the shopping centre is 4 cm from the lodge, the actual distance between the two places is 4 km. Starred exercise 5 develops this same concept.

Depending on the ability of the children, have them consider what the actual distances would be if every centimetre on the map represented 2 kilometres; in this case, one half centimetre would represent 1 kilometre. Extend this discussion according to the children's interest and ability.

A Buried-Treasure Map



When Dan was digging a hole to plant a tree, he found this map in a metal box. Answer the questions. Measure to the nearest centimetre.

1. How many kilometres from
 - A Alligator Rock to Shipwreck Beach? 3Km
 - B the cliff to the treasure? 7Km
 - C the cave to Lookout Hill? 10Km (Just under 8Km as the crow flies)
 - D Lookout Hill to the treasure? 5km
 - E the cliff to the cave? 5Km
2. Which is shorter, to go from Dead Man's Cliff straight to the treasure or to go from Dead Man's Cliff past Lookout Hill to the treasure? How many kilometres shorter?
 3 Km shorter to go straight from the cliff to the treasure
3. How far must a crow fly to go from Alligator Rock straight to the treasure? About 13Km
4. Which is shorter, to go from the cliff past the beach to Alligator Rock or to go from the cliff past Jewel Cave to Alligator Rock? How many kilometres shorter?
 1 Km shorter by way of the beach
5. ★ If you start at Alligator Rock, which is the shortest path to the treasure? How many kilometres is it?
 From the rock to the beach to the cliff to the treasure - 14Km
6. ★ What is the longest way from Alligator Rock to the treasure if you always use a path that takes you closer to the treasure?
 From the rock to the cave to the cliff to the hill to the treasure - 18 km

19



Using the Ideas

Follow-up

Suggest to interested children that they make a scale drawing of their own, such as a treasure map, a diagram for a game, or a vacation-trip map. Or they might prefer to copy another drawing by using tracing paper with a grid and figuring out the scale.

Resources for Active Learning

Nuffield Project: *Problems*—Green Set, No. 4, Wiley. [Finding the shortest path]

Franklin Series: *Learning About Measurement*, "Map and Scale Drawing," pp. 49–53, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, "Map Scales," B26, Macmillan.

Using the Exercises

The scale on the map on page 19 involves no concept more difficult than that studied for the map on page 18. Assign these exercises as independent work. Besides using the scale, the children must also make comparisons between various paths, so you should allow time for discussion when they have finished.

Assignments (page 19)

Minimum: 1–4, oral. Average: 1–4. Maximum: 1–6.

PAGES 20–21

Objective

Given a plane region and a unit of area, the child will be able to find the area of the region.

Preparation

Materials

scissors; duplicated grid of 1-cm squares or graph paper with 1-cm grid, at least one sheet per child

If you think it necessary, review with the children the idea that a square is a four-sided polygon in which all sides have equal length and all angles are right angles. If possible, prepare a duplicating master of a 1-cm grid and run off several copies. One sheet per child should be sufficient for this lesson. As an alternative you could use commercial graph paper with 1-cm grid.

Investigation

Stress with the children that they should first guess the number of one-centimetre squares they would need to cover the front of their book. Then encourage them to try as many ways as possible to find the number needed. For example, each child can place his book on 1-cm grid paper and trace around it on the paper. Then he can count the number of squares in the area which the book covered. This same method can be adapted for finding areas of other objects by having the children trace the outline of an object on newsprint and then cut out the outline on the newsprint and count how many squares on grid paper it covers. Suggest a variety of objects for children to measure in this manner, such as odd-sized books and magazines, crayon boxes, and the like.

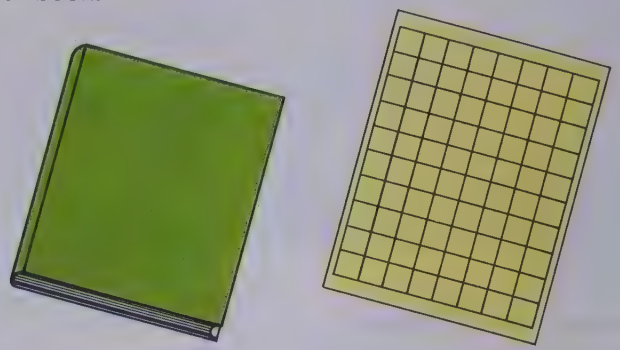
Other children may want to cut out individual one-cm squares and place them on top of their book. As children try different methods, do not show a preference for one method, but praise any reasonable ingenuity. No matter which method the children use, stress the importance of guessing the number of squares in an area before actually counting. Also, remind the children to keep a record of the object, their guess, and the number of squares they count.



Can you count squares to find area?

Investigating the Ideas

Guess how many one-centimetre squares it would take to cover the front of your book.



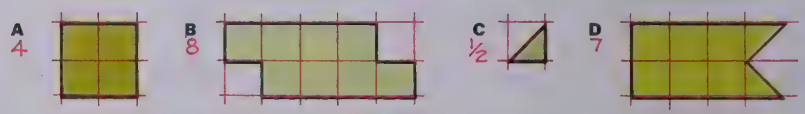
? Can you find a way to use one-cm graph paper to check your guess ?

See Investigation.

Discussing the Ideas

1. The number of squares it takes to cover a region is called the **area** of the region. The square is called the **unit of area**. What is the area of the cover of your book if the one-centimetre square is the unit ? About 458 cm²

2. If the unit of area is this square 1, what is the area of each of these figures ? (The red lines may help you.)



3. How can you prove that the area of this figure is 2 ?



See Discussion.

20

Discussion

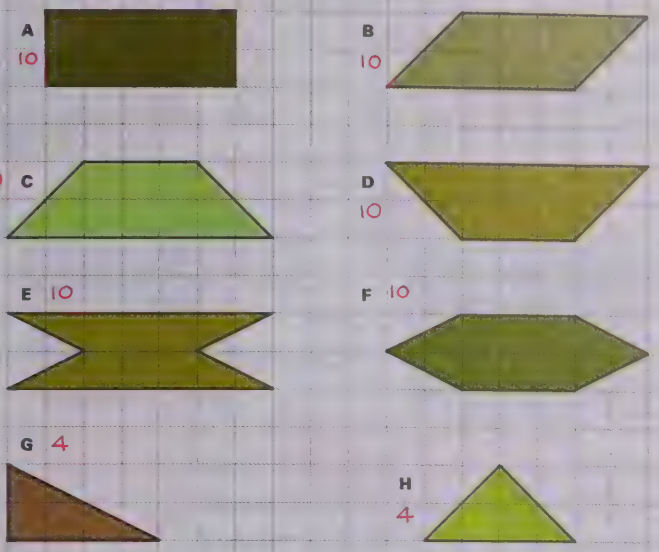
Extend the discussion of exercise 1 to include other objects the children investigated. Stress the idea that the centimetre square is the unit of area and that in giving the number of squares which cover an area, they must speak of *centimetres* or *square centimetres.*

Exercise 2 reviews the concept of counting units to find the measurement of something.

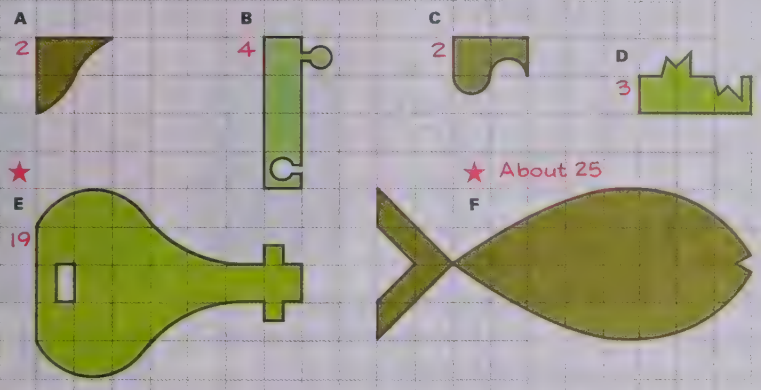
In exercise 3, the children may recognize that each of the two triangles within the shaded triangle is one-half unit, so together they make one unit. Some children may observe that the entire square is 4 units and recognize that $\frac{1}{2}$ of it is shaded, so the shaded area must be 2. Discuss how parts of a unit may be added together and considered as one unit; the children will need to do this when they estimate regions on page 21.

Using the Ideas

1. Find the number of square units (area) in each shaded region.
This ■ is the unit.



2. Estimate the area of each region.



21



Using the Exercises

Assign the exercises on page 21 as independent work. Several figures in exercise 1 require consideration of triangular regions and adding them together to count a whole unit. For example, in part E, the children must recognize that the four triangular regions, two on each end, make four square units in all. Thus, adding the two triangular regions at each end of the figure to the six square units in the centre gives a total of ten square units for this figure. Again, encourage the children to arrive at their own methods for determining the area of each figure.

Notice that parts E and F of exercise 2 are designed primarily as challenging activities. Parts A through D could also prove to be difficult for the slow learners. However, you will not want to deny these children an opportunity to discuss ways to arrive at approximations for the areas of these regions.

Assignments (page 21)

Minimum: 1. Average: 1–2D.
Maximum: 1–2.

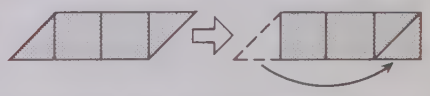
Mathematics

There are several ideas basic to finding the area of plane regions that are involved in this lesson. The idea of area, which involves the selection of a unit of area and counting, was discussed in the mathematics section at the beginning of the chapter. The selection of a square region as the unit of area is quite arbitrary. Other regions could be selected, but the advantages of the square over regions of other shapes is considerable. For example, a circular region would be a poor choice for a unit of area because of the difficulty of completely covering simple polygonal regions.



Squares, on the other hand, because of their regular properties, are easy to count and involve, at worst, problems of estimation when used with irregular regions.

Another idea concerning area that is basic to the lesson is the notion of *conservation of area*. Intuitively, this means that a plane region can be "cut up" and reassembled into a region that is a different shape without changing the area. Thus the area of the parallelogram shown below must be 3 square units since by moving the triangular region from the left side to the right side a rectangular region of area 3 square units is made. The area of the parallelogram is "conserved" in the rectangle.



Resources for Active Learning

Developmental Math Cards, F²18, G²8, Addison-Wesley.

Franklin Series: *Learning About Measurement*, "Measuring Area," pp. 20-25, Lyons and Carnahan. (Available from McGraw-Hill Rverson)

Nuffield Project: *Shape and Size* 2 , "Covering a Surface-Area," pp. 61–77, Wiley. [Appropriate for the next two lessons as well]

Math Activity Cards, "Area: Grid," B34, 35; "Area," C26, Macmillan.

Nuffield Project: *Problems*—Green Set, No. 1, Wiley.

PAGES 22–23

Objective

Given the length and width of a rectangle in whole numbers, the child will be able to find the area of the rectangle.

Preparation

Unless you think it necessary to review the meaning of "rectangle," begin immediately with the investigation.

Investigation

For this investigation, direct the children to work independently but, if you wish, allow them freedom to share ideas. The problems become progressively more difficult. For example, in part A the information is given and the entire region is shown. In B and C, parts of the regions are covered but the needed information is given. In D, again part of the region is covered, but in order to discourage counting, the children also must find for themselves how many rows and how many in each row before they can find the area of the entire region.

It is not intended that children find and formalize a particular method of finding area, such as that given by the standard formula "length times width equals area." Instead, encourage each child to figure out his own method. Some may add rows; some may multiply. It would be helpful for children to discuss and share the method they choose, but first they should spend time independently searching for a way to find the area.



Is there an easy way to find the area of a rectangle?

Investigating the Ideas

— unit of length unit of area

A 8

4 in each row

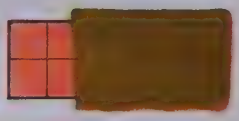
2 rows



B 10

5 in each row

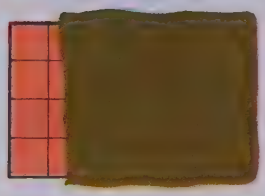
2 rows



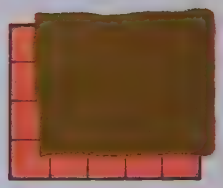
C 24

6 in each row

4



D 20



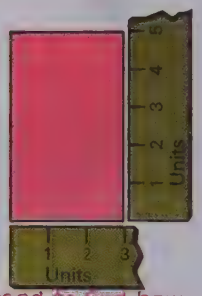
? Can you find the area of each rectangle even though part of it may be covered? See Investigation.

Discussing the Ideas

1. Can you explain an easy way to find the area of a rectangle? See Discussion.
2. What else do you need to know before you can find the area of this covered rectangle? How many rows



3 in each row



3. How can you use the rulers to help you find the area of the rectangle?

Sample answer: Rulers can be used to find how many square units in each row and each column.

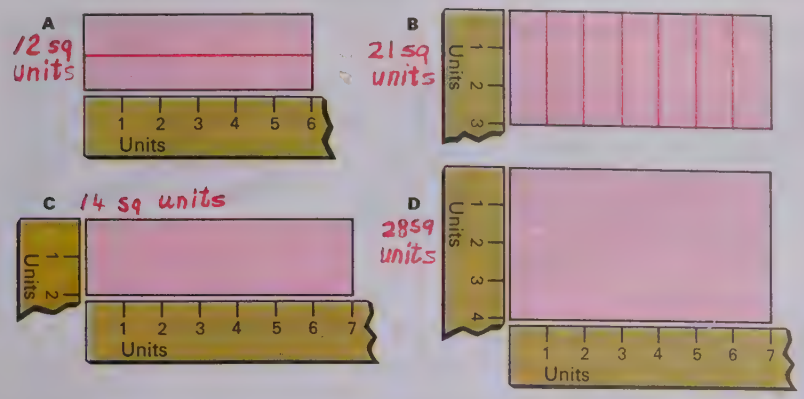
22

Discussion

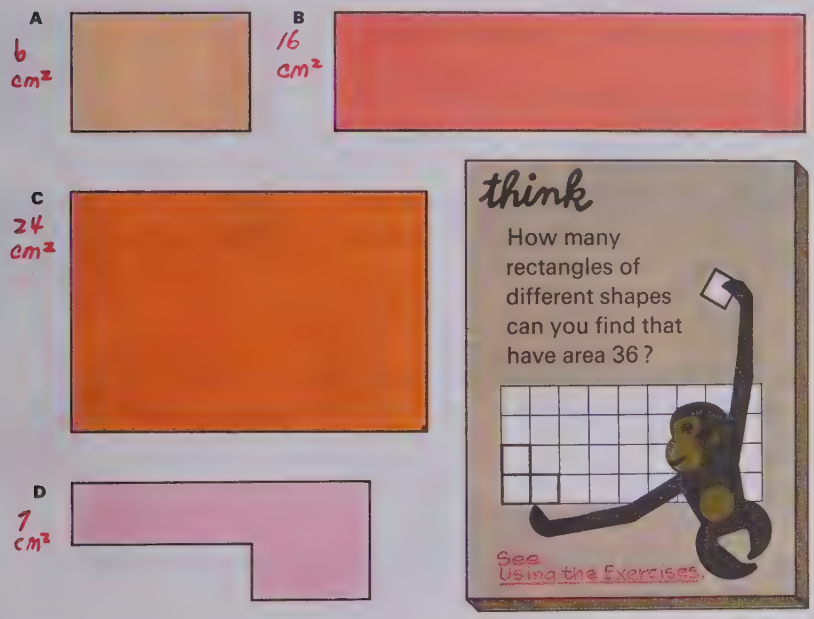
It is important that children discuss the "easy way" they found to measure area of a rectangle. To help each child participate in the discussion, stress that what might be an easy way for one, need not be so for another. Some may actually discover the standard formula, particularly as you discuss exercises 2 and 3, but do not present it formally. Children should be encouraged to make up their own rules based on the results of their investigation.

Using the Ideas

1. Give the area of each rectangle.



2. Use your centimetre ruler to find the area of each region.



More practice, page A-2, Set 3

23

Using the Exercises

Have the children work the exercises on page 23. Afterward, guide a discussion of the correct answers and the methods used for arriving at these answers. We emphasize again that we do not expect the standard formula for the area of a rectangle to be given. Rather, we expect children simply to think about the concepts and express themselves in their own words.

You may want to note that in parts A and B of exercise 1 the idea of a grid is suggested, but in parts C and D no grid is suggested.

Before directing the children to do exercise 2, be sure that all have a centimetre ruler and that all understand the directions. During the discussion of the correct answers, you will need to be sure that the children have made the correct measurements before you can judge whether or not they understand the area concept.

Ask those who try the *Think* problem to describe the rectangles by giving the length and width for each. The possible dimensions are 1×36, 2×18, 3×12, 4×9, and 6×6.

Assignments (page 23)

Minimum: 1. Average: 1-2.
Maximum: 1-2.

Follow-up

Challenge the more capable children to construct rectangles of a certain area for which they are given only area and/or the length of one side. For example, write on the chalkboard directions such as the following from which children can choose:

Draw a rectangle that has
area 10 and length 5.
area 18 and length 6.
area 20 and length 4.
How many different-shaped rectangles can you draw that have area 12?

Resources for Active Learning

Freedom to Learn, pp. 9–13, Addison Wesley. [Area and perimeter activities]

Mathematics in Modules, M7, Addison-Wesley.

Mathex: Measurement No. 10, pupil pages 33–34, Encyclopaedia Britannica Publications Ltd.

PAGES 24-25

Objective

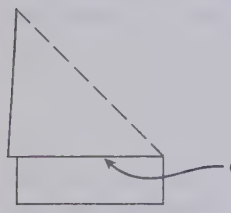
Given a region less than the unit area, the child will be able to give the area of the region as a fractional part of the unit.

Preparation

Materials

notebook paper; scissors

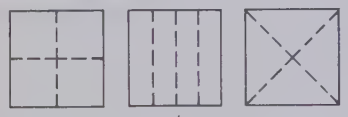
To prepare for this lesson, help the children see how to make a square from a rectangular sheet of paper. Guide them in folding over a top corner so that the top edge lines up evenly with the opposite side.



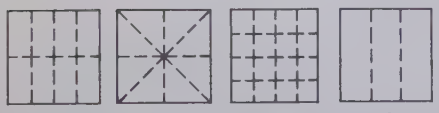
If this is done carefully, the square can be cut on the line formed by the lower edge of the folded corner.

Investigation

Unless you have previously prepared four large squares of paper for each child, guide the children in cutting out squares from notebook paper or newsprint, as suggested above. Then read the investigation question with the class. Stress the importance of having each part equal in size. This challenge is suitable for independent work, although the children would benefit from sharing results with one another. Some of the possible folds are shown below.



4



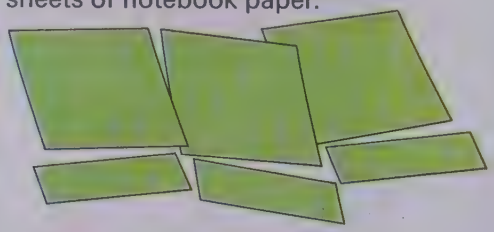
8 16 3

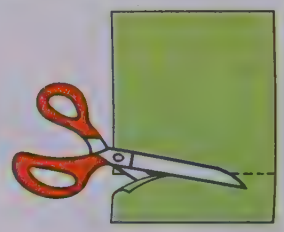
You might suggest other fractional parts into which the squares could be folded, such as sixths, fifths, ninths; these become increasingly difficult. You might also ask the children to color fractional parts of their figures and use these as colorful bulletin board displays.

Do we need fractional numbers to measure area?

Investigating the Ideas

Cut four large squares from four sheets of notebook paper.



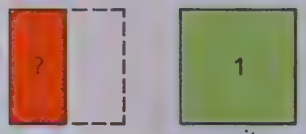


? Can you fold a square so that when you count parts of the same size you find 4? 8? 16? 3?

See Investigation.

Discussing the Ideas

The red region is 1 of 2 parts needed to match the unit. The **area** of the red region is the fractional number $\frac{1}{2}$.



The blue region is 1 of 4 parts needed to match the unit. The area of the blue region is $\frac{1}{4}$.



1. A Can you give the area of the green region? $\frac{1}{8}$
 B What do you think is the area of the pink region? $\frac{3}{8}$



2. Draw a unit square on the chalkboard so that the paper square you used in the Investigation has area $\frac{1}{4}$.
 (Sides of the square drawn should be twice the length of the sides of the square used in the Investigation.)



24

Discussion

Encourage children to show and discuss the results of the folding activity. Where possible, use examples from the children's squares to develop further the explanations in this section. It would be helpful to use demonstration squares and show regions to describe other fractions. In your discussion, associate the phrase "___of___parts" with the fraction corresponding to it.

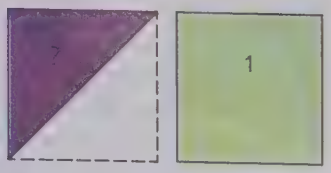
In exercise 1, help the children realize that they must count or know the number of parts into which the unit has been divided before they can name the fractional part of the area.

Exercise 2 might be a challenge to some children, so be sure to spend sufficient time discussing it. Some children should realize that the unit square needed may be drawn by using their paper square as a model four times.

Using the Ideas

1. The unit of area for each part is shown at the right. Give the missing numbers.

A The purple region is __?1__ of __?2__ parts needed to match the unit. The area of the region is __?½__.



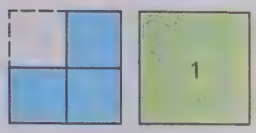
B The green region is __?1__ of __?3__ parts needed to match the unit. Its area is __?⅓__.



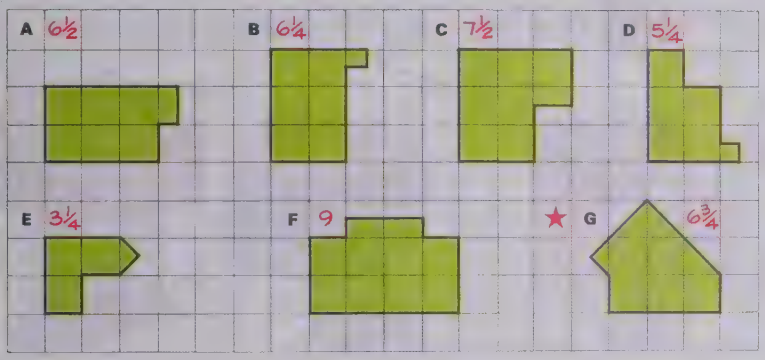
C The red region is __?1__ of __?4__ parts needed to match the unit. Its area is __?¼__.



D The blue region is __?3__ of __?4__ parts needed to match the unit. Its area is __?¾__.



2. Find the area of each figure. This ☐ is the unit.





Using the Exercises

You may choose to use the exercises on page 25 as a basis for further discussion, or you may assign them as independent work. In exercise 2, children must closely examine the parts of regions that fill only fractional parts of the unit area. In starred exercise 2G, they should think of two triangular halves as one unit. Then they will also have to think of one triangular half as two fourths in order to add it to the small triangular region that is one fourth. Some may give $6 + \frac{1}{2} + \frac{1}{4}$ as the answer.

Assignments (page 25)*_____
Minimum: 1–2F. Average: 1–2F.
Maximum: 1–2.

Follow-up

The children would benefit from using their folded squares to find the area of such classroom surfaces as the bulletin boards, chalkboard, floor, and door. When their large square does not fit exactly, they will have to estimate the fractional part of a unit. In this case their unit would be the nonstandard square which they previously cut out and folded.

Resources for Active Learning

Inquiry in Mathematics via the Geoboard. "Area," Geo-cards 11–5; 12-4; 13-3, Walker. (Available from Fitzhenry and Whiteside)

Mathex: Measurement No. 10,"Area with a Geo-board," pupil page 44, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 2
Workbook, page 5

PAGES 26–27

Objective

Given three-dimensional or pictorial representations of rectangular prisms and a unit of volume, the child will be able to find the volume of the prism by counting the number of unit cubes contained in it.

Preparation

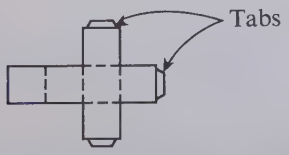
Materials

sugar cubes or blocks (or materials for constructing cubes); empty boxes of various sizes

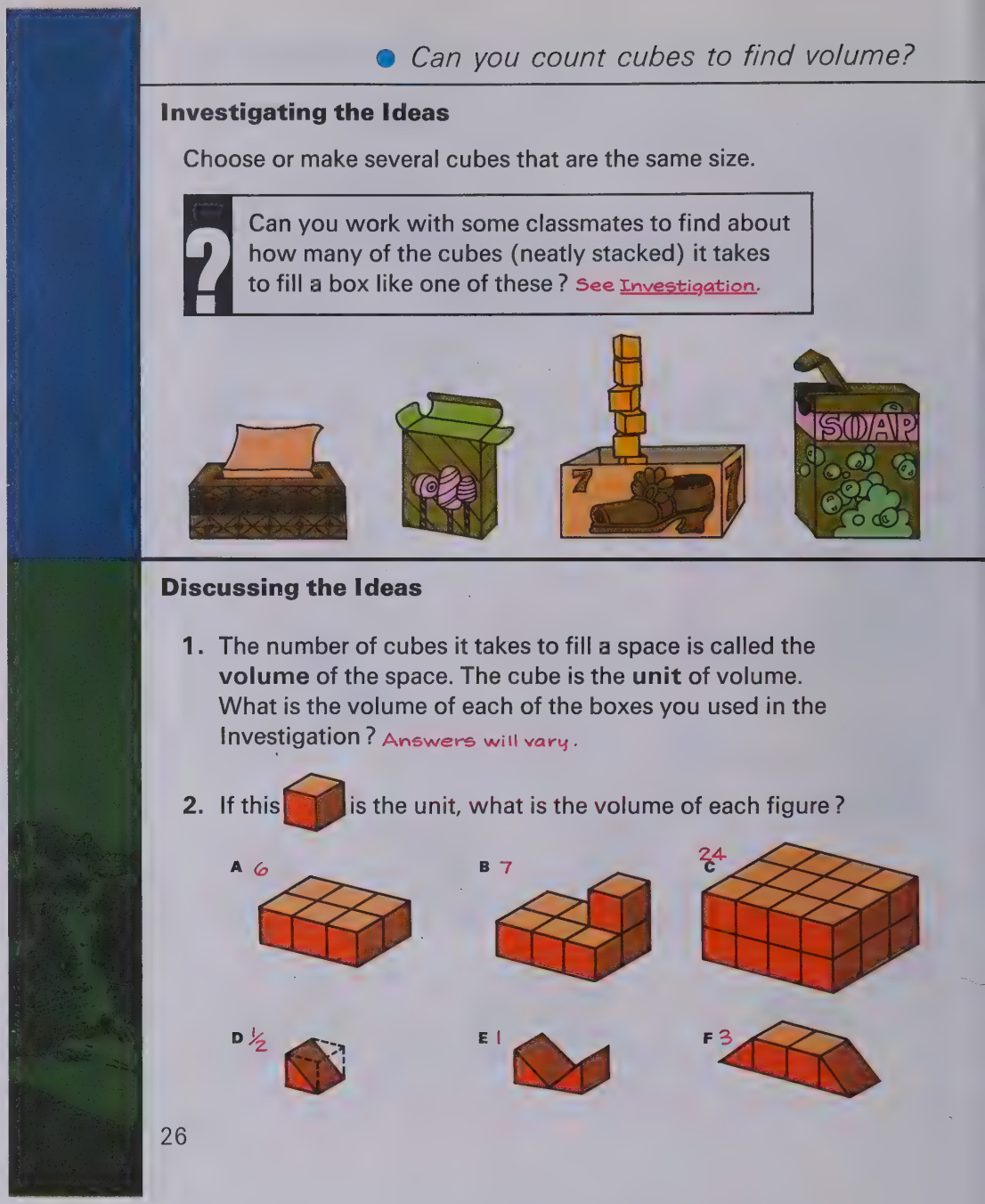
To introduce this lesson, exhibit boxes of different sizes and a cube. Point out the six faces on each as well as their shape and position. Have the children note that the faces of the cube are square. Explain that this is true for every cube. With this background and the manipulative experiences of the investigation, the children should subsequently be able to imagine cubes from pictorial representation.

Investigation

If an ample supply of sugar cubes, blocks, or other uniform-size representations of cubes is available, divide the class into small groups and direct them to work together to find the volumes of containers like those suggested in the investigation. If not enough cubes are available, provide the children with duplicated patterns which they can cut out and use to construct their own cubes. Make the pattern relatively small (about 2 or 3 cm per edge) so that measurement of small containers will not be too awkward. Guide the children in proper folding and fastening of the edges (with paste or tape) so that their cubes will be reasonably uniform in size.



When an adequate number of cubes has been provided, allow the groups of children to work independently to find the volumes of a variety of boxes. Remind them to keep a record of each measure they find. At present, they should measure only in terms of whole units.



Discussion

As you discuss exercise 1, encourage children to share not only their findings in the investigation but also the methods they used.

For parts D and E of exercise 2, it would be helpful to use clay or "play dough" to show halves of cubes cut as illustrated in the text.

Throughout the discussion of both pages of this lesson, stress the different methods which may be used to find the volume of the pictured objects. Some children may count discrete objects; others may count layers and add them; still others may count stacks and add them. Some capable children may see that the layers are rectangles and that the cubes are built on the area of the base, and thus realize that they can apply multiplication. The important point here is for children to develop a general intuitive grasp of volume concepts. Do not stress, or even tell children, the formula for finding volume. Here their discovery of the various *concepts* behind the volume formula is sufficient.

Using the Ideas

1. Find the volume of each figure.

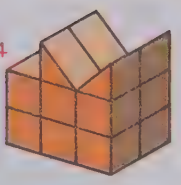
A 18



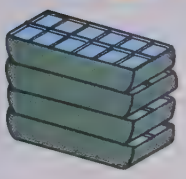
B 14



C 14



2. How many ice cubes would you have if you emptied all four of these full trays? 48



3. Find the volume of each figure even though parts of the figures are covered.

A 20



5 layers

B 24



4 layers

C 36



6 layers

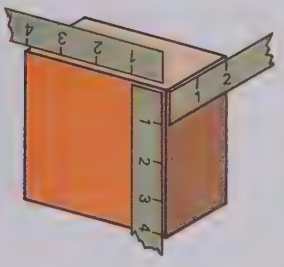
D 24



E 27



★ F 32



More practice, page A-3, Set 4



Using the Exercises

Have the children work independently through the exercises on page 27. When they have finished, allow adequate time for discussing the correct solutions. In exercise 1, help the children see that in parts B and C they must also consider half units. Encourage the children to observe that two of these half units make one full unit.

In exercise 3, parts A through F, notice with the children that as the exercise progresses, less and less information (in the form of countable units) is given, until, in part F, only the measurements of the three edges are given.

Assignments (page 27)*

Minimum: 1–3E. Average: 1–3E. Maximum: 1–3.

Resources for Active Learning

Developmental Math Cards, H²15, F³7, Addison-Wesley. [Another way to think about volume]

Franklin Series: *Learning About Measurement*, "Volume," pp. 54–59; Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Measurement No. 10, "Cubic Measure," pp. 19–21, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Green Set, No. 27, Wiley.

Nuffield Project: *Shape and Size* 2, "Volume and Capacity," pp. 16–23; five illustrations of children's activities, pp. 27–31, Wiley.

Duplicator Masters, page 3
Workbook, page 6

PAGES 28–29

Objective

Given a small unit of liquid measure and a larger unit of liquid measure, the child will be able to find the number of smaller units that are equivalent to the larger unit of measure.

Preparation

Materials

cardboard; sticky tape; scissors; water or sand (to measure volume); uniform-size blocks or sugar cubes; various sizes of containers up to 1 litre

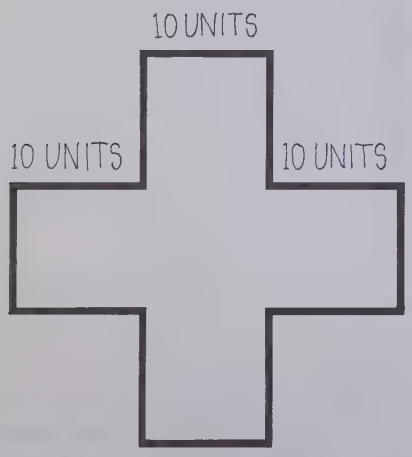
To prepare for the investigation, put distinct labels on the containers and display them.

Investigation

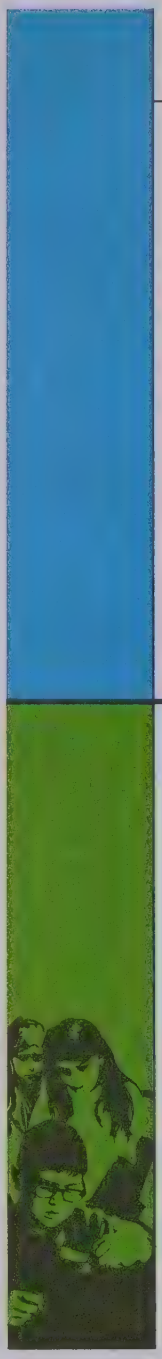
To minimize the quantity of containers needed for the investigation, have the children work in groups of three or four.

Even though they are working in groups, each child should keep an individual record of his findings. They could do this in the form of a chart, for example.

Instruct each group to cut the following pattern from cardboard, after you have drawn it on the board. The sides can then be taped up to make a box of side 10 units, where one unit is the size of the cubes they are using.



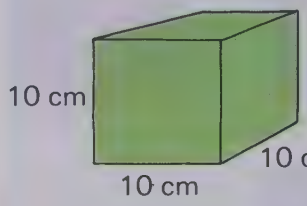
If centimetre blocks are not available, you will have to remind them of the idea of a *scale* developed earlier, so that they can correlate their results with the illustrated 10-centimetre cube.



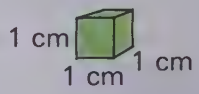
What are some units for liquid measure?

Investigating the Ideas

A litre is the unit used for liquid measure.



A box 10 centimetres on each edge holds about 1 litre. How many blocks 1 centimetre on each edge would the litre box hold? 1000



A millilitre is a much smaller unit used for liquid measure. It takes one thousand millilitres to make a litre.

? What size cube would you need to hold a millilitre of liquid?
One that is 1cm on each edge.

Discussing the Ideas

1. If a recipe calls for 1½ litres of milk and you have 1500 millilitres, do you have enough for the recipe? Yes
2. If a jug holds a litre of syrup and it is ¾ full, how many more millilitres must be added to fill it? 250 ml
3. The volume of a container is 900 cubic centimetres. Will it hold a litre of water? No

28

Discussion

The children should check their answers for exercise 1 by measuring, and this will give an opportunity to develop the idea that measurements are not exact. They should suggest that differences can be accounted for by skimping or overfilling the measuring containers. Also, solicit the idea that in some instances the measuring devices themselves may not hold the same amounts.

For discussion exercise 2, again encourage the children to find the answer without measuring, but let them "prove" their answer by measurement if they wish. Similarly, for exercise 3, the children should determine the answer with reasoning before demonstrating that a litre holds more than 900 cubic centimetres.

Using the Ideas

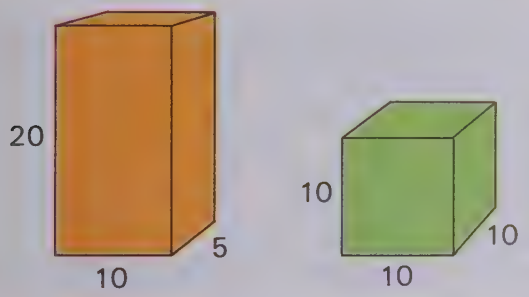
1. Give the word [millilitre(s) or litre(s)] that best completes each sentence.
 - A A glass holds 200 ___ ? ___ of milk. millilitres
 - B A tablespoon holds 20 ___ ? ___ of soup. millilitres
 - C A car's tank holds 80 ___ ? ___ of gasoline. litres
 - D A boy who drinks 4 big cups of cocoa drinks about 1 ___ ? ___ of cocoa. litre
 - E A perfume bottle might hold about 15 ___ ? ___. millilitres
 - F An engine might use 4 ___ ? ___ of oil. litres
 - G A pitcher that holds 1½ litres of tea holds 1500 ___ ? ___ of tea. millilitres

2. The volume of a 3-litre container is 3000 cubic centimetres. Estimate (in cubic centimetres) the volume of a milk carton that holds half as much. 1500 cm³



★ 3. Karen made a box to hold one litre. Tom did too. The boxes did not look the same. Can you explain why?

(NOTE: Be sure the children understand that the shape of the container is not important as long as it holds 1000 cubic centimetres.)





29

Using the Exercises

Unless the exercises on page 29 are likely to present reading difficulties, assign them as independent work. If necessary, read the exercises with the children, but let them write down the answers independently. Though starred exercise 3 is intended primarily as a special challenge, it should be included with the other exercises in the discussion held when children have completed their written work.

Assignments (page 29)* _____
Minimum: 1–2, oral. Average: 1–2.
Maximum: 1–3.

Follow-up

Some children may need further experience in working with liquid measure. If so, direct them to use the containers to check selected parts of exercise 1 on page 29.

Resources for Active Learning

Applied Mathematics Cards, "Measuring Liquids," Group 1/13, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Franklin Series: *Learning About Measurement*, "Liquid Measure," pp. 60-65, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, B31, C28, Macmillan. [Liquid measure problems]

Mathematics in Modules, M10, Addison-Wesley.

Nuffield Project: *Computation and structure* 2, "Capacity," pp. 33–41, Wiley.

Workbook, page 7

PAGES 30–31

Objective

Given a measurement which is expressed as a combination of units (such as 4 metres and 119 centimetres), the child will be able to regroup the units so that the number of smaller units is less than its equivalence to the larger unit (such as 5 metres and 19 centimetres).

Preparation

To prepare for this lesson, direct the children to the Tables of Measures in the back of the book (page A–38). Have them note that equivalents are available from a table, but they should eventually be known from memory.

Investigation

Some children will benefit from making actual comparisons with the quantities indicated on the chart. For example, two or three children might pour 5 litres into as many containers as they need. When they see that one container is not completely full they can consider ways of describing their quantity of liquid.

● *Can you write measurements in different ways?*

Investigating the Ideas

3 METRES	
METRES	CENTIMETRES
1	200
2	100
2½	? 50

5 LITRES	
LITRES	MILLILITRES
3	? 2000
3½	? 1500
4½	? 500

MAY 25 DAYS	
WEEKS	DAYS
1	? 18
2	? 11
3	? 4

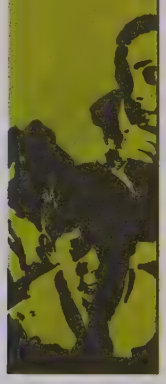
4 KILOGRAMS	
KILOGRAMS	GRAMS
2	? 2000
2½	? 1500
3	? 1000

Use any measuring devices that might help you.

Can you copy each table and give the missing numbers so that each row shows the same amount as the measurement given at the top?

Discussing the Ideas

1. If you are telling the height of a door that is 300 centimetres tall, which measurement in the table above will you probably use? 3 metres
2. Suppose you have 6 tall glasses, each containing 250 millilitres. How many litres of milk would you have? 1½ litres
3. The month of May has 31 days. Explain how to write this using weeks and days. 4 weeks 3 days
4. How many 1-kilogram bags could you fill with 1037 grams of peanuts? How many grams would be left? 1 37



Discussion

During this discussion, you will want to allow children the opportunity to explain the expressions of measurement they used in completing the tables in the investigation.

Then present other examples involving units with which children are fairly familiar. For instance, help them see how 17 days may be thought of as 2 weeks and 3 days; and how 300 centimetres may be thought of as 3 metres. Also, present examples of combinations of units in which the number of smaller units may be converted into larger units, such as 3 h and 75 min. Help the children see that 3 h and 75 min is the same as 3 h + 60 min + 15 min or 4 h 15 min. Work through each problem in the text slowly to enable children to understand the regrouping process and to recognize that smaller units should be converted into larger units when their number is equal to or greater than the equivalence of the two units.

Using the Ideas

1. Copy the exercises below on your paper, and give as many of the missing numbers as you can. Look in the Tables of Measures at the back of the book to find the others.

A 100 cm = ▮ m 1	E 1 min = ▮ s 60	I 1 wk = ▮ days 7
B 100 m = ▮ km 1/10	F 1000 ml = ▮ l 1	J 12 mo = ▮ yr 1
C 1 day = ▮ h 24	G 1000 g = ▮ kg 1	K 1 yr = ▮ weeks 52
D 1 h = ▮ min 60	H 1000 kg = ▮ t 1	L 2 yr = ▮ days 730

2. Give the missing numbers.



A 2 h 75 min = 3 h ▮ min 15

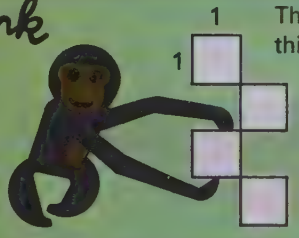
B 15 min 90 s = ▮ min 30 s 16

C 3 days 30 h = 4 days ▮ h 6

★**3.** Change each measure so that you have the greatest number of the larger unit. (Example: 3 weeks 10 days → 4 weeks 3 days)

A 2 days 30 h 3days 6hr	E 3 kg 300 g 3kg 300g	I 3 m 800 cm 11 m
B 5 l 1000 ml 6 l	F 8 min 80 s 9min 20s	J 9 min 75 s 10 min 15s
C 3 wk 20 days 5wk 6 days	G 3 h 62 min 4h 2min	K 7 wk 21 days 10 wk
D 1 m 200 cm 3 m	H 0 l 2000 ml 2 l	L 5 days 25 h 6days 1 h

think



The perimeter of this figure is 16.

Arrange the squares to get a perimeter of:

A 14 B 12 C 10

D 8 E 9 F 15

See Solution, T.E. page 31.

Using the Exercises

After the children complete exercise 1 by using the Tables of Measures, you might want to work through a few parts of exercise 2 together. Starred exercise 3 is optional.

All may try the *Think* problem. If possible, provide everyone with four heavy paper squares (about 5 centimetres on a side) and suggest that they write "1 unit" on each side and then move the squares around to help them find the solution. Except for part D, several solutions are possible.

Assignments (page 31)*

Minimum: 1. Average: 1–2.
Maximum: 1–3.

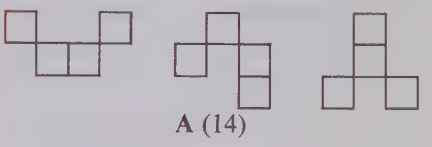
Follow-up

Encourage children to make displays showing equivalences of different units of measure. They might include drawn or printed pictures of quantities that could be measured by the different units.

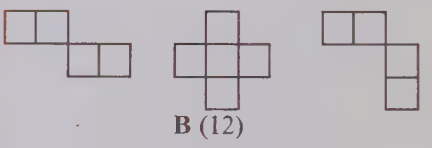
Suggest to others that they write a report describing uses of the different kinds of measures found in the Tables of Measures.

Solutions, Think

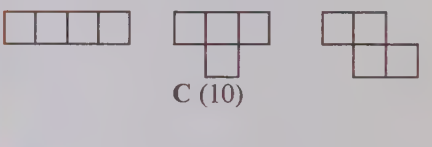
Some of the possible solutions to the *Think* problem are shown below.



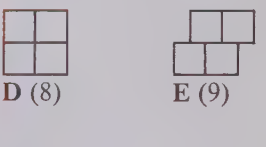
A (14)



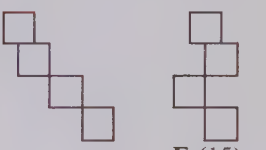
B (12)



C (10)



D (8) E (9)



F (15)

Resources for Active Learning

Developmental Math Cards, F²22, Addison-Wesley.

Franklin Series: *Patterns and Puzzles* "A Timely Puzzle," pp. 47–48, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Franklin Series: *Learning About Measurement*, pp. 66-84, Lyons and Carnahan. [Measurement of all kinds] (Available from McGraw-Hill Ryerson)

Math Activity Cards, B21, 22, 25, 27–30; C29-31, 34, Macmillan. [Measuring problems]

Nuffield Project: *Problems*—Green Set, No. 15, Wiley. [A challenging problem using a calendar]

PAGES 32–33

Objective

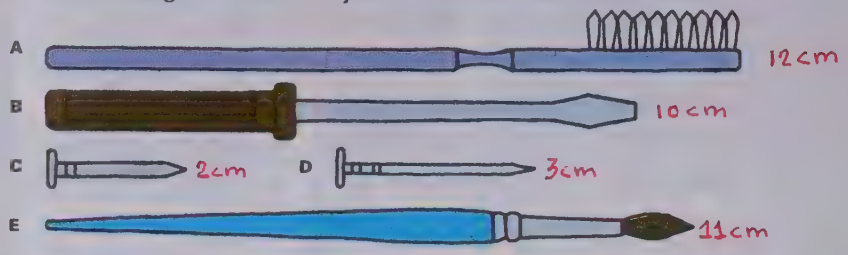
The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

To prepare for this lesson, write the words *length*, *area*, and *volume* on the chalkboard. Ask volunteers to give a unit of measure used in finding each, and to name the kind of objects they might measure in terms of length, area, or volume.

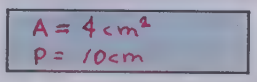
Reviewing the Ideas

1. Give the length of each object to the nearest centimetre.

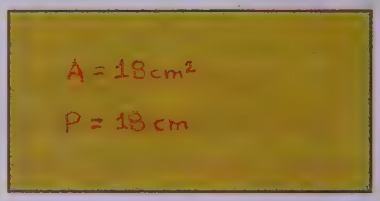


2. Find the area and perimeter of each rectangle. Use the centimetre and square centimetre as your units.

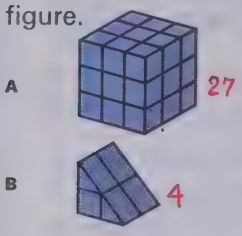
A



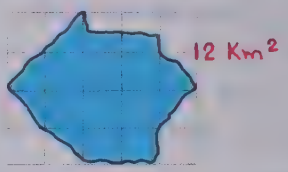
B



3. Give the volume of each figure.

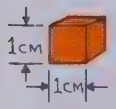


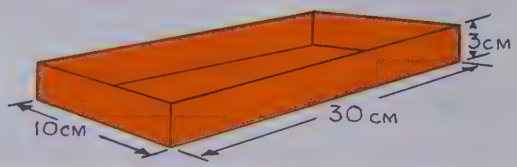
4. Estimate the area of this lake. 1 square unit on the map means 1 square kilometre of the lake.



★ 5. How many cubes, 1 centimetre on an edge, would fit in this shoe box cover?

900



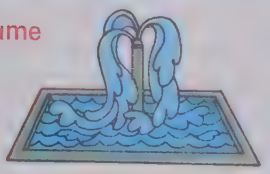


32

Discussion

You may use these pages for testing or as a chapter review. If you use it for testing, you will want to evaluate the papers yourself to decide which concepts caused difficulty. If you are using the pages primarily as a review, it is desirable to have the children check their own papers and thus recognize their mistakes and be able to make the proper corrections. Rather than stressing mastery of any particular skill, assist the children in understanding the concepts. The concepts of perimeter, area, and volume should be clarified even if the children have not mastered the skill in finding each.

6. Do you need to find length, area, or volume to answer each question?



A How deep is the pool? length

B How much water does the pool hold? volume

C How large a piece of plastic must we buy to cover the pool? area

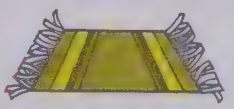


D How tall is the rocket? length

E How many boxes will the truck hold? volume



F What size rug shall we buy to cover the floor? area



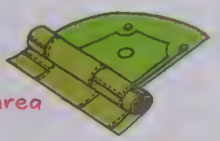
G How high is the basket? length



H How much room is inside the space capsule? volume



I How large is the canvas that covers the baseball diamond? area



J What size belt do you wear? length



Follow-up

To give the children more practice in finding lengths and areas using unit lengths and unit squares, draw the same segments and regions on different grids made on duplicating masters. Pass these sheets out, and ask the children to find the measure of each object, first with one scale and then with the other. Discuss the relation of the units to the total measure and the relation of kinds of units to each other.

Resources for Active Learning

Nuffield Project: Problems—Green Set, No. 11, Wiley. [A challenging problem]

Workbook, page 8

General Objectives

To provide a thorough understanding of place-value concepts

To provide practice in working with hundreds, thousands, and millions

To introduce billions and trillions

To review and extend work with inequalities

A clear understanding of place value is one of the most important objectives in the *Investigating School Mathematics* program. If the children lack this knowledge, it is not likely that they will comprehend the algorithms of arithmetic and decimal notation. For this reason, we have treated place value carefully in the earlier books of the series and follow it up in this book and throughout the remaining books in the *Investigating School Mathematics* series.

To remind children of the basic ideas of place value, Chapter 2 begins by reviewing the idea of grouping by tens and of using 2-digit numerals. Concepts of hundreds and thousands are reintroduced and carefully explored. Then, the children are given many opportunities to work with place value by identifying the number a given digit represents and by working with inequalities.

At this time, we do not teach the meaning of individual places in the place-value system beyond the thousands' place. For 5- and 6-digit numerals, we simply point out that the three digits in the fourth, fifth, and sixth places name the number of thousands. Naming groups of three places called *periods* makes it easier for children to read large numbers. In this way, they learn that the digits in the third group to the left of the decimal point, the seventh, eighth, and ninth places, tell the number of millions. Learning to read large numbers is more important than concentrating on each individual place after the fourth, or thousands', place. The more able children may recognize that the fifth place is the ten-thousands' place and so on, but it is not necessary to stress this point in their work.

Following the introduction of millions, the children work with large numbers in contexts especially designed to stimulate and maintain their interest.

Mathematics

The *digits* are the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. A *numeral* is any symbol that stands for a number. Thus, the symbol 4 is both a digit and a numeral. The symbol 57 consists of two digits, and it is a numeral for the number *fifty-seven*.

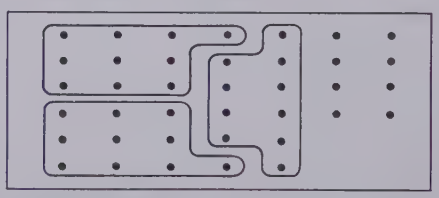
Correct usage of the words *number*, *numeral*, and *digit* is often quite awkward. For example, you might ask a child to write the "number" 1486, or you might refer to 1486 as a "four-digit number." Technically, these are abuses of the language, because the *number* cannot be written and the *number* does not have four digits. You write the *symbol*, or the *numeral*, for the number; and it is the symbol, or the numeral, which has four digits. However, both of the preceding expressions are acceptable, since children will understand that when you say "write the number," you mean for them to write the symbol for the number and that when you call it a "4-digit number," you are referring to the symbol for the number. But there are certain abuses of the number-numeral terminology which are objectionable and may confuse the children. For example, you should not ask the children to perform operations on *digits* or *numerals*. Emphasize that we *add* numbers, but we *write* digits and numerals.

The most important thing to remember about number-numeral terminology is to keep your language simple so that it has meaning to the children. Whenever you are uncertain whether to say number or numeral, say *number*, and avoid making an issue of these words. If a child points to a symbol, or numeral, and calls it a number, do not criticize his remark. Children have an intuitive grasp of the difference between number and numeral, and a lengthy discussion may serve only to confuse what was previously clear.

When we write the numeral 4357, the digits 4, 3, 5, and 7 stand for 4000, 300, 50, and 7, respectively. This illustrates an important property of our numeration system: place value. Place value means that the number a digit represents depends on the digit's place in the numeral.

Another property, which simplifies our calculations and makes it easy to learn arithmetic, is that we can represent any number by using only ten symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each digit used by itself represents a single number; it is only when we write symbols for numbers greater than nine that a given digit may stand for two or more numbers. Thus, in 636, one 6 stands for 600 and the other stands for 6.

The place-value scheme that we use has a base of ten, which means that we group by tens. That is, given a collection of objects, we might ask how many disjoint sets (sets that have no common elements) of ten can be formed. Consider the collection of dots shown in the illustration; we see that there are 3 sets of ten, and 8 left over.



The importance of place value becomes evident when we attempt to write a numeral for the illustrated number of dots. Instead of writing "3 groups of ten, and 8 more," we simply write "38" and agree that the digit in the "second place" (3 in this case) is to represent groups of ten.

When we work with larger numbers, we group the sets of 10 by tens. Then we call each group of 10 tens *one hundred*. For the set of objects grouped as 5 groups of one hundred, 3 groups of ten, and 7 more, we write 537.

Teaching the Chapter

Materials

Atlases
Auto and camper ads from the newspaper, or catalogues
Books, five different sizes
Base-ten blocks in units, rods of 10 units, layers of 10 rods, and blocks of 10 layers (Sugar cubes may be attached to one another with white glue to make such blocks.)
Calendars
Checkers (at least 10)
Encylopedia
Eyedropper
Index cards, small (1 per child)
Magazines
Mail-order catalogues
Paper bags, nine small, nine medium, nine large (for demonstrations)
Rice, corn kernels, peas, beans, jellybeans, or substitutes
Scissors
Tape measure or metre stick
Tennis ball or handball
World Almanac

Vocabulary

century	millenium	place value
decade	million	set
digit	numeral	thousand
hundred	period	

If children have previously used base-ten arithmetic blocks, such as those illustrated in the lesson on page 34 (or others like them), they will probably be able to work from pictorial representations in the text. Many children who are familiar with the centimetre strips will be able to apply their understanding to the blocks. Demonstrations will help develop understanding of place value, particularly for children who have had physical activity with blocks, but they will not replace actual manipulation of blocks by the child himself.

In this chapter the children should become accustomed to distinguishing between the words *digit* and *numeral*. They should realize that when you talk about a digit you mean one of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 and that when you talk about a numeral you mean any symbol, or combination of symbols, that stands for a number.

In the lessons covering numerals having more than four digits, the vocabulary problem increases. It is not easy for children at this level to read numerals naming large numbers unless you stress that the three digits in the fourth, fifth, and sixth places to the left of the decimal point name the thousands, and that the three digits in the seventh, eighth, and ninth places name the millions. The children should then find that reading numerals with four to nine digits is no more difficult than reading hundreds. They only need to recognize whether the number is in the millions or the thousands.

Although all the children may not master reading large numerals, all of them should understand elementary place-value concepts.

Lesson Schedule

The time the children should spend on this chapter will depend primarily on their background. If they have used this series prior to beginning Book 4, they will be able to cover this chapter in two weeks, but if necessary, allow two-and-a-half to three weeks.

Evaluation of Progress

The chapter review on page 56 should serve as a measure for evaluating the children's progress. The ability to recognize place value in 2- and 3-digit numerals is one of the most important skills to evaluate in this chapter.

There are many lessons containing review material on place value throughout the text. Children who need more practice on the ideas presented in this chapter will have an opportunity to attain better understanding of them as they move through Book 4.

Although the pages of word problems provide stimulating opportunities for working with large numbers, do not expect all the children to be able to work all the problems. You may choose to treat some of the pages of word problems as a class-discussion activity. However, let students who are oriented toward working independently explore the problems by themselves prior to the class discussion.

Resources for Active Learning

General Activities

Chip Trading Activities, Set 1, Sigma, Scott Scientific
Freedom to Learn, "Place Value," pp. 116–117, Addison-Wesley
Nuffield Project: *Computation and structure* 2 , "Place Value," pp. 70–77, Wiley
Nuffield Project: *Computation and Structure* 4 , "Extension of Place Value," pp. 1–12; "Large Numbers and Indices," pp. 50–58, Wiley

Manipulative Devices

Abacus or Abacus Board (school supplier)
Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)
Dienes Multibase Arithmetic Blocks (Herder and Herder)
Peas and Particles—a unit which deals with estimation and large numbers (Selective Educational Equipment; Webster, McGraw-Hill)
Unifix Math Lab Kit (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Commercial Games

Kalah—a game of counting and strategy (Creative Publications; World Wide Games)
Oh-Wah-Ree (Selective Educational Equipment)
Ranko (Midwest Publications)

PAGES 34–35

Objective

Given base-ten units, rods, layers, and blocks, the child will be able to use number blocks, cubes, or their pictorial representations to regroup ones, tens, hundreds, and thousands.

Preparation

Materials

number blocks like those illustrated on page 34, or sugar cubes

The materials at your disposal and the background of the children will determine how you handle this lesson. If you have no number blocks, you might use sugar cubes as a substitute. (White glue may be used to fasten cubes together to form rods, layers, and blocks.)

If neither number blocks nor cubes are available, you can easily use 10-by-10 centimetre grids to make the layers on stiff paper. These may then be used with the white and orange strips which are available with this series. (In this case, change the investigation so that the children are asked to try to find the *area* of each instead of the volume. A set of 10 layers may be used to represent 1000, but we would not speak of "volume" in this case.) Many children, however, will be able to work quite satisfactorily from pictorial representations if necessary.

Investigation

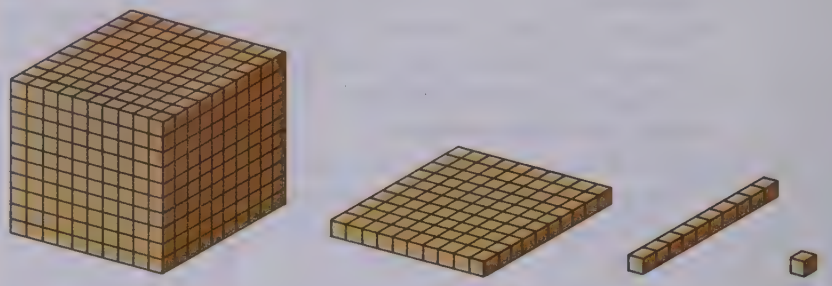
If the three-dimensional materials are available, encourage the children to work with them to answer the challenge questions. Even if the manipulative materials are not available, most children should be able to interpret the pictures well enough to answer the questions and benefit from the ideas being explored in this investigation. Encourage comparisons such as 10 layers are the same as 1 block; 100 rods are the same size as 1 block.

2 Numbers and Numerals

Let's explore units, rods, layers, and blocks.

Investigating the Ideas

The block, layer, rod and unit are made of cubes which are the same size.



Block 1000 Layer 100 Rod 10 Unit

? Can you find the volume of the rod? the layer? the block?

See Investigation.

Discussing the Ideas

1. A How many units placed together make a rod? 10
 B How many rods placed together make a layer? 10
 C How many layers placed on top of each other make a block? 10

2. Here is a record of the number of **blocks, layers, rods,** and **units** Joe has. If he exchanges them all for units, how many units will he have? 378

blocks	layers	rods	units
0	3	7	8



34

Discussion

The questions in discussion exercise 1 may be used as a summary of the ideas introduced in the investigation. Emphasize that the pictured rods, layers, and blocks are all based on the number 10. For example, if the child realizes that the rod has 10 units and the layer has 10 rods, he should see that he can use 1 layer in place of 10 rods.

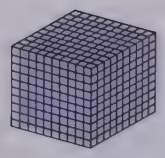
Exercise 2 develops the important idea of recording the various groups so that the placement of the number for each group corresponds to the way we order numerals to show place value in our numeration system, that is, with the largest group on the left and the units on the right. It would be helpful to set up a similar chart on the chalkboard and present various examples to give children practice in making exchanges among the various groups. For example, you might present the following:

	Blocks	*Layers*	*Rods*	*Units*
A	2	5	7	1
B	0	8	2	5
C	1	0	1	0

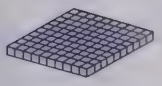
Ask questions such as: "If you exchanged the blocks in set A for as many layers as you could, what would you have?" (0 blocks, 25 layers, 7 rods, 1 unit) "If you exchanged layers for rods in set B?"

Using the Ideas

1.



Thousand-block



Hundred-layer



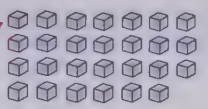
Ten-rod



One-unit

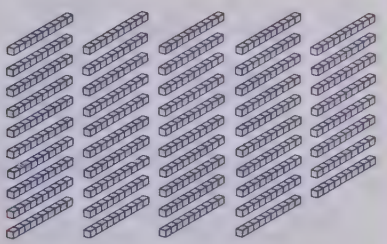
A You can trade in a ten-rod and get __?__ one-units. 10
B You can trade in a hundred-layer and get __?__ ten-rods. 10
C You can trade in a thousand-block and get __?__ hundred-layers. 10
D You can trade in __?__ one-units and get a ten-rod. 10
E You can trade in __?__ ten-rods and get a hundred-layer. 10
F You can trade in __?__ hundred-layers and get a thousand-block. 10

2. A How many one-units are shown here? 27



B If you trade in as many as you can for ten-rods, how many ten-rods would you get? 2 How many one-units would you have left? 7

3. A How many ten-rods are shown here? 48



B If you trade in as many as you can for hundred-layers, how many hundred-layers would you get? How many ten-rods would you have left? 4 8

★ **4.** Suppose this is a record of all the Thousand-blocks, Hundred-layers, Ten-rods, and One-units you have. What will the record look like after you make all of the trades that can be made?

Th	H	T	O
2	26	24	32
3 4	16 8	24 7	32 2

(0 blocks, 0 layers, 82 rods, 5 units) "If you exchanged both blocks and layers for rods in set C?" (0 blocks, 0 layers, 101 rods, 0 units)

When appropriate during this discussion, associate the place-value terms *thousands*, *hundreds*, *tens*, and *ones* with the terms *blocks*, *layers*, *rods*, and *units* respectively.

Using the Exercises

Exercises 1 through 3 on page 35 may be assigned as independent work. The children who need to should be allowed to use number blocks, cubes, or centimetre grids to help them work out the trade-in values.

Starred exercise 4 is intended especially for more capable children, but all the children would benefit from an explanation of the "best trades" that can be made. If the children seem capable, you might provide other, similar examples.

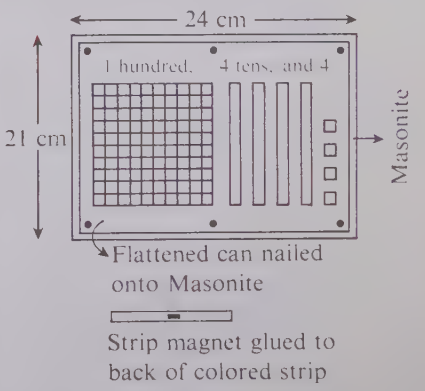
Assignments (page 35)
Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Follow-up

On the chalkboard, display a chart such as the one in starred exercise 4. Ask the children to give the base-ten numeral that represents the total for each set that you show on the chart; that is, ask them to show what the record would be after they made all the trades necessary to get a 1-digit numeral in each column.

Some children may need work with concrete materials to reinforce their understanding of base-ten numerals. If no number blocks are available, provide sugar cubes, or use the following suggestion for these children. To represent layers, 10-by-10 centimetre grids may be made on a duplicator master and run off on relatively stiff paper. Likewise, 10-by-1 centimetre strips can be run off from a duplicator master, or the orange centimetre strips may be used, to represent the rods. The individual 1-centimetre strips can be used as units, and groups of 10 of the centimetre grids (layers) can be clipped together in place of blocks.

For children who would benefit from use of the strips or centimetre grids, you might want to make some "strip boards." A homemade board can be constructed by cutting both lids off a tennis can (or other appropriate can), cutting the vertical seam, flattening it, taping the edges, and tacking it to a piece of Masonite or other board. It can then serve as a convenient base for working with arrangements of magnetized grids and strips. (Strip magnets can be purchased in variety stores and glued onto the strips or grids.)



Resources for Active Learning

Nuffield Project: *Problems*—Green Set, No. 36, Wiley.

PAGES 36–37

Objective

Given a 4-digit numeral in base ten, the child will be able to give the place value for each numeral.

Preparation

If you do not wish to begin immediately with the investigation, show an arrangement of blocks, sugar cubes, or grids and ask a child to tell you what number the arrangement represents to him. Continue for just a few minutes with this procedure, altering the arrangement and giving several children an opportunity to respond.

Investigation

After reading through the directions for the investigation with the children, encourage them to work independently to find the solution to the challenge question. As they work, observe that they recall what numbers are represented by blocks, layers, rods, and units. If necessary, suggest that they refer to the illustration of these terms on page 34 of the preceding lesson.



How is place value used in writing numerals?

Investigating the Ideas

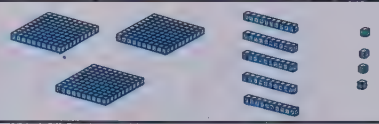
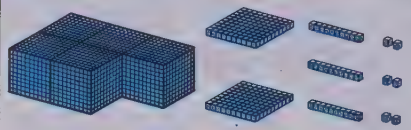
Suppose you have 367 wooden cubes and could glue them together to end up with block pieces, layer pieces, rod pieces, and unit pieces.



? How many blocks, layers, rods, and units would you have if you tried to end up with the fewest possible pieces?
3 layers, 6 rods, 7 units

Discussing the Ideas

1. We use the idea of **place value** and the **digits** (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) to write a **numeral** that tells how many units are in each set below.

	We see	We think	We write
A		T(tens) \| O(ones) 4 \| 7 Read: forty-seven	47
B		H(hundreds) \| T \| O 3 \| 5 \| 4 Read: three hundred fifty-four	354
C		Th(thousands) \| H \| T \| O 5 \| 2 \| 3 \| 6 Read: five thousand two hundred thirty-six	5236

How would you read the numerals in A, B, and C? See above.

2. A What digit is in the tens' place in A? 4 in B? 5 in C? 3
 B What digit is in the hundreds' place in B? C? B-3; C-2
 C What digit is in the thousands' place in C? 5

Discussion

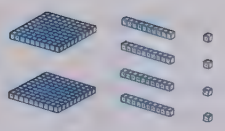
Allow ample time for children to compare and discuss their investigation results. Then direct their attention to the discussion exercises.

The discussion exercises in this lesson relate to the ideas illustrated in the chart. Point out to the children that not only do we group by tens when we write a numeral in base ten, but also we have ten digits to choose from in writing the numeral. An important point to stress is that a digit has not only its own number value but also a value according to where it is *placed*. After discussing the exercises, ask the children to explain what the 4 in A represents and what the 4 in B represents. Similarly, let them consider the 3 in B and the 3 in C, and the 5 in B and the 5 in C.

You might then write several numerals on the chalkboard to give children further practice in identifying place value. You might also use the suggestion in the follow-up section at this time.

Using the Ideas

1. Write a numeral that tells how many units all together. 244



2. Each bundle contains ten sticks. Each box contains one hundred sticks. Write a numeral that tells how many sticks all together. 175



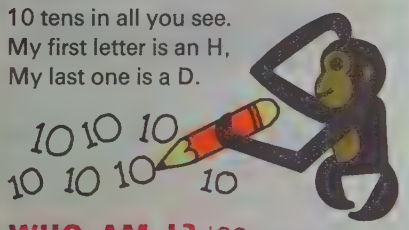
3. Write a numeral for each exercise.
 - A 5 tens and 9 ones 59
 - B 6 tens and 4 ones 64
 - C 3 tens and 0 ones 30
 - D 3 hundreds, 9 tens, and 4 ones 394
 - E 6 hundreds, 0 tens, and 7 ones 607
 - F 5 thousands, 7 hundreds, 4 tens, and 8 ones 5748

4. Give the missing digits.
 - A 39 means ▮ tens and ▮ ones. 3, 9
 - B 93 means ▮ tens and ▮ ones. 9, 3
 - C 597 means ▮ hundreds, ▮ tens, and ▮ ones. 5, 9, 7
 - D 8723 means ▮ thousands, ▮ hundreds, ▮ tens, and ▮ ones. 8, 7, 2, 3

5. Solve the equations.
 - A $32 = 30 + n$ 2
 - B $98 = n + 8$ 90
 - C $324 = 300 + 20 + n$ 4
 - D $836 = n + 30 + 6$ 800
 - E $409 = 400 + n + 9$ 0
 - F $380 = 300 + 80 + n$ 0
 - G $277 = n + 70 + 7$ 200
 - H $699 + 1 = n$ 700
 - I $999 + 1 = n$ 1000

think

I'm one more ten than 90,
10 tens in all you see.
My first letter is an H,
My last one is a D.



WHO AM I? 100

More practice, page A-3, Set 5

Using the Exercises

You may choose to work through exercises 1 and 2 on page 37 with the class. Assign the remainder of the exercises as independent work. To check the answers for exercise 3, have children write their numerals on the chalkboard.

Give all the children an opportunity to try to do the *Think* problem before they are given the correct answer. Most children will understand it once the correct answer is given.

Assignments (page 37)

Minimum: 1–3. Average: 1–4.
Maximum: 1–5.

Mathematics

The *digits* are the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. *A numeral* is a symbol that stands for a number. The symbol 5, for example, is both a digit and a numeral. The symbol 83 consists of two digits and is a numeral for the number *eighty-three*. Keep number-numeral terminology simple so that it is meaningful to the children. Be careful not to use such phrases as "*add* the *digits*" or "*add* the *numerals*." Emphasize that we *add* numbers and *write* digits and numerals.

Follow-up

Evaluate the children's understanding of place value by asking them to write numerals from your oral directions.

Sample oral directions:

Write the 3-digit numeral for each of these numbers.

1. 700 plus 30 plus 5
2. 900 plus 8
3. 800 plus 90 plus 8

Write the next group of numbers in expanded notation. For example, 452 should be written as 400 + 50 + 2.

1. 475
2. 660
3. 308

You might also include examples such as the following:

Write the numerals for the following numbers.

1. Ten more than 76
2. Ten less than 38
3. Eight tens and eight

Some children might benefit from continued use of blocks or cubes. A worksheet such as the following may be used to show an arrangement of units, rods, layers, and blocks for each:

	Blocks	Layers	Rods	Units
563				
702				
4395				
4593				

Duplicator Masters, page 4
Workbook, page 9
Skill Masters, page 4

PAGES 38–39

Objective

Given a whole number between one and one thousand, the child will be able to write a base-ten numeral for it and will be able to read the numeral.

Preparation

A brief review of writing numerals using expanded notation would be suitable if you prefer not to begin immediately with the discussion. Pattern the review on the suggestions given in the follow-up section of the previous lesson. Limit such a review to four or five minutes at the most, keeping a brisk pace throughout.



Do you know the number names up to one thousand?

Discussing the Ideas

1. A Count by tens to ninety. Then count by ones to ninety-nine.

B **Before** the extra stick is added, there are

9 ? tens and ? ones. 9



After the extra stick is added, there are

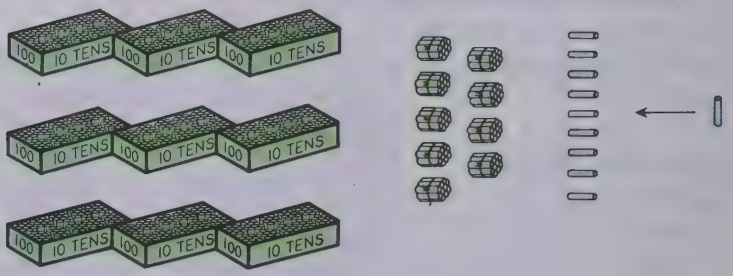
10 ? tens and ? ones. 0

C How do you read and write the numerals for the number of sticks before and after? Read: ninety-nine; write: 99
Read: one hundred; write: 100

2. A Count by hundreds to 900, by tens to 990, and by ones on to nine hundred ninety-nine.

B **Before** the extra stick is added, there are

9 9 9
? hundreds, ? tens, and ? ones.



After the extra stick is added, there are

10 0 0
? hundreds, ? tens, and ? ones.

C Can you write the numerals for the number of sticks before and after? 999, 1000

38

Discussion

To begin the discussion, you might ask children about the kind of counting they do when they play a game like hide-and-seek. Encourage them to count by tens to ninety as directed in exercise 1A. As you discuss this exercise, help them see that $90 + 10$ is 100 or 10 groups of ten and that $99 + 1$ is also 100 or 10 groups of ten. The chief point to emphasize here is that 100 is 10 groups of ten. When the children write the numerals for exercises 1C and 2C, show them how we can think of 100 as a 10 in the tens' place, and of 1000 as a 10 in the hundreds' place.

If the children have had earlier opportunities to work with number blocks, you might not spend much time with demonstrations. However, some classes may find demonstrations helpful. To demonstrate 1000, you can have the children pretend that certain labelled containers such as paper bags hold 100 each; then you can have them think about ten such containers, or about one large container holding 1000 objects.

Using the Ideas

1. Write the numeral for each exercise.

A three hundreds, five tens, and two ones 352

B eight hundreds, three tens, and seven ones 837

C nine hundreds, eight ones, and three tens 938

D six tens, four ones, and eight hundreds 864

E five ones, four tens, and three hundreds 345

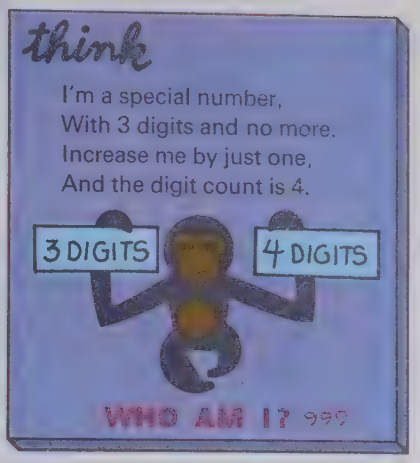
F one hundred, nine tens, and four ones 194

G three hundred ninety-four 394

H seven hundred sixty-eight 768

I four hundred forty 440

J four hundred four 404



2. Write the numeral for the number that is

A just before 200. 199

B just after 799. 800

C just before 250. 249

D just after 899. 900

E just before 610. 609

F just after 990. 991

3. Write the numeral for the number with

A 3 hundreds, 5 tens, 7 ones. 357

B 8 tens, 7 hundreds, 6 ones. 786

C 8 hundreds, 0 tens, 0 ones. 800

D 9 hundreds, 5 ones, 0 tens. 905

★**4.** Solve the equations.

A $99 + 1 = n$ 100
$100 + 99 = n$ 199

B $199 = 100 + n$ 99
$199 + 1 = n$ 200

C $299 = 200 + n$ 99
$299 + 1 = n$ 300

D $399 = 300 + n$ 99
$399 + 1 = n$ 400

E $599 = n + 99$ 500
$599 + 1 = n$ 600

F $899 = 800 + n$ 99
$899 + 1 = n$ 900

Using the Exercises

Assign the exercises on page 39 as independent work. To check exercises 1 through 3, call on children to write their answer for an exercise on the chalkboard. Also, give the children opportunity to read the 3-digit numerals. If necessary, provide more numerals on the chalkboard for the children to practice reading.

Although exercise 4 is starred, all should benefit from a discussion of this exercise. These equations are sequentially ordered so that the children begin by finding the sum of 99 and 1, then the sum of 100 and 99, the sum of 199 and 1, of 299 and 1, and so on.

Again, give all the children an opportunity to try to do the *Think* problem before they are given the correct answer. Most children will benefit from a discussion of the correct answer.

Assignments (page 39)

Minimum: 1, oral. Average: 1–3. Maximum: 1–4.

Mathematics

Once a transition has been made from tens to hundreds, extending the idea of place value to include thousands is little more than a formality. If the children understand that 100 is 10 groups of ten, they should be able to extend this easily to an understanding that 1000 is ten groups of 100. Of course, an understanding of this extension should be quite simple with larger and larger numbers. The idea is simply that, for each place to the left of a given place, we think of ten groups of the number of the given place.

Follow-up

If the class is fairly proficient in place value, divide it into two closely matched teams and have a relay. Give each team leader chalk and an eraser. As you give directions, the leaders at the chalkboard should race to write the correct numeral. Award one point for speed and two points for accuracy. The game goes on as the children next in line replace the leaders at the board and write numerals according to your instructions. Directions for the relay could be similar to these.

1. Write the numeral for 100 more than 806.
2. Write the numeral for 10 less than 205.
3. Write the numeral for 30 more than 303.
4. Write the numeral for 50 less than 605.

Resources for Active Learning

Mathex: Operations No. 3, "Numbers in the Thousands—Activities 1–4," pp. 23–25, Encyclopaedia Britannica Publications Ltd.

Teaching Aids for Elementary Mathematics, "Numeral Expanders," pp. 24–25, Holt, Rinehart and Winston.

PAGES 40–41

Objective

Given appropriate physical objects, or standard units, the child will be able to estimate 1000 of the objects or units.

Preparation

Materials

5 books, each with a different number of pages; rice, corn, navy beans, lima beans, water, or substitutes; paper cup, 100 ml beaker, can lid, pop bottle, 500 ml container, 1 litre container; 10 checkers; calendar; eyedropper

To prepare for this lesson, write the word *estimate* on the chalkboard. Ask the children to tell you what the word means to them. Make sure they have the idea that to make an estimate of something is to make a careful guess, to give an approximation. Then, direct the children to the text and title of the lesson.

Investigation

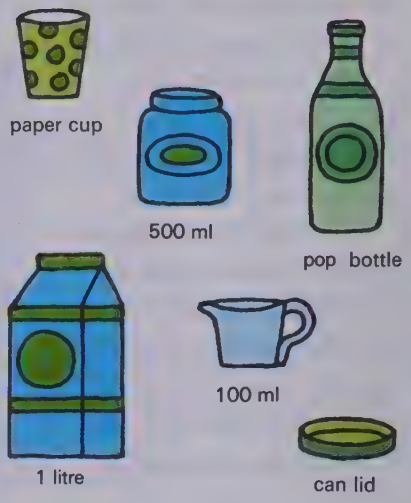
Your method of handling this lesson will depend on your class and the kind of freedom you are accustomed to giving the children. It would be possible to let the children work in groups of three or four. Give the children time to make and record their guesses, and then work together to check at least one of the guesses. If necessary, suggest to some groups that measurement or actual counting is possible but tedious. If children want to make an accurate check, they could divide the counting among 4 or 5 children, each counting 250 or 200 grains of rice or beans.

• *Can you estimate 1000 things?*

Investigating the Ideas

Guess which container shown would most nearly be filled by

- 1000 grains of rice.
- 1000 grains of popcorn.
- 1000 drops of water.
- 1000 navy beans.
- 1000 lima beans.



? Can you find a way to check one of your guesses?
See Investigation.



Discussing the Ideas

1. If you could count to 100 in about 1 minute, about how long do you think it would take you to count to 1000? About 10 min
 How many hundreds are in 1000? 10
 See Discussion.

2. About how many of your classmates would have to get on the scales together to weigh 1000 kilograms? About 28
 See Discussion.

3. If 100 centimetres is 1 metre, about how many metres make 1000 centimetres? Can you throw a ball 1000 cm?
 10
 See Discussion.

40

Discussion

Have children explain how they checked their guesses. Then use the discussion exercises as models of ways to think through such problems. For example, if you can count to 100 in about 1 minute you should be able to count to 200 in about 2 minutes, to 300 in about 3 minutes, to 600 in 6 minutes, and to 1000 in 10 minutes.

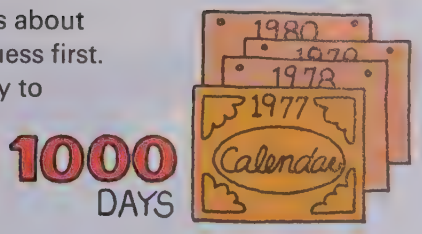
For exercise 2, ask several medium-sized children to give their weights. Then their average weight can be used in repeated addition until you get a sum close to a multiple of 10. If weights are around 35 or 40 kilograms, for example, about 7 children would be needed for 250 kilograms; so about 4 times as many, or 28, would be needed to make 1000 kilograms.

You might prefer to use the second part of exercise 3 as an investigative activity, encouraging the children to find out on the playground who can throw a ball 1000 cm. Throughout the discussion, stress both the concept of how large 1000 is and the "thinking through" approach which should be used in making estimates.

Using the Ideas

1. One thousand days is about how many years? Guess first. Then try to find a way to check your guess.

About $2\frac{2}{3}$ or 3 yr



2. How thick is a book with 1000 pages? Choose 5 books of different sizes. See how close you can come to guessing the number of pages in each book. Does the thickness of the page make a difference in your guess?

160 pages are about 1cm thick, so 1000 pages are about $6\frac{1}{4}$ cm thick; yes



3. How far is 1000 centimetres? Make a guess and then check your guess.

10 metres



4. Is 1000 seconds more than one hour? Find a way to decide.

Sample: 60 seconds are 1 minute. 240 seconds are 4 minutes. 1000 seconds are about 5 x 4 minutes, or 20 minutes.



★5. How tall is a stack of 1000 checkers? Can you measure the height of a stack of 10 checkers and use this information to figure out the answer?

Answer will depend on size of checker.





Follow-up

Children might enjoy illustrating the results of their investigations and displaying them on a bulletin board or on a shelf with containers holding 1000 of the items they counted. Or they might show two containers side by side and have others try to guess which one holds 1000 items.

Resources for Active Learning

Developmental Math Cards, H17, Addison-Wesley. [Estimation and frequency]

Using the Exercises

Read through each exercise on page 41, including starred exercise 5, and have the children record their guesses. Then assign three or four children to a group. Have materials ready for each exercise. You might also encourage groups to check the questions in the discussion section by measuring and by counting seconds.

As the children work, do not suggest methods of counting. However, after they finish the exercises and you allow time for discussion, point out that usually the best method is to count multiples of ten and figure out from a few countings how many they need for 1000.

Assignments (page 41)

Minimum: 1–4, oral. Average: 1–4. Maximum: 1–5.

PAGES 42–43

Objective

Given a 4-, 5-, or 6-digit numeral, the child will be able to read it, write it, and explain the place-value meaning of each digit.

Preparation

Materials

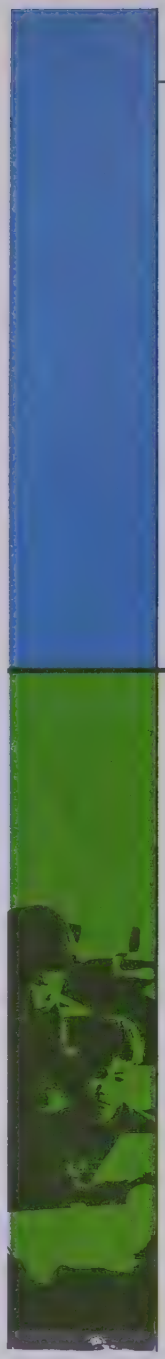
magazines; atlases, almanacs; encyclopedias; auto and camper ads in newspapers, mail-order catalogues, etc.

To prepare for this lesson, write the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 on the chalkboard. Review the word *digit* and ask the children to write or think of a 4-digit numeral. Call on one child to write his 4-digit numeral on the chalkboard. Ask how many others wrote the same numeral. It is unlikely that any other child wrote the same numeral. Mention that there are nine thousand 4-digit numerals and that in this lesson they will investigate how these, and some even larger, numerals may be used.

Investigation

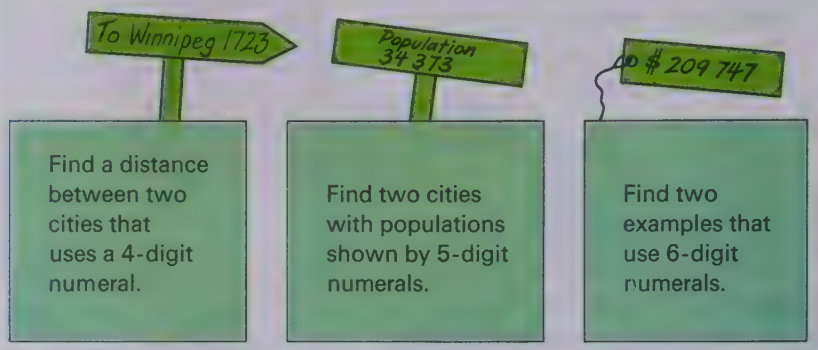
Have an adequate supply of advertisements, catalogues, encyclopedias, and atlases. If necessary, have the children work in pairs; but, if your supply of materials is adequate, encourage them to work independently. Ask questions such as: "Can you find famous dates shown by 4-digit numerals?" "To find 5- and 6-digit numerals, look for areas of provinces and populations of cities in your geography book." Some ways in which 4-digit numerals are used are: dates, weights, speeds, altitudes of mountain ranges, lengths of long rivers, etc. The best source for finding the 4- and 5-digit numerals used in populations and distances are the charts printed in most atlases and almanacs. If your supply of such sources is limited, you might have the children look for other uses you suggest on the chalkboard. (See comments under "Investigation" on page 44.)

If any children give numerals representing amounts of money, encourage them to use only whole numbers of dollars, not including any amounts such as $362.70.



How are 4-, 5-, and 6-digit numerals used?

Investigating the Ideas



? Can you use a reference book to find the information needed in the boxes above? See Investigation.

Record the names and numbers you find.

Discussing the Ideas

1. In a 4-digit numeral, which digit tells how many thousands? 1st digit
 hundreds? 2nd digit tens? 3rd digit ones? 4th digit
2. Read the numerals you found above. Answers will vary.
3. Write five 4-digit numerals on the chalkboard. Tell how many thousands, hundreds, tens, and ones. Then read the numerals. Answers will vary.
4. A What is the largest number named by a 4-digit numeral? 9999
 B What is the largest number named by a 5-digit numeral? 99 999
 C What is the largest number named by a 6-digit numeral? 999 999
5. What are the hundreds' and thousands' digits in a certain 4-digit numeral that appears in the newspaper every day? Hundreds' digit – 9; thousands' digit – 1 (in publication date)

42

Discussion

To introduce discussion exercise 1, write a 4-digit numeral on the chalkboard; then discuss the place value of each numeral. For exercise 2, have children write on the chalkboard the numerals they found in the investigation and then have children read them and discuss the place value of the various digits.

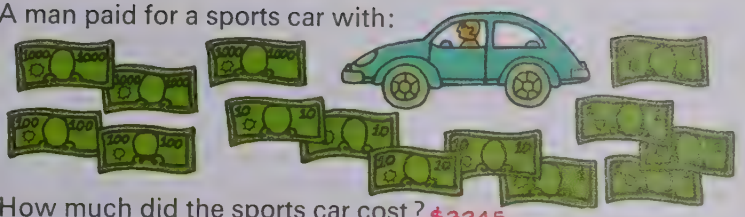
Exercise 4 may challenge some children. The activity suggested in the follow-up section may give children practice in recognizing smaller and larger numerals. Notice, however, that the emphasis in this lesson is on understanding the place values in 4-digit numerals.

Exercise 5 should prompt a discussion of a numeral the children themselves use very frequently—that which represents the date of the current year. So the numerals in the hundreds' and thousands' places are 9 and 1, respectively.

Throughout the discussion, use demonstration materials if you think it necessary. For example, you can illustrate a given 4-digit numeral by exhibiting a large sack labelled "1000," several small sacks labelled "100," a few bundles of 10, and a few objects left over. Continue to stress that 100 is 10 tens and that 1000 is 100 tens or 10 hundreds

Using the Ideas

1. A man paid for a sports car with:



How much did the sports car cost? $3245

2. Give the missing digits in the order indicated.
 - A 3476 means ▮ thousands, ▮ hundreds, ▮ tens, and ▮ ones. 3,4,7,6
 - B 4007 means ▮ thousands, ▮ hundreds, ▮ tens, and ▮ ones. 4,0,0,7
 - C 25 479 means ▮ ten thousands, ▮ thousands, ▮ hundreds, ▮ tens, and ▮ ones. 2,5,4,7,9
 - D 681 493 means ▮ hundred thousands, ▮ ten thousands, ▮ thousands, ▮ hundreds, ▮ tens, and ▮ ones. 6,8,1,4,9,3

3. Write the correct numeral for each exercise.
 - A six thousands, five hundreds, four tens, and three ones 6543
 - B three thousand seven hundred sixty-two 3762
 - C nine thousands, zero hundreds, four tens, and five ones 9045
 - D nine thousand thirty-five 9035
 - E eight ten thousands, four thousands, six hundreds, zero tens, and seven ones 84 607
 - F seven hundred thousands, five ten thousands, nine thousands, zero hundreds, zero tens, and three ones 759 003

4. Solve the equations.
 - A $2856 = 2000 + 800 + 50 + n$ 6
 - B $3495 = 3000 + 400 + n + 5$ 90
 - C $9765 = n + 700 + 60 + 5$ 9000
 - D $82\,763 = n + 2000 + 700 + 60 + 3$ 80 000
 - E $765\,114 = n + 60\,000 + 5000 + 100 + 10 + 4$ 700 000

think

How many different 4-digit numerals can you write by using each of the digits 1, 2, 3, and 4 only once in each numeral?

See Solution, T.E. p. 43.

More practice, page A-4, Set 6



Using the Exercises

After discussing exercise 1 on page 43, assign the remaining exercises as written work. When the children have finished, have them check their work together. To check exercise 3, have children write their answers on the chalkboard. When children write a 4-digit numeral, it is not necessary for them to leave a space. In this book, the space will generally be used only in numerals of five or more digits. A space is occasionally used in a 4-digit numeral, however, to illustrate a certain point, or to maintain consistency in a context where numerals with more than four digits occur.

Encourage all the children to try the *Think* problem, even though most will not find all of the 24 possible numerals.

Solution, Think, page 43

1234	2143	3124	4123
1243	2134	3142	4132
1324	2341	3241	4213
1342	2314	3214	4231
1423	2431	3412	4321
1432	2413	3421	4312

Assignments (page 43)

Minimum: 1–2, oral. Average: 1–3. Maximum: 1–4.

Follow-up

To give children practice in reading 4-digit numerals, play a roll-a-die game with small groups of four or five children. Instruct children to make a place-value chart on their paper such as the following:

1000's	100's	10's	1's

One player rolls a die. The roller of the die announces whether the goal of the game is the largest possible number, or the smallest possible number. Each player records the first digit rolled in any of the four positions, the second digit in any of the remaining three positions, the third in one of the two remaining positions, and the fourth in the last. The placement of a digit may not be changed after it is written in position. Children should keep their own score, noting how many times they have written the numeral announced by the roller of the die.

This game helps children realize the importance of place value. If they are aiming for the greatest possible number and a "2" appears on the first throw, they should soon realize that it would be unwise to place the 2 in the hundreds' or thousands' place.

Resources for Active Learning

Mathex: Operations No. 3, "Boxes and Tickets," pp. 25–26, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Green Set, No. 44, Wiley.

Duplicator Masters, page 5
Workbook, page 10
Skill Masters, page 5

PAGES 44–45

Objective

Given two different numerals of less than 7 digits, the child will be able to state which represents the larger number and to write a statement of the inequality using the correct symbol (> or <).

Preparation

Materials

atlases, almanacs; encyclopedias; automobile advertisements

Since no specific preparation is necessary, it would be appropriate to begin immediately with the investigation.

Investigation

If a sufficient quantity of resource materials is available, have the children work independently to find pairs of numerals to compare. Since the emphasis of this lesson is on comparing numerals, and supplying enough resource materials might be difficult, you might find it necessary to write numerical data on the chalkboard, such as the following:

diameter of earth—12 672 km
diameter of Mars—6720 km
diameter of Mercury—4960 km
distance from Toronto to New York —600 km
distance from Vancouver to Calgary —1027 km
distance from Montreal to Halifax —835 km
distance from Ottawa to Winnipeg —2040 km
population of Vancouver—1 071 081
length of Churchill River—1600 km
length of Mackenzie River —4050 km
length of St. Lawrence River —3040 km
altitude of Regina—570 m
height of Mt. St. Elias—5430 m
height of Commerce Court building, Toronto—236 m



● *Which of two numbers is larger?*

Investigating the Ideas

Can you complete one of these investigations?
See Investigation.

Find the prices of two cars you like. Decide which costs more.



Find the heights of two of the tallest buildings in the world. Decide which is taller.



Find the distances from your home to two places you would like to visit. Decide which is farther away.





Find the lengths of two rivers. Decide which is longer.

Discussing the Ideas

1. Three digits are covered in each salary number. Could Mr. A's salary possibly be more than Mr. B's? Explain.
No. See Discussion.

Mr. A's salary: $8
Mr. B's salary: $9

2. Each of these numbers has the same thousands' digit. If you could see one more digit in each to find out who made more hits, which digit would you choose? Explain.
Hundreds' digit. See Discussion.

Total number of major league hits	
Willie Mays:	3
Hank Aaron:	3

3. How do you decide which of these statements is true?
See Discussion.

A 35 798 > 35 801 False
B 35 798 < 35 801 True

44

Discussion

Discuss exercises 1 through 3 before discussing the numbers the children found in the investigation. Emphasize that they should compare first the digits in the thousands' place. Only if these are the same should they compare the digits in the hundreds' place and so on. Review the meaning of the inequality symbols, ">" (is greater than) and "<" (is less than). Children might share their own clues for distinguishing between the symbols, or you might suggest that, if we think of the symbol as an arrowhead, the arrow is always pointed toward the lesser number.

When the children discuss their numerals from the investigation, have them show their comparisons with the inequality symbol on the chalkboard. Continue to point out the method of examining first the digits in the place of greatest value, that is, the place furthest to the left; then if necessary, the digits in the next place to the right; and so on.

Using the Ideas

1.



The numerals on the cans tell how many marbles are inside.
Which picture (A or B) has the greater number of marbles? A

2. In each exercise, write the sign (<, >) that should go in each ◍.
Then write the words (greater than, less than) that should go in each blank.

A 730 ◍ 73 >
730 is __?__ 73. greater than

B 703 ◍ 730 <
703 is __?__ 730. less than

C 7300 ◍ 7030 >
7300 is __?__ 7030. greater than

D 42 593 ◍ 42 583 >
42 593 is __?__ 42 583. greater than

E 65 423 ◍ 65 523 <
65 423 is 100 __?__ 65 523. less than

F 126 742 ◍ 125 742 >
125 742 is 1000 __?__ 126 742. less than

G 483 762 ◍ 483 672 >
483 762 is __?__ 483 672. greater than

H 95 461 ◍ 95 164 >
95 164 is __?__ 95 461. less than

3. Give the number that is 100 more than

A 7. 107 C 327. 427 E 32 007. 32 107 G 2341. 2441 I 438 900. 439 000
B 27. 127 D 5327. 5427 F 72 700. 72 800 H 900. 1000 ★ J 999 900. 1 000 000

4. Give the number that is 1000 less than

A 2000. 1000 C 5003. 4003 E 61 004. 60 004 G 310 000. 309 000
B 5234. 4234 D 31 900. 30 900 F 1000. 0 H 845 900. 844 900

★ **5.** A Give the largest 3-digit number that has the digits 2 and 5. 952
B Give the smallest 3-digit number that has only one 0 digit. 101
C Give the smallest 4-digit number that has the digits 5 and 3. 1035
D Give the largest 5-digit number that has the digits 4, 7, 8, 0, 5. 87 540
E Give the largest 6-digit number with no two digits alike. 987 654

More practice, page A-5, Set 7

45

Follow-up

Prepare a worksheet similar to the following to help children become more familiar with comparisons of numerals in the thousands.

1. Use one of the inequality symbols (> or <) to complete the following statements.

A 2536 ◯ 2653	F 5861 ◯ 5860
B 1984 ◯ 1899	G 4935 ◯ 4953
C 7562 ◯ 6751	H 7812 ◯ 7795
D 9119 ◯ 9191	I 8111 ◯ 1118
E 1040 ◯ 1004	J 5059 ◯ 5111

2. For each of the exercises above, give the name of the place (thousands', hundreds', tens', ones') which enabled you to tell which was the greater number.
Example:

A *hundreds*	F_____
B_____	G_____
C_____	H_____
D_____	I_____
E_____	J_____

Duplicator Masters, page 6
Workbook, page 11
Skill Masters, page 6

Using the Exercises

Work through exercise 1 together. It is not essential that children write out the numerals for A and B in order to compare them. They might match the objects in the two sets first and then, when they have decided which has the greater number of marbles, show the comparison by writing the two numerals with the inequality symbols.

Assign the remaining exercises as independent work. Explain that in exercises 3 and 4 they are to think about counting (by hundreds and thousands) and about place value, rather than about addition. In other words, to give the number that is 100 more than 5327, the children should count by hundreds starting with 5327 (i.e., fifty-three hundred twenty-seven, fifty-four hundred twenty-seven). To find a number that is 1000 less than a given number, they should think about counting backward by 1000.

Note that exercise 5 is a challenge for the capable children, although nearly everyone will enjoy discussing it.

Assignments (page 45)
Minimum: 1–2, oral. Average: 1–4. Maximum: 1–5.

PAGES 46–47

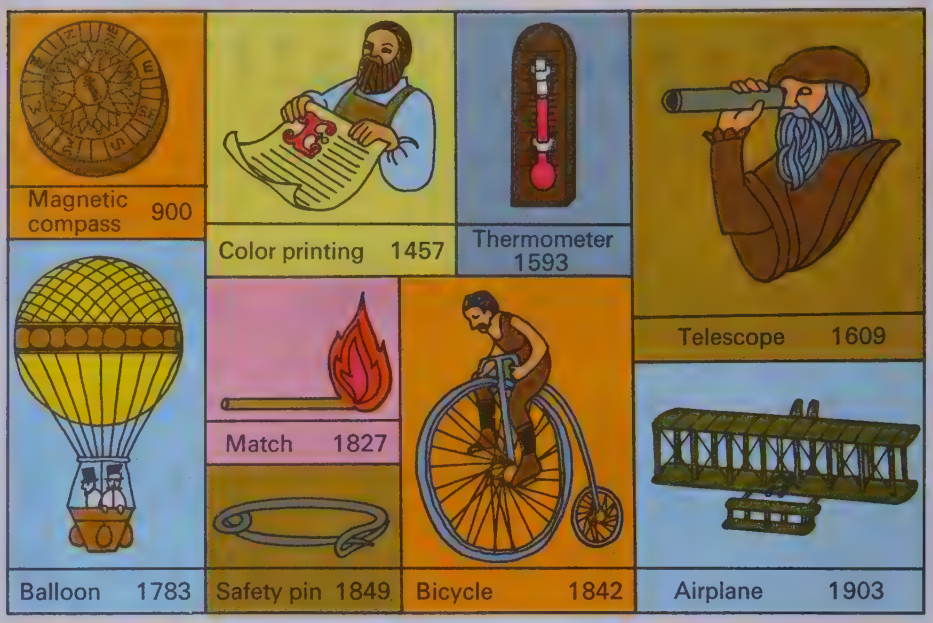
Given the number of years in a decade, century, *and* millennium *and word problems using these terms, the child will show an understanding of the terms and of place value by answering the questions in the problems.*

Preparation

To prepare for this lesson, write the words *decade*, *century*, and *millennium* on the chalkboard. Explain the meaning of each term, emphasizing that a century is ten decades and that a millenium is ten centuries.

Solving Story Problems

INVENTIONS



A decade is a period of 10 years.
A century is a period of 100 years.
A millennium is a period of 1000 years.

You have lived about a decade. Very few people have lived for a century. Use the chart and your knowledge of place value to answer the questions.

1. The sewing machine was invented 3 years later than the match. When was the sewing machine invented? 1830

2. The bicycle tire was invented 4 decades later than the safety pin. When was the bicycle tire invented? 1889

3. The adding machine was invented 2 centuries earlier than the bicycle. When was the adding machine invented? 1642

46

Discussion

Give the children an opportunity to discuss the chart listing various inventions and the years they were invented. Review the meaning of decade, century, and millennium with them. The sentences below the definitions should give the children some notions about the time spanned by a decade and by a century. Ask selected children to read the sentences aloud.

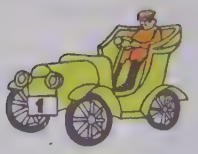
Again, emphasize to the children that they should solve these problems by counting and thinking about place value rather than by addition and subtraction.

Depending on the reading ability of the children, direct the children to do the problems on these pages on their own. You may, however, choose to discuss the first three before assigning those on page 47 as independent work. If necessary, help poor readers by reading the problems for them in a small group; however, let them do their own thinking to answer the questions.

Assignments (pages 46–47)

Minimum: 1–11, oral. Average: 1–11. Maximum: 1–11.

4. The match was invented 6 decades before the gasoline auto. When was the gasoline auto invented? 1887



5. The piano was invented 10 decades after the telescope. Give the year in which the piano was invented. 1709



6. The fountain pen was invented 1 century and 1 year after the balloon. In what year was the fountain pen invented? 1884



7. The movie machine (projector) was invented 4 centuries and 1 year after Columbus discovered America. When was the projector invented? 1893



8. Leonardo Da Vinci learned about flight by watching birds. He drew a picture of an airplane about 400 years before the airplane was invented. In what year did Da Vinci draw the picture? 1503



9. The first modern naval submarine was built about 10 centuries after Chinese sailors invented the magnetic compass. About when was the first naval submarine built? 1900



10. What will be the date one millennium after the invention of the thermometer? 2593

11. Suppose that when the first sheet of color printing was made it was sealed in a block of stone. If it is discovered 3000 years later, what year will that be? 4457

47

PAGES 48–49

Objective

Given a base-ten numeral having no more than 9 digits, the child will be able to read the numeral and give the place value for each digit.

Preparation

Materials

index cards, small (*Duplicator Masters*, page 61); scissors

It would be appropriate to begin immediately with the investigation. However, if you prefer, introduce the investigation by suggesting that the children imagine some "large numbers" and things which occur in large numbers. Then explain that in this lesson they will learn to read large numerals, that is, the names for such numbers.

Investigation

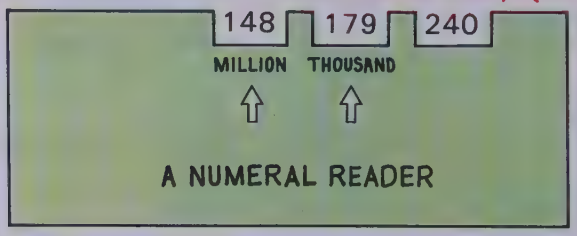
Guide the children in studying the numeral and numeral reader pictured in the text. Following this, give them specific, close guidance in making their numeral readers. Direct them to place their index card slightly below the illustrated card in the text, and to mark the sections they are to cut out. After the children read the numerals in the challenge question, have them work with several other numerals. For example, write a dozen or so numerals of 5 to 9 digits on the chalkboard and instruct the children to write these (or others of their own choice) on paper, using their numeral reader as a guide. Assign partners and direct the partners to take turns reading each other's numerals, using the numeral reader.

Can you read large number names?

Investigating the Ideas

Use the numeral reader to read the sentence below.
The average distance from earth to our sun is:

One hundred forty-eight million, one hundred seventy-nine thousand, two hundred forty (kilometres)



A NUMERAL READER

? Can you make a copy of the numeral reader and use it to read these numerals? See Investigation.

Eight thousand, four hundred twenty-seven	8	427	
Two hundred thirty-seven thousand, eight hundred nine	237	809	
Fifty-three million, seven hundred forty-five thousand six hundred twenty-six.	53	745	626

Discussing the Ideas

1. How would you use your numeral reader to read these numerals?

A 5 280	C 29 002	E 46 719 845
B 36 198	D 2 592	F 971 635 887

See Discussion.

2. What is the largest number you can read with your numeral reader? 999 999 999

3. Explain how you can use your numeral reader to help you write the numeral for these names. See Discussion.
 - A seventy-two thousand, one hundred fifty-four 72 154
 - B nine million, four hundred fifty-three thousand, eight hundred nineteen 9 453 819

48

Discussion

The purpose of the numeral reader is to help children recognize that large numerals are grouped in blocks of three places called *periods*. Thus, the stress should be not on the special names for individual places such as the fifth and sixth places, but on the fact that the three places (fourth, fifth and sixth), as a group, name the number of thousands. Explain that the blocks of three places, the *periods*, extend, as a way of counting, through the millions, billions, and so on. Point out that they can easily read larger numerals by naming the period to the left of each space.

In exercise 3, help the children observe that the numeral reader tells them the proper order in which to write the digits which represent millions, thousands, and so on.

Using the Ideas

1. Give the number of thousands. Read the numeral by using your numeral reader.

A	3	942	3	D	409	636	409
B	10	536	10	E	82	746	962 746
C	408	212	408	F	438	986	503 986

2. Use your numeral reader to help you write the numeral for each exercise.

A eight thousand, six hundred thirty-five 8635

B twenty-eight thousand, four hundred twenty-seven 28 427

C nine hundred thirty-six thousand, two hundred five 936 205

D four million, seven hundred ninety-six thousand, seventy-three 4 796 073

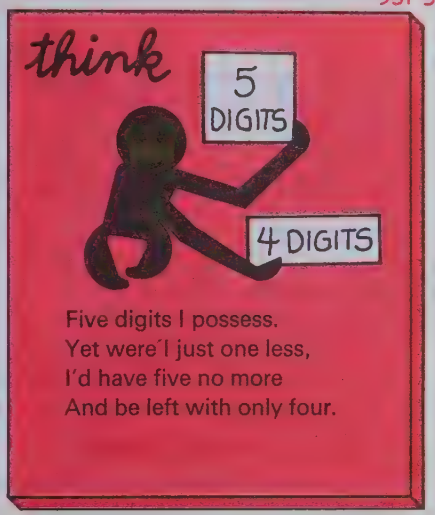
3. Give the number 1000 greater than each of these.

A 1342 2342 C 249 361 250 361 E 409 317 410 317 G 300 876 301 876

B 23 157 24 157 D 399 264 400 264 F 300 007 301 007 H 950 304 951 304

4. In each numeral below, one digit is red. Give the number the red digit stands for. See Answers, T.E. page 49.
Example: 27 341
(Answer: 7000)

A	28 436	H	622 317
B	382 984	I	307 460
C	675 832	J	999 999
D	637 243	K	436 872
E	513 200	L	923 465
F	983 741	M	345 872
G	200 700	N	407 600



More practice, page A-5, Set 8

Using the Exercises

Exercise 1 is an oral exercise. It would be helpful to include other numerals such as those in exercise 3, or some you write on the chalkboard, so all would have an opportunity to participate.

Assign the remaining exercises as independent work. For exercise 3, remind the children not to add but to think about counting by thousands.

All of the children will benefit from a discussion of the *Think* problem. Be sure that they are given an opportunity to think about the riddle before the answer is given. Most of the children will understand the problem once they know the answer.

Answers, exercise 4, page 49

A	400	H	2000
B	80	I	0
C	5000	J	900 000
D	600 000	K	30 000
E	10 000	L	900 000
F	3000	M	5000
G	700	N	600

Assignments (page 49)

Minimum: 1–2. Average: 1–4.
Maximum: 1–4.

Mathematics

In the place-value scheme, a problem occurs in naming 5- and 6-digit numerals. Through the fourth place, each place is given a new name. (The first place is the ones' place; the second, the tens' place; the third, the hundreds' place; the fourth, the thousands' place.) However, we use no new names from the fourth place to the seventh place, where we introduce the name *millions*. It might be argued that the fifth place is called the ten-thousands' place and that the sixth place is called the hundred-thousands' place, but the term *thousands* still applies to these two places.

Some inquisitive children may ask about the names of some of the larger periods. The names of the periods following millions are given here as reference, even though they are seldom used.

billion, trillion, quadrillion, quintillion, sextillion, septillion, octillion, nonillion, decillion, undecillion, duodecillion, tredecillion, quattuordecillion, quindecillion, sexdecillion, septendecillion, octodecillion, novemdecillion, vigintillion

Follow-up

To give children an opportunity to become more familiar with larger numerals, play a place-value card game. Have printed on fairly large-sized cards two sets of the digits 0, 1, 2, . . . 9. Have the children form two teams of ten children each. Give each team a set of digits, so that each team member has a card to hold. As you call out a numeral, each child who holds one of the digits called stands with the other members of his team who also have a digit in the numeral, to form the numeral for the remaining members of the class, those not on a team, to see. The team whose members are the first to position themselves correctly gets a point. Those children not on a team may judge which team was quicker in forming the correct numeral.

PAGES 50–51

Objective

Given the numeral 1 000 000, the child will have at least a limited grasp of how large it is in terms of association with smaller numbers.

Preparation

Materials

numeral reader (from previous lesson)

Since this investigation is intended to be treated in a light manner, you might begin by writing the numeral 1 000 000 on the chalkboard. Ask the children to count the number of digits in the numeral. Explain that in this lesson they will have a chance to think about 1 000 000.

Investigation

The questions on this page are intended for enjoyment and to create curiosity about the size of 1 000 000. Answers are expected to be purely guesses rather than true estimates. There is, of course, a correct answer for each question but no one is expected to answer most of the questions correctly. The children should understand this from the outset. You might read the questions aloud and have the children raise hands to show their chosen answer for each one. Encourage all children to participate in the discussion of these questions.

Investigating the Ideas

Can you guess the correct answer for each question?



A There would be about 1 000 000 hairs on the heads of __?__ students. (2; 10; more than 20)

B The weight of __?__ busloads of students would total 1 000 000 kilograms. (less than 20; about 90; more than 200)



C A mathematics book with 1 000 000 pages would need a bookshelf as long as __?__ parked cars? (1; 5; 10)


?

D A person who has lived 1 000 000 hours is __?__ years old. (10; 50; over 110)



See Investigation and Discussion.



Discussing the Ideas

1. A How long do you think it would take you to count to 1 million if you said one number each second? About 12 days
 B Was your guess too small or too large? See Discussion.

2. What number comes just before 1 million? Show how to write this number. 999 999

Discussion

The following remarks might help you answer some of the children's queries in regard to the questions.

Research has shown that, though great variations occur in the number of hairs on a person's head, the average total is between 100 000 and 150 000 hairs (see "Skin," *Encyclopaedia Britannica*). Thus, to count about 1 000 000 hairs, you would have to count the hairs on 10 heads.

An empty school bus which holds 79 passengers weighs about 8200 kg. If an approximate weight of one fourth-grade child is 36 kg, then one busload of fourth-grade children is about 8200 + 2900 or 11 100 kg and about 90 such busloads would total 1 000 000 kg.

A book of 160 pages is almost one centimetre in thickness, so about 6000 centimetres would be the thickness of 1 000 000 pages.

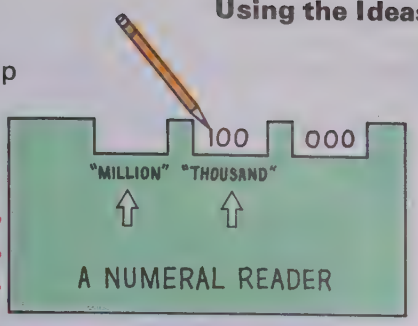
An average car length may be considered about 6 metres, so a bookshelf about as long as 10 cars would be needed for the 1 000 000 page book.

Since there are 24 hours in a day and 365 days in a year, 110 years contain 963 600 hours, so the correct answer is "over 110 years old."

You might use discussion exercise 1 as a sample of how to make an estimate for these kinds of questions. If you said one number each second, you would count to 60 in a minute, to 600 in 10 minutes, to 3600 in an hour, to 360 000 in 100 hours or about 4 days. You would need about $3 \times 360\,000$ to reach 1 000 000, so it would take about 12 days.

Using the Ideas

1. Use your numeral reader to help you write these numerals.
 - A five hundred thousand 500 000
 - B six hundred thousand 600 000
 - C seven hundred thousand 700 000
 - D eight hundred thousand 800 000
 - E nine hundred thousand 900 000
 - F ten hundred thousand (one thousand thousand) 1 000 000

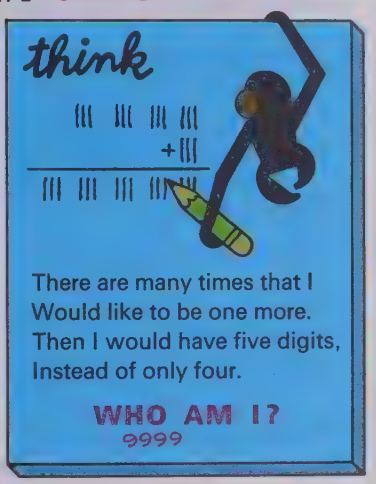




2. How many thousands does it take to make 1 000 000?
 One thousand or ten hundred

3. Use your numeral reader to help you write these numerals. 999 998
 - A nine hundred ninety-nine thousand, nine hundred ninety-eight
 - B nine hundred ninety-nine thousand, nine hundred ninety-nine 999 999
 - C the next numeral after the one in B 1 000 000

4. Write the number that is
 - A 100 more than one million. 1 000 100
 - B 1000 more than one million. 1 001 000
 - C 10 more than one million. 1 000 010
 - D 1 more than one million. 1 000 001
 - E 10,000 more than one million. 1 010 000
 - F three million more than 324 562 218. 327 562 218
 - G thirty million more than 812 469 855. 842 469 855
 - H five hundred million more than 283 618 962. 783 618 962





51

Using the Exercises

Assign the exercises on page 51 as independent work and direct the children to use the numeral reader made in the previous lesson. When the children finish these exercises, check their work carefully by having volunteers write their answers on the chalkboard. Remind the children of the proper use of the space.

Note that exercises 1 through 3 present different ways to help the children understand one million. Exercise 1 considers skip counting, exercise 2 concerns 1000 addends, and exercise 3, counting. Point out these approaches to the children as you discuss their answers.

Assignments (page 51)*

Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

Follow-up

If the children are capable, direct them to make up more problems using very large numbers. You can expect them to present problems on multiples of ten and one hundred that require finding whether a number is more or less than such multiples by using place value, *not* operations.

A sample worksheet might look like this.

Use > or < for each ◯.

1.	4 007 250	◯	4 007 025
2.	387 200 246	◯	387 020 426
3.	26 190 378	◯	26 109 378
4.	17 349 520	◯	17 349 502
5.	509 721 864	◯	590 721 864

Resources for Active Learning

Developmental Math Cards, F[18], Addison-Wesley. [Thinking about large numbers]

Duplicator Masters, page 7
Workbook, page 12
Skill Masters, page 7

PAGES 52–53

Objective

Given a pictorial "base-ten machine," the child will be able to interpret the place value of 9-digit base-ten numerals.

Preparation

To prepare for this lesson, write several 6-, 7-, 8-, and 9-digit numerals on the chalkboard for the children to practice reading. Call attention to particular digits in some numerals and ask children to give their place-value meaning. For example, if you have written on the chalkboard 28 372 500 you might ask what the digit 3 represents here, and you are expecting the response 300 000. However, as with all preparatory activities, limit such a review to a maximum of five minutes.

Investigation

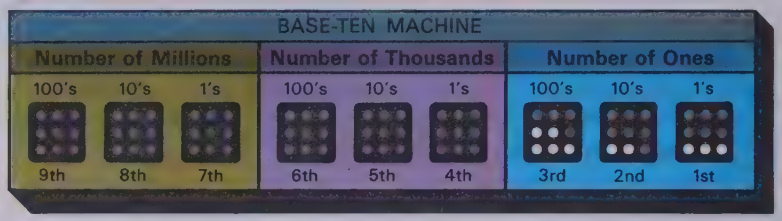
Encourage children to study the machine and share reactions to it. Try to get the children to describe how they think the machine works with regard to lights on and lights off in the various positions. When the children understand how the machine works, have them try to answer the challenge question on their own. Then proceed to the discussion section.



Let's explore the "Base-Ten Machine."

Investigating the Ideas

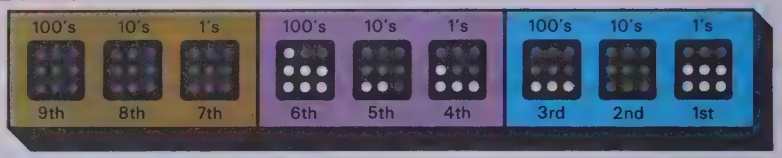
A card gave the machine a signal to remember the number 523. The lights show how the machine works.



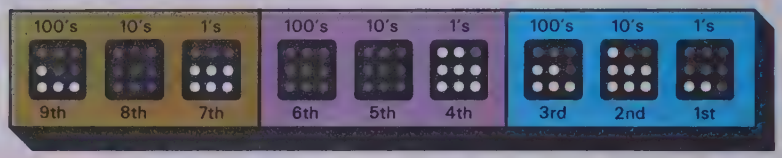
Can you give the number that the machine was signalled to remember in each part below ?

724 306

A



B



406 008 572

Discussing the Ideas

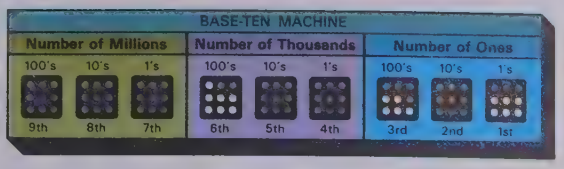
1. The 1st, 2nd, and 3rd places on the machine tell the number of ones. Which places on the machine tell how many thousands? millions? 7th, 8th, and 9th
 4th, 5th, and 6th
2. Explain what lights would be on if the machine were signalled to remember 9 009 009.
 9 in the 7th place, 0 in the 6th and 5th places, 9 in the 4th place, 0 in the 3rd and 2nd places, 9 in the 1st place

52

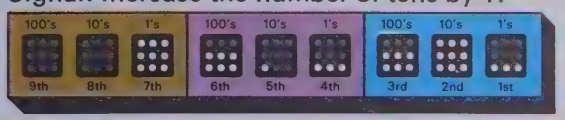
Discussion

During the discussion, remind the children of the purpose of the spaces and how they should be used to designate the millions' period and thousands' period. You need not stress the name of the "ones" for the first period since it is not spoken when reading a numeral. That is, we do not say 76 thousand, 435 ones; we say 76 thousand, 435. As mentioned in exercise 1, use the machine to stress the role of the periods and how they help to make the number easier to read and understand.

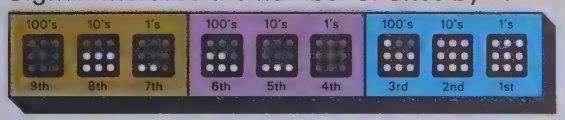
★ Using the Ideas



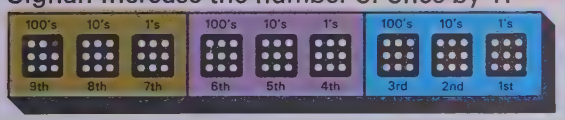
1. Give the number the machine was signalled to remember. 700 506
2. A card is put into the machine. It signals the machine to increase the number of ones by 4.
 - A How many lights are now on in the 1st place? 0
 - B How many lights do you think are on in the 2nd place? 1
3. Another card signals the machine to increase the number of hundreds by 5.
 - A How many lights are now on in the 3rd place? 0
 - B How many lights are on in the 4th place? 1
4. Another card signals the machine to increase the number of hundred thousands by 5.
 - A How many lights are now on in the 6th place? 2
 - B How many lights are now on in the 7th place? 1
5. Give the number that results when each signal is given to the machine shown.
 - A Signal: Increase the number of tens by 1. 9 613 703



 - B Signal: Increase the number of ones by 1. 62 530 800



 - C Signal: Increase the number of ones by 1. 900 000 000



More practice, page A-6, Set 9



Using the Exercises

Note that this exercise set in the student book is starred in its entirety. Depending on the ability of your class, you might use some of these exercises for discussion, encourage faster children to work through them, or assign some of the exercises for all to try. If more capable children try exercise 5C, all would benefit from a discussion of how they arrived at the correct answer; this exercise presents a fine example to show ten as the base of our numeral system.

Assignments (page 53)*
Minimum: 1–4. Average: 1–4.
Maximum: 1–5.

Follow-up

Some children might enjoy making a "base-ten machine" either by using the machine in the text as a model or by making up one of their own. Large cardboard boxes, small toy batteries and lights may even be used by some more energetic children. A less complicated machine might include sets of 9 cards for each place, each marked to show a number from 0 to 9.

Workbook, page 13

PAGES 54–55

Objective

Given population figures expressed in millions and thousands, the child will be able to answer questions concerning the comparison and rounding off of such numerals.

Preparation

If children have made a base-ten machine, you might use it to review the reading and understanding of large numerals. You might prefer simply to write on the chalkboard pairs of large numerals which have the same number of digits. Then have the children read them and tell which number in each pair is larger. Recall that a review such as this should take only a very short time.

Solving Story Problems

Populations of 10 Canadian Metropolitan Areas*

Metropolitan Area	Number of People
Calgary, Alberta	400 154
Edmonton, Alberta	490 811
Hamilton, Ontario	495 864
Montreal, Quebec	2 720 413
Niagara-St. Catharines, Ontario	301 108
Ottawa-Hull, Ontario-Quebec	596 176
Quebec, Quebec	476 232
Toronto, Ontario	2 609 638
Vancouver, British Columbia	1 071 081
Winnipeg, Manitoba	534 685

1. Is the number of people in the Montreal area closer to 2 000 000 or 3 000 000 ? 3 000 000

2. Is the number of people in the Ottawa-Hull area closer to 500 000 or 600 000 ? 600 000

3. Is the number of people in the Vancouver area closer to 1 000 000 or 2 000 000 ? 1 000 000

4. We say: The number of people in the Hamilton area **(to the nearest thousand)** is 496 000.
 The number of people in the Calgary area (to the nearest thousand) is 400 000.
 Give the number of people (to the nearest thousand) in these areas.

 A Winnipeg 535 000 C Toronto 2 610 000 E Edmonton 491 000
 B Quebec 476 000 D Vancouver 1 071 000 F Montreal 2 720 00

*All population data are based on preliminary figures from the 1971 census, Statistics Canada.

Discussion

Read and study with the children the data in the population chart. Have them read the populations of the various cities. Discuss with the children what it means to give the population figure to the nearest 1000 people. Ask the children to tell what they think this means. You can draw an analogy to the idea of measuring to the nearest centimetre, and the children should quickly understand that when a population figure is given to the nearest 1000, it simply means that the population is closer to that number of thousands than it is to the one before or the one after.

Have the children work the exercises. When they have finished, give each child a chance to check his paper and to participate in a discussion of the ideas involved.

If your school is not in one of the metropolitan areas listed in the chart, you might have the children do some research regarding the population of your city or metropolitan area, or your province. Even if you do live in one of the areas listed in the chart, the children can still do research that interests them.

Assignments (pages 54–55)

Minimum: 1–11, oral. Average: 1–11. Maximum: 1–11.



5. A Which area has the greatest number of people? Montreal
 B Which area has the least number of people? Niagara - St. Catherines

6. The areas are listed in alphabetical order. List them according to population. Place the name of the area with the most people at the top of your list. Montreal, Toronto, Vancouver, Ottawa-Hull, Winnipeg, Hamilton, Edmonton, Quebec, Calgary, Niagara-St-Catherines

7. Montreal has about 2 500 000 more people than Halifax, Nova Scotia. What is the population of Halifax? About 220 000

8. A Which areas have between 2 000 000 and 3 000 000 people? Montreal, Toronto
 B Which areas have between 500 000 and 600 000 people?
 C Which areas have more than 450 000 and less than 500 000 people? Ottawa-Hull, Winnipeg, Edmonton, Hamilton, Quebec

9. Which area on the list has a population about 2 000 000 greater than the Ottawa-Hull area? Toronto

10. A The province of Quebec has about 3 000 000 more people than the Montreal area. About how many people live in Quebec? About 5 720 000
 B The province of New Brunswick has about 400 000 fewer people than the Vancouver area. About what is the population of New Brunswick? About 670 000

11. The population of Canada is about 19 000 000 more than the number of people living in the Montreal area. About what is the population of Canada? About 21 720 000

Follow-up

To give the children more experience with place-value concepts, prepare an exercise sheet like the one shown.

How many?	
376 921	—thousands
76 380 540	—millions

Round as directed.	
35 498 657	—to the nearest million
9 745 321	—to the nearest million
609 890	—to the nearest thousand
79 099 895	—to the nearest million
300 751 642	—to the nearest million

PAGES 56–57

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

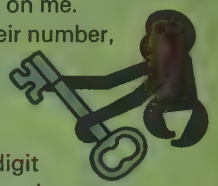
Write several large numbers on the chalkboard, and ask the children to list these numbers in order. Focus attention on the place-value concepts shown by having the children read the number of ones, tens, hundreds, and so on. Following this, pick a particular digit from each number and have one of the children in the class tell the number that the digit stands for.

1. Write the correct numerals.
 - A 7 tens and 8 78
 - B 9 tens and 3 93
 - C 8 hundreds and 9 tens 890
 - D 4 thousands and 5 4005
 - E 7 thousands and 4 tens 7040

2. Give the missing numbers.
 - A One hundred is ▒ tens. 10
 - B One thousand is ▒ hundreds 10
 - C One thousand is ▒ tens. 100
 - D ▒ thousands are 1 million 1000
 - E 99 + 1 = *n* 100
 - F 999 + 1 = *n* 1000

think

When digits get together
They always count on me.
If you must find their number,
I really am the key.



WHO AM I?
Place value

Now I'm the only digit
Whose number stays the same
No matter where I am
In our place-value game.

WHO AM I? 0

3. Study the numeral. Then give the correct word for each blank.

$\underbrace{436}_{\text{group 3}}\ \underbrace{507}_{\text{group 2}}\ \underbrace{246}_{\text{group 1}}$

The digits in group 1 tell how many ones.
 - A The digits in group 2 tell how many __?__. thousands
 - B The digits in group 3 tell how many __?__. millions

4. Write the sign (<, >) that should go in each ▒.
 - A 83 762 ▒ 84 672 <
 - B 342 968 ▒ 351 000 <
 - C 687 234 572 ▒ 687 243 572 <
 - D 999 999 ▒ 1 000 000 <

5. Give the number that is 1 000 000 more than
 - A 5 000 000. 6 000 000
 - B 23 560. 1 023 560
 - C 4862. 1 004 862
 - D 343 000 000. 344 000 000

6. A Give the smallest 2-digit numeral that has 3 as a digit. 13
 - B Give the smallest 3-digit numeral that has the digits 1 and 9. 109
 - C Give the largest 4-digit numeral that has 9 as a digit in three places. 9998

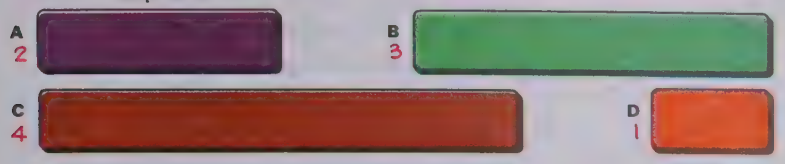
56

Discussion

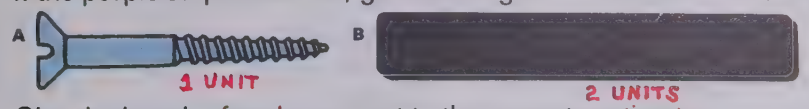
Page 56 provides a review of the material covered in the chapter. You may choose to use this page as a chapter test, in which case you will want to check the papers yourself. However, you may choose to use these exercises as a review, thoroughly discussing the exercises after children have worked them on their own, or working through them with the class. In either case, attempt to give explanations which will strengthen their understanding of the ideas of the chapter. Focus particular attention on any topics with which the children have had difficulty.

You can use the *Think* riddles either as a part of the chapter review or simply as enrichment for the more able children, who may finish the page early. All children should be given an opportunity to consider the riddles before the correct answers are given.

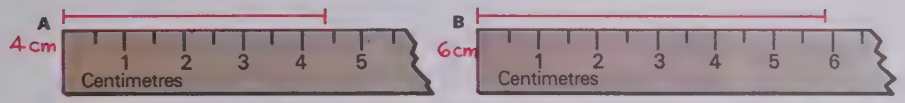
1. If the red strip  is the unit, what is the length of each strip below?



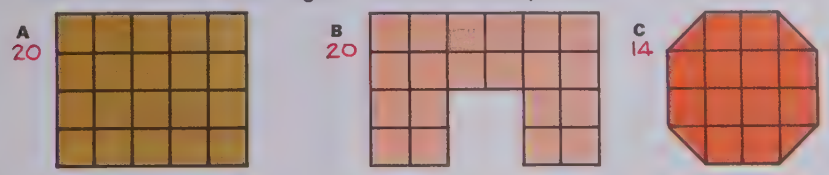
2. If the purple strip is the unit, give the lengths to the nearest unit.



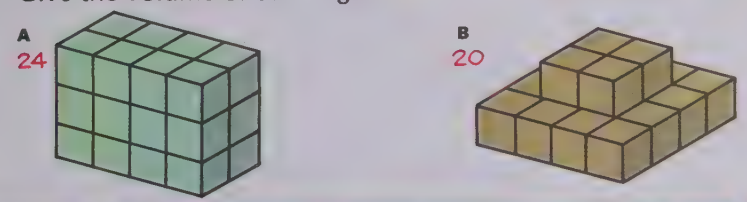
3. Give the length of each segment to the nearest centimetre.

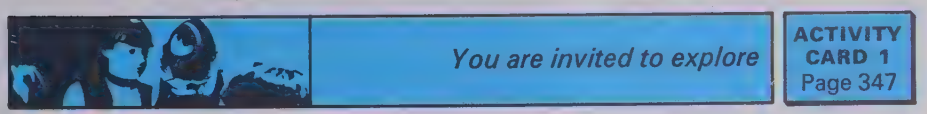


4. Find the area of each region. Each small square is a unit.



5. Give the volume of each figure.





Resources for Active Learning

Developmental Math Cards, H49, Addison-Wesley. [A good place-value game]

Workbook, page 14

Using the Exercises

Page 57 reviews the measurement ideas of Chapter 1. Children will need a centimetre ruler for these exercises. When children finish them and check their work, be particularly mindful of measurement to the nearest centimetre.

General Objectives

To focus attention on addition and subtraction concepts

To provide experiences in working with equations

To improve skills with addition and subtraction facts for sums through 18

To develop skills in adding and subtracting

To focus attention on the inverse relationship between addition and subtraction

To provide experiences in reasoning

To provide experience with word problems including work with coins

The initial pages of this chapter provide a review of the basic meaning of addition and subtraction and the relationship between these two operations. Following this, the basic principles for addition are explored as background for introducing column addition and working with the regrouping algorithm for addition. Column addition is extended to include numbers having up to four digits. Word problems, including some money problems, are provided.

After addition is thoroughly explored, the lessons review some basic ideas about place value. Next, subtraction with regrouping is introduced, including subtraction of tens, hundreds, and thousands. Considerable work on word problems is provided together with material designed to strengthen the children's understanding of subtraction when zeros occur in various positions.

Mathematics

To gain a clear understanding of the whole-number system, it is important to become aware of its structure. Much of this structure is inherent in the basic principles that govern the operation of addition on the whole numbers. The first such principle is the *commutative* (order) principle for the addition of whole numbers which states that

> We can change the order of the addends, and the sum is the same.
>
> Examples:
>
> $6 + 8 = 8 + 6$
>
> $175 + 26 = 26 + 175$

In general, if a and b are whole numbers, then

$$a + b = b + a$$

Another important principle that governs the addition of whole numbers is the *associative* principle. Since addition is an operation on two numbers at a time (a *binary* operation), we have an option concerning the grouping when three numbers such as 6, 4, and 5 are to be added.

The essence of the associative principle (often called *the grouping principle*) is that the final sum is the same, no matter how the numbers are grouped.

The associative principle for the addition of whole numbers states that

> We can change the grouping of the addends, and the sum is the same.
>
> Examples:
>
> $(6 + 4) + 5 = 6 + (4 + 5)$
>
> $31 + (7 + 16) = (31 + 7) + 16$

In general, if a, b, and c are whole numbers, then

$$a + (b + c) = (a + b) + c.$$

Other important principles are easily introduced in the course of normal development. From the beginning, it is intuitively obvious to children that for every pair of whole numbers there is *exactly one whole* number that is their sum. Formally stated this property is referred to as the *closure principle*. The child may not think about it, but certainly he feels that when any two numbers are added there can be only one correct answer.

Children quickly discover the special additive property of zero through carefully planned sequences of exercises. A formal statement of the *zero principle* follows:

> For each whole number a,
>
> $a + 0 = 0 + a = a.$

Children may express the principle this way: "If I add zero to a number, I get that same number."

Rearranging addends is very important in developing the addition algorithm. By utilizing a generalization of the order (commutative) and grouping (associative) principles for addition, we can show the following steps for 2-digit addition.

$$\begin{aligned} 23 + 64 &= (20 + 3) + (60 + 4) \\ &= (20 + 60) + (3 + 4) \\ &= 80 + 7 \\ &= 87 \end{aligned}$$

Consider the mathematics of this simple exercise in regrouping.

$$\begin{array}{r} 28 \\ +64 \\ \hline 92 \end{array} \qquad \begin{aligned} 28 + 64 &= (20 + 8) + (60 + 4) \\ &= (20 + 60) + (8 + 4) \\ &= 80 + 12 \\ &= 80 + (10 + 2) \\ &= (80 + 10) + 2 \\ &= 90 + 2 \\ &= 92 \end{aligned}$$

Until we get to the expression 80 + 12, we use the same procedures as we do for sums like 23 + 64. For 80 + 12, we write 12 as 10 + 2 and then regroup to complete the problem.

An intermediate step between the preceding kind of display and a short form for adding two 2-digit numbers may be helpful. The following example shows how the two processes are related, and it also shows how to explain the algorithm to children. Italic numerals show the relation between the processes.

Intermediate Step	*Algorithm*
$\begin{array}{r} 28 \\ +64 \\ \hline 12 \\ 80 \\ \hline 92 \end{array}$	$\begin{array}{r} {}^{1}28 \\ +64 \\ \hline 92 \end{array}$

We rely heavily on the relationship between addition and subtraction in developing subtraction of 2-digit numbers. By pointing out that finding a difference is the same as finding a missing addend, we present the idea that we can subtract in columns because we can add in columns.

Consider the following example of

2-digit subtraction in which regrouping is unnecessary.

$$\begin{aligned} 75 - 23 &= (70 + 5) - (20 + 3) \\ &= [(70 + 5) - 20] - 3 \end{aligned}$$

We can see that in order to subtract 23, or $20 + 3$, we must subtract both the 20 and the 3. (See the second equation.) The equations illustrate why this situation is often confused with regrouping.

Now think through a subtraction exercise that *does* require regrouping.

$$\begin{array}{r} 34 \\ -\ 6 \\ \hline 28 \end{array} \qquad \begin{aligned} 34 - 6 &= (30 + 4) - 6 \\ &= (20 + 14) - 6 \\ &= 20 + (14 - 6) \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

In the first example, subtraction in the ones' column was possible without regrouping. However, regrouping is necessary in the second example, since 6 is greater than 4. So $30 + 4$ is regrouped as $20 + 14$, and then 6 is subtracted from 14.

Teaching the Chapter

Materials

Advertising circulars
Mail-order catalogues
Newspaper advertisements
Overhead projector (if available) and transparencies
Play coins and currency
Slips of paper, 5 by 10 centimetres, 7 per child

Vocabulary

add	matching lines
addend	minus
addition	number line
compare	order principle
difference	plus
dollar	regrouping
equals	set
equation	solution
greater than	solve
grouping principle	subtract
inequality	subtraction
less than	sum

In the earlier books, the children have often worked with manipulative materials to understand addition and subtraction concepts. Most 9- and 10-year-olds will be able to master the ideas and skills in this chapter without resorting to manipulative activities. However, some children might benefit from using materials such as cubes on page 34 to understand the regrouping used in subtraction. If cubes or number blocks are not available, use items such as cans or paper sacks of three or four different sizes for the different place values. You might choose to use some set demonstrations if you think the children are finding a specific concept particularly troublesome.

The children will be quite familiar with many words in the vocabulary list, and should be able to use them in the proper contexts as the need arises.

Lesson Schedule

Plan to cover this chapter in about four to five weeks; the time required will depend largely on the background and abilities of your children.

Evaluation of Progress

There are two important points to remember when you evaluate the children's progress in this chapter. Part of the chapter presents addition and subtraction; the other part develops an understanding of algorithms and skill in using them. Therefore, the two broad classifications to be evaluated are (1) understanding the concepts and (2) developing computational skills.

Evaluating the children's comprehension of the concepts can best be done on a day-to-day basis by noting how fully the children participate in and contribute to class discussions involving key ideas and discoveries. You might also use an interviewing technique: at convenient times see each child individually and ask him to "think aloud" through a few exercises for you. In this way you will be able to determine if the child is using the algorithm with understanding.

Do not expect all the children in your class to master completely the addition and subtraction algorithms presented in this chapter. The "Keeping in Touch" lessons, which are presented at intervals throughout the text, will give the children an opportunity to review the addition and subtraction algorithms and will also allow further opportunity to work with the children who have not yet attained these skills.

Resources for Active Learning

General Activities

Developmental Math Cards, "Nim," F42, Addison-Wesley

Early Number Multi-Group Lab, Teacher's Guide, Responsive Environments Corp.

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs," pp. 19–24, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Math Activity Cards, "Addition Slide Rule," B6, Macmillan. [Using number lines to solve addition and subtraction equations]

Mathex: Operations and Problems Solving No. 8, pp. 1–6 (pupil pages 1, 2, 6), Encyclopaedia Britannica Publications Ltd. [Four skill-drill games].

Nuffield Project: *Computation and Structure* 2, "Addition," pp. 2–18, Wiley. [Ideas for tables, problems, games and assignment cards]

Nuffield Project: *Computation and Structure* two, "The Operation of Addition," pp. 48–69, Wiley

Manipulative Devices

Abacus or Abacus Board (school supplier)
Cuisenaire Rods (Cuisenaire Co.)
Dienes Multibase Arithmetic Blocks (Herder and Herder)
Grid Kit (Sigma, Scott Scientific)
"Invicta" Math Balance (Math Media; Selective Educational Equipment)
Papy Minicomputer (Macmillan)
SEE Calculator (Selective Educational Equipment)
Unifix Math Lab Kit (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Commercial Games

Cover-up (Selective Educational Equipment)
Equations (Creative Publications)
TUF—based on a number sentence (Creative Publications; Cuisenaire Co.; TUF)

PAGES 58–59

Objective

Given related addition and subtraction equations, the child will be able to find differences by finding missing addends.

Preparation

Materials

slips of paper approximately 5 by 10 centimetres, 7 per child (Children may cut their own out of tablet paper folded into eighths.)

Most children will know the addition combinations through 18. However, if they do not, spend time in these early lessons to see that they are learned. A short, brisk review before each lesson would usually be appropriate, but it should not exceed five minutes. To prepare for the lesson on these pages, review the term *equation* with the children. Write on the chalkboard the symbols +, —, =, and ask the children what you have written and how these symbols are used. Then direct them to the text.

Investigation

Read the top of the text page with the children. Point out the equation $5 + 3 = 8$, as a sample of what they are to write. Then help the children in cutting out their slips of paper. This may be done easily by folding a piece of tablet or binder paper lengthwise in half and then widthwise twice. When the children have prepared their slips of paper, direct them to search for the equations independently. Stress the importance of recording each equation they find.

The possible equations follow.

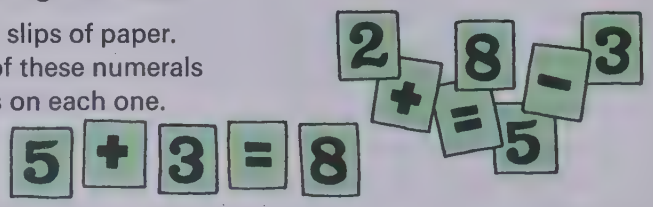
$2 + 3 = 5$ (or $3 + 2 = 5$)
$5 - 3 = 2$
$5 - 2 = 3$
$3 + 5 = 8$ (or $5 + 3 = 8$)
$8 - 5 = 3$
$8 - 3 = 5$

3 Addition and Subtraction

Are addition and subtraction related?

Investigating the Ideas

Cut out 7 slips of paper.
Put one of these numerals and signs on each one.



How many different equations can you "write" with your slips of paper?
See Investigation.

Record each equation you find.

Discussing the Ideas



1. A How many of these ▲? 2
 B How many of these ■? 4
 C How many shapes in all? 6
 D Can you give two addition and two subtraction equations for these three numbers?
 A A S A A S S A A S A A
 2+4=6 4+2=6 6-2=4 6-4=2

2. The Addends and Sum are marked in each equation. 4 is also called the difference of 7 and 3. Give the names for the numbers in the equations you found for exercise 1D.
 See above, exercise 1D.

 A A S
 4 + 3 = 7

 S A A
 7 − 3 = 4

3. Find the missing addend and difference.

You find this difference

8
n + 7 = 15

15 − 7 = n
8

when you find this addend.

Discussion

One of the main purposes of this lesson is to review the concepts of addition and subtraction and to study the relations between them. As you discuss parts A, B, and C of exercise 1, encourage children to talk about the process of putting things together to find the total. Also, encourage discussion of the subtraction process by saying something like this: "If we know that the total of red and blue objects is 6 and the number of red objects is 2, how do we find the number of blue objects?" You might find it helpful to use demonstration materials to show addition and subtraction and have children write the related equations.

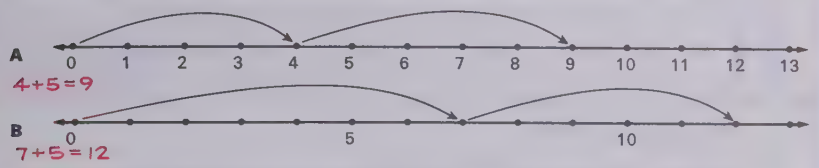
As you discuss exercise 2, use examples to call attention to the terms *addend*, *sum*, and *difference*. Similarly, for exercise 3, give the children other examples until they see the relation between addition and subtraction and realize that they can find the difference by thinking of a missing addend. During the discussion, include examples written in vertical notation.

Using the Ideas

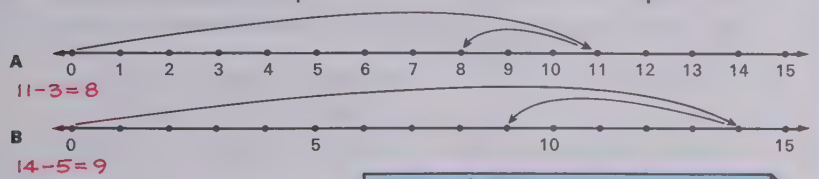
1. Find the differences by finding the missing addends.

A	$n + 5 = 9$ 4 $9 - 5 = n$ 4	E	$n + 6 = 15$ 9 $15 - 6 = n$ 9	I	$n + 7 = 16$ 9 $16 - 7 = n$ 9
B	$n + 6 = 11$ 5 $11 - 6 = n$ 5	F	$n + 8 = 17$ 9 $17 - 8 = n$ 9	J	$n + 9 = 16$ 7 $16 - 9 = n$ 7
C	$n + 8 = 13$ 5 $13 - 8 = n$ 5	G	$n + 5 = 12$ 7 $12 - 5 = n$ 7	K	$n + 8 = 14$ 6 $14 - 8 = n$ 6
D	$n + 9 = 15$ 6 $15 - 9 = n$ 6	H	$n + 9 = 18$ 9 $18 - 9 = n$ 9	L	$n + 6 = 13$ 7 $13 - 6 = n$ 7

2. Write an addition equation for each number-line picture.

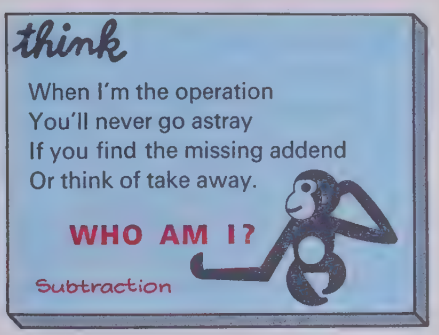


3. Write the subtraction equation for each number-line picture.



4. One exercise below has no whole-number answer. Find it. Then find the differences for the others.

A	14 −8 6	B	13 −5 8	C	14 −6 8
D	13 −0 13	E	7 −9 No whole-number answer	F	12 −7 5





Using the Exercises

Before assigning the exercises on page 59, emphasize for the children that they can find differences by thinking about missing addends. Use examples in exercises 2 and 3 to review the use of the number line for addition and subtraction. Note that exercise 4E has no whole-number answer.

It would be helpful to accompany this page with oral and/or written practice on addition and subtraction facts. More practice will follow in subsequent lessons, but you should begin here to sharpen "fact skills" even though in this lesson the addition and subtraction relations and concepts are the chief apparent concern.

Assignments (page 59)

Minimum: 1–3. Average 1–4.
Maximum: 1–4.

Mathematics

Although addition may be defined as the union of two disjoint sets and subtraction may be defined in terms of set, subset, and the complement of a set, this terminology is not important for the fourth grader.

Besides reviewing addition and subtraction concepts, this lesson stresses the point that finding a difference is equivalent to finding a missing addend. Such an idea directly relates to the meaning or definition of subtraction. That is, when we define subtraction, we define the number $a - b$ as the number that adds to b to give a. Thus,

$$(a - b) + b = a.$$

Therefore, to find the difference $a - b$, we find the number that added to b gives a.

This approach to subtraction eliminates the need for children to memorize subtraction combinations. If children have a thorough knowledge of addition combinations, they can arrive quickly at given differences. Later, this same method is used again when quotients are treated as missing factors.

Follow-up

To help the children evaluate their skill with addition facts, give them a timed test on which they need write only the answers. Use a duplicated sheet with four small addition tables, or with a single table such as the one illustrated.

+	5	2	9	0	3	6	8	1	7	4
4										
7										
1										
8										
6										
3										
0										
9										
2										
5										

Resources for Active Learning

Developmental Math Cards, G42, Addison-Wesley. [A game to review previously learned facts]

Workbook, page 15

PAGES 60–61

Objective

Given addition and subtraction word problems involving only basic number facts, the child will be able to identify the appropriate operation and solve the problems.

Preparation

So that the children will be able to write problems like those suggested in the investigation, you might present a review of oral word problems. For example, make up two or three oral problems and have the children solve them, writing an equation for each. The problems should be simply stated, thus:

> 13 children sat in two rows. 7 were in one row. How many in the other?

Following this, direct the children to the investigation, for which they will write problems of their own.

Investigation

In this investigation, children are to study the illustrations and write a word problem for each. Then, before solving the problems they write, they should first write an equation for each. It would be best to have the children work independently, waiting until the discussion section to develop the various possibilities.

For example:

A. Bob lives 7 blocks from school. Ted lives 6 blocks farther from school than Bob does. How far from school does Ted live? $7 + 6 = 13$

B. A boy sawed off 1 metre from a board 4 metres long. How long was the remaining piece of board? $4 - 1 = 3$

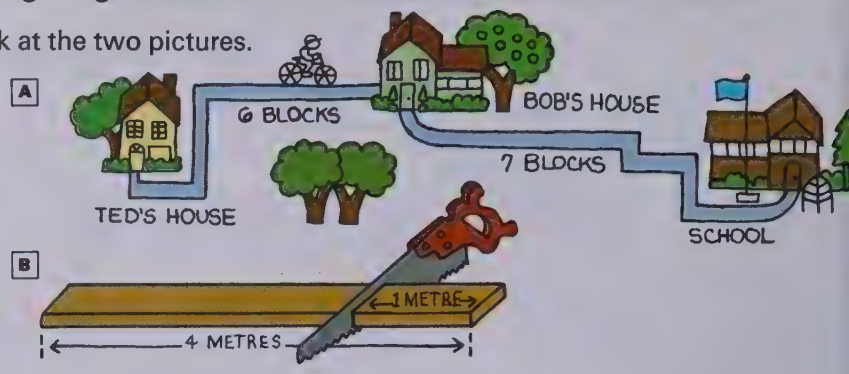
Encourage children to write *short* problems; the stress should be on identifying an additive and subtractive situation, not on writing stories.



Can you write and solve problems?

Investigating the Ideas

Look at the two pictures.



? Can you write and solve an addition problem for picture A and a subtraction problem for picture B ? See Investigation.

Discussing the Ideas

1. The numbers in these problems are missing. Explain how you would find the answers if the numbers were given.

 A Had ▮ apples. Ate ▮ of them. How many left? Subtract

 B Had ▮ cents. Earned ▮ cents more. How much now? Add

 C ▮ boys. ▮ girls. How many more boys than girls? Subtract

 D Spent ▮ cents. Have ▮ cents left. Had how much to start? Add

 E Won ▮ games. Lost ▮ games. No ties. Played how many games? Add

 F Jim is ▮ centimetres tall. Tom is ▮ centimetres tall. How much taller is Jim? Subtract

2. Make up a problem of your own. Can you solve your own problem? Problems will vary. See Discussion.

60

Discussion

Have several children read the problems they wrote and then write their equations on the chalkboard. Accept any correct variations. For picture B, for example, $4 - 1 = n$, $n + 1 = 4$, and $1 + n = 4$ are all acceptable (see the note in the Mathematics section of this lesson) even though, as written, two of them appear as addition equations.

Discussion exercise 1 presents examples which concentrate on problem-solving skills and not computational skills. It is important for the child to interpret the physical situation from the word problem and recognize it as being either additive or subtractive. After the physical situation of each problem is discussed, suggest numbers for the problems and have children write and solve the equations.

As children work exercise 2, some may use numbers beyond their computational skill. They should be able to write an equation to describe the problem, but it is not necessary that they be able to compute the answer.

Short Story Problems

1 6 boys in the game. 8 boys on the bench. How many boys? 14



2 13 slow turtles. 7 slow snails. How many more turtles than snails? 6

3 16 papers. Delivered 7. How many more to be delivered? 9



4 Magician has 12 rabbits. 7 disappear. How many remain? 5

5 15 dogs. 7 tails. How many dogs have no tails? 8

6 6 hockey cards. Need 15 in all. How many more needed? 9



7 13 jungle elephants. 7 taken to the zoo. How many left? 6

8 9 girls with dolls. 5 girls without dolls. How many girls? 14

9 5 eggs. How many more needed to make a dozen? 7





10 16 cowboys. 9 horses. How many more cowboys than horses? 7

12 14 girls. 6 go home. How many left? 8

11 4 cows. 8 horses. How many animals? 12



13 17 chickens. 9 eggs. How many more chickens than eggs? 8

14 14 kites in the air. 9 boys each flying one kite. How many kites got away? 5

★15 17 airplanes on the ground. 9 flew away. 6 others landed. How many planes now on the ground? 14

More practice, page A-6, Set 10



Using the Exercises

Note that all the problems on page 61 involve only the basic facts. Have the children work the exercises independently. When they have finished, allow time for discussion. Exercise 15 is a straightforward two-step problem and all the children would benefit from a discussion of it.

Assignments (page 61)

Minimum: Odd-numbered problems 1–13. Average: 1–14. Maximum: 1–15.

Mathematics

Because of the way subtraction is related to addition, one can come up with many different devices to trigger the subtraction operation. The following five examples are different situations which necessitate the subtraction operation.

1. We have a number, such as 8, and we want to know how much more it takes to get to another number, such as 17. That is, $8 + n = 17$.
2. We have a certain amount, and we add 6 to this amount and get 14. With how much did we start? Or, $n + 6 = 14$.
3. How much more is 16 than 7? This could trigger $7 + n = 16$, or simply $16 - 7 = n$. This, of course, is an example of the simple comparison idea in subtraction.
4. We start with 12, and 7 are taken away. How many are left? This indicates the equation $12 - 7 = n$. This is the "take away" idea of subtraction and is basically the simplest form of interpreting subtraction.
5. Starting with 16, some are taken away and 9 remain. How many were taken away? This prompts the equation $16 - n = 9$.

The point we wish to make in describing these five ways to trigger the subtraction operation is that, regardless of the equation indicated in these situations (note that in examples 1 and 2 addition equations are triggered), the same general idea can be used to solve any of these equations.

Follow-up

The children would benefit from an opportunity to make up several problems of their own. You might have them illustrate some problems for display on the bulletin board. In this case, encourage them to write the equation for the problem and its solution with the illustration. If they write short story problems, it would be appropriate to have them exchange papers and solve each other's problems.

PAGES 62–63

Objective

Given addition and subtraction exercises and problems using the basic facts, the child will be able to find the sums and differences and solve the problems.

Preparation

Spend three or four minutes reviewing basic facts with a brisk oral activity. For example, start a chain reaction game by asking a child for the sum of 6 and 8. When a child correctly responds "14," he in turn asks for a sum or a difference from another child, and so on. Remind the children that only combinations from the basic facts should be used. If you prefer to spend a longer time in oral review, begin immediately with the discussion and save the oral practice until the end of the lesson.

Do you know your facts?

1. Find the missing numbers in the tables.

	Add 5	
	6	11
	9	14
A	8	13
B	4	9

	Add 8	
	2	10
C	6	14
D	4	12
E	9	17

	Add 6	
	7	13
F	8	14
G	3	9
H	9	15

	Add 9	
I	2	11
J	5	14
K	8	17
L	7	16

2. Copy each addition table and give the missing numbers.

A

+	3	5
7	10	12
6	9	11

B

+	3	8
4	7	12
9	12	17

C

+	5	4
9	14	13
8	13	12

D

+	5	9
7	12	16
6	11	15

3. Find the sums and differences.

A 8 +5 = 13	B 10 −7 = 3	C 16 −8 = 8	D 11 −8 = 3	E 2 +8 = 10	F 13 −7 = 6	G 12 −9 = 3
H 15 −8 = 7	I 7 +9 = 16	J 15 −7 = 8	K 11 −3 = 8	L 13 −5 = 8	M 2 +7 = 9	N 11 −4 = 7
O 3 +9 = 12	P 14 −7 = 7	Q 18 −0 = 18	R 11 −5 = 6	S 16 −8 = 8	T 9 +9 = 18	U 14 −6 = 8

4. Give the missing numbers.

	Addend	Addend	Sum
	5	8	13
A	7	4	11
B	6	8	14
C	4	9	13
D	8	3	11
E	5	7	12

think

Walking on a Path

One girl in front of two girls.
One girl behind two girls.
One girl between two girls.
How many girls? 3



62

Discussion

Both of these pages give the children practice with the basic facts in addition and subtraction exercises.

Although the tables in exercise 1 differ slightly from those in exercise 2, children should be able to figure them out by studying the examples that are already completed. If necessary, explain one or two examples of each exercise before assigning the page as independent work.

When the children have finished and as you check their work, use these exercises to review the terms *sum*, *difference*, and *addends*.

All would benefit from a discussion of the challenging *Think* problem.

Assignments (page 62)

Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

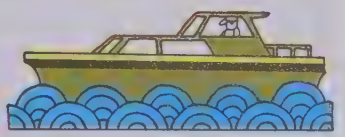
AT THE SERVICE STATION

See Using the Exercises.

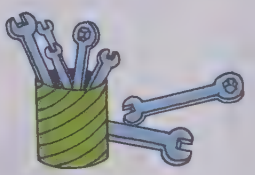


1. Mr. Blue greased 9 cars before lunch and 8 cars after lunch. How many cars did he grease? 17 (9+8=17)

2. A gas tank on a sports car holds 47 litres when full. 8 litres have been put in. How many more are needed to fill the tank? 39 (47-8=39)

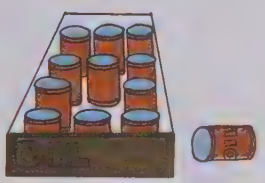


3. The gas tank on Mr. Brown's boat holds 54 litres. When 6 litres were put in, the tank was full. How many litres were already in the tank? 48 (54-6=48)



4. Mr. Blue had a set of 7 wrenches. He bought a new set of 16 wrenches. How many more wrenches are in the new set than in the old? 9 (16-7=9)

5. On Friday 5 tires were sold, and on Saturday 9 tires were sold. How many tires were sold on these two days? 14 (5+9=14)



6. There are 12 cans of oil in a box. 7 cans are sold. How many cans are left? 5 (12-7=5)

7. There are 16 cans of auto polish in a box. Some are sold. 9 cans are left. How many cans were sold? 7 (16-9=7)

More practice, page A-7, Set 11

Resources for Active Learning

Developmental Math Cards, F15, Addison-Wesley. [A riddle]

Discovery, Section II, Unit 3/3, Encyclopaedia Britannica Educational Corp. [A "difference" game]

Franklin Series: *Patterns and Puzzles*, "Magic Stars," pp. 27–29, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Nuffield Project: *Problems*—Green Set, No. 50, Wiley.

Duplicator Masters, pages 8–10
Workbook, page 16
Skill Masters, pages, 8, 9

Using the Exercises

Assign the problems on page 63, suggesting that children write an equation for each. Allow time for a thorough discussion of each problem when they have completed the page.

Do not make an issue of the various devices for the subtraction operations that are illustrated on this page. Rather, be sure the children are aware of the flexibility in equations that might be written for various problems. For example, in problem 2 the children should understand that it is perfectly acceptable for them to write $n + 8 = 17$ or to write $17 - 8 = n$. Notice with them that no matter which equation they use, they must think about the same thing in order to find the correct answer.

Assignments (page 63)

Minimum: 1–7, oral. Average: 1–7. Maximum: 1–7.

PAGES 64–65

Objective

Given any two of the three function machine components—input number, function rule, or output number—the child will be able to give the third component.

Preparation

If children have used the preceding book in this series they will probably recall the function game. For adaptations of it, see the follow-up section. With some groups of children, you might want to work through a sample function game.

Investigation

For this investigation, it would be suitable to have the children work in groups of three or four. Suggest that they first read together the description of the game as Kay and Paul were playing it, and try to figure out Paul's rule. Then they should each think of a rule and respond to numbers their group members give them. Move around the room to make sure that they understand how to play the game and have correctly figured out Paul's rule. Also, some may need suggestions for inventing a rule of their own.

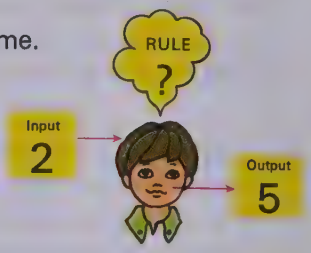
It would also be possible to read together with the class the description at the top of the page and agree on Paul's rule before the children try to invent rules of their own. You might then have them work in groups as mentioned above.



How does the function machine work?

Investigating the Ideas

Kay and Paul were playing the function game.
When Kay said 2, Paul answered 5.
When Kay said 3, Paul answered 7.
When Kay said 5, Paul answered 11.



When Kay said 6, what do you think Paul answered? 13

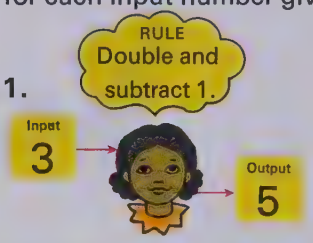
What is Paul's rule? Double and add 1

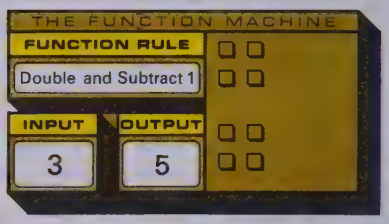
? Can you invent a rule and play the function game with some of your classmates? See Investigation.

Discussing the Ideas

The function game helps you understand the **function machine.** Study the pictures to see how it works. Give the output number for each input number given.

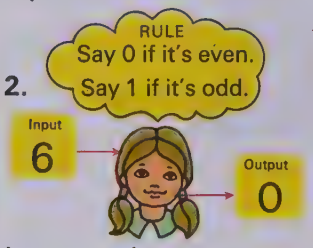
1.

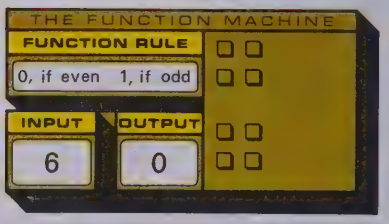




Input numbers → A 23 B 59 C 815 D 1019 E 917

2.





Input numbers → A 20 B 40 C 51 D 371 E 1000

64

Discussion

One of the important points of this lesson is to present children with an intuitive introduction to the concept of function. Encourage children to talk about the function machine. Work through exercises 1 and 2 together. It would be helpful to review even and odd numbers as you begin exercise 2. Remind the children that an even number is two times a whole number and always ends in 0, 2, 4, 6, or 8, and that an odd number is one more than an even number and always ends in 1, 3, 5, 7, or 9.

Using the Ideas

Think about the function machine and tell what you think should go in each gray space.

1. Function Rule: Add 7

	Input	Output
A	4	11
B	7	14
C	9	16
D	5	12

2. Function Rule: Double

	Input	Output
A	2	4
	3	6
	5	10
B	9	18
C		8

3. Function Rule: Subtract 8

	Input	Output
A	16	8
	12	4
	8	0
B	13	5
C	17	9

4. Function Rule: Subtract 9

	Input	Output
A	10	1
	18	9
	12	3
B	16	7
C		6

5. Function Rule: Add 6

	Input	Output
A	4	10
	7	13
B	9	15
C		14
D	5	11

6. Function Rule: Subtract 5

	Input	Output
A	10	5
	7	2
B	14	9
C	5	0
D	8	13

★**7.** Function Rule: Add 1 and double

	Input	Output
A	2	6
	5	12
	3	8
	9	20
B	7	16

★**8.** Function Rule: Add 7 if odd, Add 8 if even

	Input	Output
A	5	12
B	6	14
C	7	14
D	8	16
E	9	16

★**9.** Function Rule: 7 or less: add 8; > 7: subt. 8

	Input	Output
A	4	12
B	15	7
C	10	2
D	7	15
E	17	9

65



Using the Exercises

Before assigning the exercises on page 65 as independent work, make sure that the children understand the relationship between these tables and the function machine exercises on page 64.

Although exercises 7, 8, and 9 are primarily for the faster children, all will benefit from a discussion of them, and should understand them once correct answers are given.

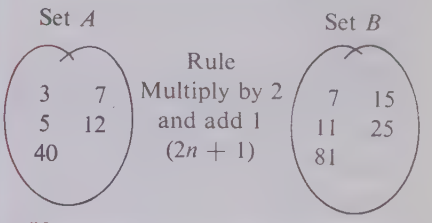
Suggest that those who finish these exercises quickly make up function rules and partially completed tables of their own. They may then exchange these tables among themselves and try to complete them.

Assignments (page 65)

Minimum: 1–6. Average: 1–6.
Maximum: 1–9.

Mathematics

The concept of a function is one of the most important ideas in mathematics, yet it can be presented on an intuitive basis very early in a child's experiences. Rather than present a precise mathematical definition of function, we give examples and point to some significant features.



If we take a number from set A, and apply the rule, we get exactly one number in set B. Hence, we get the set of pairs, (3, 7), (7,15), (5,11), (12,25), and (40, 81). One essential feature of these pairs is that for each first number, there is exactly one second number.

It is important for you, the teacher, to recognize that the function rules in this lesson are by no means unique. Each table leads the children to a particular rule. However, other rules are possible, and it is not improbable that some child will invent a rule that works for a given table of values but is different from the one suggested in the answers.

Follow-up

The function game or "What's My Rule" game may be played in a variety of ways. You may wish to record the answers in tables such as in the text or on a grid. For example, draw a grid on the chalkboard, filling in one row, row *a*, with numbers 10 or less. Form row *b* by using a rule on the numbers in row *a*. Fill in enough numbers to enable children to discover the rule and supply the missing numbers

a	3	1	4	6	5	7	8	2	9
b	12	10	13	15	?	?	?	?	?

Resources for Active Learning

Franklin Series: *Patterns and Puzzles*, "Switch 'Em," pp. 24–25, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, "The Switcheroo Function," D44, Macmillan.

Workbook, page 17

PAGES 66–67

Objective

Given 2, 3, and 4 addends, the child will be able to recognize and apply the following basic principles of addition: order principle, zero principle, rearranging principle, and grouping principle.

Preparation

Materials

slips of paper approximately 5 by 10 centimetres, 7 per child (Children may cut their own out of tablet paper folded into eighths.)

Unless you prefer to begin with a brief oral review, it would be suitable to begin this lesson immediately with the investigation.

Investigation

Unless you have slips of paper previously cut and ready to distribute, guide the children in folding and cutting tablet paper into eighths. Instruct them to work independently in writing their equations. Remember them to record their equations carefully. There are six different equations for the children to find using 3 addends:

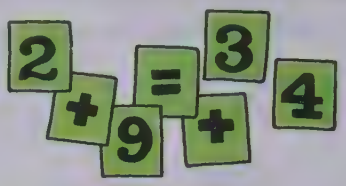
$2+3+4=9$ $\quad 3+4+2=9$
$2+4+3=9$ $\quad 4+3+2=9$
$3+2+4=9$ $\quad 4+2+3=9$

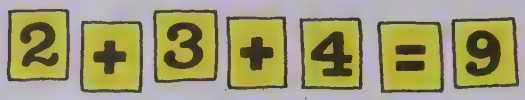


What are the basic principles for addition?

Investigating the Ideas

Cut out 7 slips of paper.
Put one of these numerals or one of these signs on each one.





? How many different equations with 3 addends can you "write" with your slips of paper?
See Investigation.

Record each equation you find.

Discussing the Ideas

1. Solve these equations.
 A $5+3=n$ 8 B $3+5=n$ 8 C $6+4=n$ 10 D $4+6=n$ 10
2. Can you change the order of two addends and get the same sum? Yes
3. Solve these equations. Add the shaded numbers first.
 A $(2+3)+1=n$ 6 $\quad 2+(3+1)=n$ 6
 B $2+(5+4)=n$ 11 $\quad (2+5)+4=n$ 11
4. Can you change the grouping of addends and get the same sum? Yes
5. When you add three or more numbers, can you **rearrange** them in any way you wish and get the same sum? Yes
6. What is the result when you add zero to any number?
 The number to which you added zero.
7. Explain each of these principles in your own words.
 Order principle See ex. 2. Grouping principle See ex. 4.
 Rearrangement principle See ex. 5. Zero principle See ex. 6.

66

Discussion

One of the main points of this lesson is to show how the order principle and the grouping principle may be combined into a "rearrangement" principle whereby the sum of the addends does not depend upon the arrangement of the addends.

In working through the discussion exercises with the children, you may find it worthwhile to extend exercises 1 and 3 by presenting other equation pairs that exemplify the order principle and the grouping principle, before children encounter the generalization of each in exercises 2 and 4. In exercise 5, be sure the children understand that the term *rearrange* can mean either to change the order or the grouping or both. Provide specific examples of equations with addends of 0 in connection with exercise 6.

Using the Ideas

1. Each exercise suggests an example of the order principle. Give the example.

A 

(Answer: $4 + 2 = 2 + 4$)

B Start at 9. Count forward 4. Start at 4. Count forward 9.

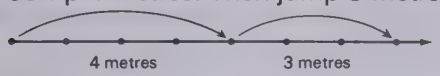
$9+4=4+9$

C Jump 3 metres. Then jump 4 metres.



$3+4=4+3$

Jump 4 metres. Then jump 3 metres.





2. In each exercise, the two sums are the same. To find the sum, use the grouping that is easier for you.

A $(2 + 8) + 7$ 17
$2 + (8 + 7)$

B $(9 + 7) + 3$ 19
$9 + (7 + 3)$

C $(99 + 1) + 17$ 117
$99 + (1 + 17)$

3. For each part of exercises A and B, add the red numbers first. Then find the sum. Is each sum the same in A ? in B ? Yes

A $3 + 4 + 5$ 12
$3 + 4 + 5$
$3 + 4 + 5$

B $2 + 5 + 4 + 6$ 17
$2 + 5 + 4 + 6$
$2 + 5 + 4 + 6$

4. Solve the equations.

A $9 + 0 = n$ 9
B $0 + 56 = n$ 56
C $0 + 99 = n$ 99
D $784 + 0 = n$ 784

5. Find the sums. Look for ten.

A $2 + 7 + 8$ 17
B $3 + 9 + 1$ 13
C $56 + 9 + 1$ 66
D $5 + 20 + 5$ 30
E $6 + 88 + 4$ 98
F $75 + 5 + 5$ 85

think

For each exercise, give the pair of numbers that should go in the gray spaces.

	Sum	+ −	Difference	
	15	9 +	6	3
	7	5 +	2	3
A	12	8 +	4	4
B	15	8 +	7	1
C	18	9 +	9	0
D	80	50 +	30	20



Using the Exercises

If necessary, you may use exercise 1 as a basis for further discussion of the order principle, and exercise 2 as a basis for further discussion of the grouping principle. Assign the remaining exercises as independent work.

Encourage many children to try the *Think* problem. They may need an explanation of the chart and of the addition and subtraction symbols on it. After they have been given an opportunity to try the *Think*, draw one of these tables on the chalkboard and conduct a class discussion. The children must look for a pair of numbers whose sum will be a certain number and whose difference will be a certain number, according to the table entries. If several children explain how they arrive at a correct answer for a given part of this exercise, it may encourage those who were unsuccessful to attempt another part of the exercise.

Assignments (page 67)

Minimum: 1–4. Average: 1–5.
Maximum: 1–5.

Mathematics

The commutative, or order, principle for addition is stated as follows.

For any whole numbers a and b,
$$a + b = b + a.$$

The associative, or grouping, principle for addition is stated:

For any whole numbers a, b, and c,
$$(a + b) + c = a + (b + c).$$

The fact that we can change the order and the grouping of numbers in addition permits any rearrangement that is convenient for three or more addends.

Since three numbers can be ordered six ways and since each of these orderings can be grouped in two ways, three addends such as a, b, and c can be ordered and grouped in 12 ways:

$(a + b) + c$ $(c + a) + b$ $(b + c) + a$
$a + (b + c)$ $c + (a + b)$ $b + (c + a)$
$a + (c + b)$ $c + (b + a)$ $b + (a + c)$
$(a + c) + b$ $(c + b) + a$ $(b + a) + c$

We can proceed down this list and verify the equality of these expressions by alternately applying the grouping and order principles for addition. Even without this complete list, we can prove the equality of any two of these expressions by using the order and grouping principles. To show that $c + (b + a) = b + (c + a)$, we offer two proofs.

First proof:

$c + (b + a) = (c + b) + a$ grouping
$= (b + c) + a$ order
$= b + (c + a)$ grouping

Second proof:

$c + (b + a) = (b + a) + c$ order
$= b + (a + c)$ grouping
$= b + (c + a)$ order

Another important idea in this lesson is that the grouping principle allows the omission of parentheses from expressions involving the sum of three numbers. Since we can group the numbers in any convenient way, the expression will be just as meaningful without parentheses (we will get the same sum regardless of how we consider the grouping).

Resources for Active Learning

Developmental Math Cards, F5 10, G5 9, G5 19, G5 10, Addison-Wesley.

Duplicator Masters, page 11
Workbook, page 18
Skill Masters, page 11

PAGES 68–69

Objective

Given 2-digit addends which require regrouping, the child will be able to find the sum by using the addition algorithm to do the required regrouping.

Preparation

Use a short oral drill to prepare for this lesson. For example, review the basic facts to 18 by using the "What's My Rule" game described on page 65, or quickly call out facts and ask the children to respond with the sums. Also, you might wish to present as review a few problems involving 2-digit addends where no regrouping is required. Though the exercises provided for this lesson contain enough such problems to meet review needs, the emphasis is on problems which do require regrouping. In all cases, keep the preparation activity short and lively.



Do you understand regrouping in addition?

Discussing the Ideas

1. Which of these sums are "in the 70's"? Which of these sums are "in the 80's"?

70's: 76 + 2, 76 + 3

80's: 76 + 4, 76 + 5, 76 + 6

2. Can you tell quickly, without actually finding the sum, which of the sums below are in the 70's and which are in the 80's?

77	74	75	79	72	76	78
+8	+1	+9	+2	+6	+3	+3
80's	70's	80's	80's	70's	70's	80's

3. Solve and explain each equation.
 - A Since 8 + 4 = 12, we know 8 + 14 = *n*. 22
 - B Since 9 + 5 = 14, we know 9 + 25 = *n*. 34
 - C Since 7 + 6 = 13, we know 7 + 56 = *n*. 63

4. Explain each step in the example below.
See Discussion.

Step 1	Step 2
$\begin{array}{r} {\scriptstyle 1} \\ 58 \\ +\ 36 \\ \hline 4 \end{array}$	$\begin{array}{r} {\scriptstyle 1} \\ 58 \\ +\ 36 \\ \hline 94 \end{array}$
8 + 6 = 14	10 + 50 + 30 = 90

Discussion

Allow sufficient time for a thorough discussion of each exercise. Have the children quietly study an exercise before discussing it. In exercises 1 and 2, help the children to realize that the sums are in the "80's" only when the sum of the addends in the ones' column is 10 or greater than 10. Exercise 3 gives the children an opportunity to reason from a known addition fact to an unknown fact.

Exercise 4 presents the traditional addition algorithm. According to the needs of your class, you may want to present a longer algorithm first. In such a case, explain the steps of the following algorithm:

Step 1	Step 2	Step 3
58	58	58
+36	+36	+36
14	14	14
	80	80
		94
8+6=14	50+30=80	14+80=94

Some children may need to depend on this method, but most should be encouraged to use the shorter algorithm. Present other examples of the algorithm as it is shown in exercise 4, and have children explain each step.

Using the Ideas

1. Find the sums.

A 55 +34 = 89	B 27 +60 = 87	C 63 +25 = 88	D 51 +76 = 127	E 34 +85 = 119	F 63 +54 = 117
G 45 +79 = 124	H 88 +66 = 154	I 63 +49 = 112	J 74 +58 = 132	K 85 +67 = 152	L 96 +76 = 172
M 91 +19 = 110	N 34 +87 = 121	O 43 +99 = 142	P 55 +88 = 143	Q 56 +89 = 145	R 99 +89 = 188

2. Find the sums.

A 11 23 +35 = 69	B 67 20 +31 = 118	C 72 31 +34 = 137
D 42 20 +86 = 148	E 30 19 +24 = 73	F 26 37 +12 = 75
G 37 16 +28 = 81	H 28 27 +26 = 81	I 67 27 +58 = 152

think

On your paper, draw a figure like the one below. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so the sum along any line is 21.



3. Solve the equations.

A 54 + 37 + 28 + 9 = *n* 128	D *n* = 64 + 9 + 78 + 8 159
B 4 + 17 + 36 + 8 = *n* 65	E *n* = 85 + 9 + 7 + 39 140
C *n* = 76 + 58 + 29 + 7 170	F 76 + 84 + 7 + 3 = *n* 170

4. Copy each problem. Give the missing digit for each ▮.

A 4▮2 +▮53 = 77	B 8▮▮3 +45 = 128	C 5▮9 +35 = 94	★ D 6▮7 +4▮4 = 111	★ E 56 8+▮▮9 = 145	★ F 7▮6 +▮76 = 143

More practice, page A-7, Set 12



Using the Exercises

Assign the exercises on page 69 carefully. Consider asking slower children to do only alternate rows so that they can try each type of problem without bogging down. Notice that exercise 4 is the reconstruction type in which the children must find certain missing digits in addition problems. Parts D, E, and F are starred, and they are intended, along with the *Think* problem, for more capable children. Because this material is difficult, not all your children will be successful with it, but most should benefit from an explanation of the strategies used by those who do solve these problems.

One strategy for solving the *Think* problem would be to consider that the "middle" number of any three which add to 21 would be 7, so 7 should be written in the centre circle. This leaves 14 for the sum of the two numbers opposite each other. The smallest in the set, 2, and the largest, 12, total 14, so 2 and 12 are one of these pairs, and so on.

Assignments (page 69)

Minimum: 1. Average: 1–3.
Maximum: 1–4.

Follow-up

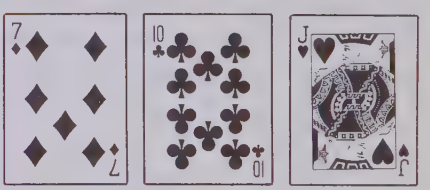
To give the children additional practice, prepare a worksheet similar to the following:

Find the missing digits.

▮4 +2▮ = 97	36 +▮▮ = 61	5▮7 +▮9▮ = 933	▮5 +2▮ = 90
6▮ +▮7 = 134	3▮ +▮7 = 95	▮9 +4▮ = 127	6▮ +▮7 = 94

To improve children's understanding of place value and regrouping, small groups might play the following game.

Have a deck of cards from which the jokers have been removed. Each of two, three, or four children should label place-value spaces in front of them on a piece of paper. Each child picks three cards from the deck placing them in order in the ones', tens', and hundreds' space. Each of the cards ace through ten has its numerical value and all face cards have the value of ten. The child must write a base-ten numeral to show what number his cards total. For example, the following arrangement would be written as 810.



(Notice the opportunity for practice in regrouping.) The child with the highest number after three hands wins.

Resources for Active Learning

Discovery, Section II, Unit 10/3, Encyclopaedia Britannica Educational Corp.

Mathematics in Modules, WN11, Addison-Wesley.

Nuffield Project: *Problems*—Green Set, No. 19, Wiley.

Duplicator Masters, page 12
Workbook, page 19
Skill Masters, page 12

PAGES 70–71

Objective

Given a magic square with sufficient addends shown, the child will be able to find the missing addends.

Preparation

It would be appropriate in this lesson to begin immediately with the investigation. However, if you prefer a warm-up activity, use a short oral drill of the basic facts. You might include sums of three addends, such as $8 + 7 + 9$, for which the child might think, "Eight plus seven equals 15; 15 plus nine equals 24."

You will find it quite helpful to provide the children with this type of oral practice from time to time throughout the school year. Most children enjoy this type of work more than written work. You will find that with some practice they become quite good at mental addition of three or four numbers.

Investigation

For this investigation, the children should work directly from the text. Make sure that they understand the significance of the arrows and realize how to find the sums. For those who finish quickly as well as for those who need other examples, write a few more magic squares on the chalkboard, such as the two below.

6	13	5
	8	
	3	

(Sum: 24)

	6	
12		
4	8	9

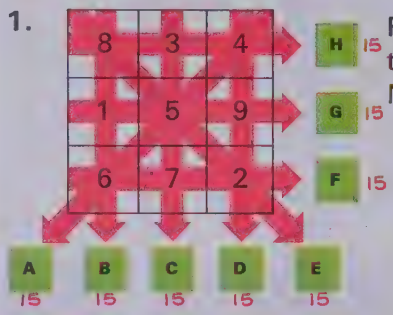
(Sum: 21)

Try to heighten the children's enthusiasm for the investigation by encouraging them to find out why a magic square is called "magic." Of course, the response should be that it is a magic square because the sum of the numbers in any row, column, or diagonal is the same.



What is a magic square?

Investigating the Ideas

1. 

Find the sums A through H to see why this is a MAGIC SQUARE.

2.

3	8	1
2	3	7
7	1	4

? The sums of the diagonal numbers are not the same as the sums of the rows and columns. Can you write a sentence that tells why 2 is not a MAGIC SQUARE?

Discussing the Ideas

1. Copy this figure. Add 2 to each number in the magic square above and put the sums in the same positions in your square.
 - A Do you still have a magic square? Yes
 - B What is the "magic sum"? 21

2. Find the sum along the colored arrow.
 - A What is the magic sum? 15
 - B Can you find A, B, and C so this will be a magic square?

4	9	2A
3	5B	7
8	1	6C

70

Discussion

Encourage children to share their reasons for believing that the squares in the investigation are or are not magic squares. The completed square in exercise 1 will be a magic square with the "magic sum" of 21. Carefully work through exercise 2 with the children. For example, you might show them how to add along the row in which all three addends are given to find the magic sum. Then help them see that they must add the two addends in a row and use that sum and the magic sum to find the missing addend. Note that the magic squares provide an excellent opportunity to practice addition and to find differences by thinking of missing addends.

Using the Ideas

1. A What is the magic sum? 18
B Find numbers for A, B, and C that make this a magic square.

5	B 8	C 5
6	6	6
A 7	4	7

2. Copy and complete each square to make it a magic square.

A

6	9	6
7	7	7
8	5	8

B

5	9	10
13	8	3
6	7	11

C

4	8	9
12	7	2
5	6	10

★ **3.** See if you can find the missing numbers that will make this a magic square.

18	4	5	15
7	13	12	10
11	9	8	14
6	16	17	3





71

Using the Exercises

Assign the exercises on page 71 as independent work. Help any children who have difficulty mastering the method of finding the missing addends. Those who finish quickly might enjoy making up magic squares of their own. If they find such a suggestion difficult, tell them that studying the diagonal rows might give them a hint on how to begin. Note that one of the diagonal rows always contains a consecutive sequence such as consecutive whole numbers, consecutive even numbers, or consecutive odd numbers. This is not true, however, of squares with an even number of rows.

Assignments (page 71)*
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Follow-up

Encourage children to make up magic squares of their own. If you have them shade the squares with crayon they would make an attractive display. Also it would be wise to have two or three children exchange magic squares and check each other's work before you put the squares up for display.

Resources for Active Learning

Experiments in Mathematics, Stage 1, pp. 50–51, Houghton Mifflin. (Available from Thomas Nelson & Sons)

Math Activity Cards, C40, D38, Macmillan.

Mathex: Operations and Problem Solving No. 8, pupil page 10, Encyclopaedia Britannica Publications Ltd.

Teaching Aids for Elementary Mathematics, pp. 102–108, Holt, Rinehart and Winston.

Workbook, page 20

PAGES 72–73

Objective

Given 3-, 4-, or 5-digit addends, the child will be able to find the sum by regrouping.

Preparation

To prepare for this lesson, plan a short warm-up activity to provide practice in the basic facts and in using the basic facts in adding tens and hundreds. For example, ask the children for sums of pairs such as $6 + 8$ and $60 + 80$, $7 + 9$ and $70 + 90$, $8 + 3$ and $800 + 300$, $7 + 6$ and $700 + 600$.

What is new about finding larger sums?

Discussing the Ideas

1. You can use what you know about adding 2-digit numbers to help you add 3-digit numbers. Study the example. Give the missing digits.

Adding ones	Adding tens	Adding hundreds
Step 1 1 5 6 4 + 3 8 9 3	Step 2 1 1 5 6 4 + 3 8 9 5 3	Step 3 1 1 5 6 4 + 3 8 9 9 5 3

Try these.

A
$$\begin{array}{r} 528 \\ +235 \\ \hline 763 \end{array}$$

B
$$\begin{array}{r} 785 \\ +368 \\ \hline 1153 \end{array}$$

2. Copy the example and give the missing digits.

Adding ones	Adding tens	Adding hundreds
Step 1 2 2 3 6 3 5 8 + 2 8 9 3	Step 2 1 2 2 3 6 3 5 8 + 2 8 9 8 3	Step 3 1 2 3 6 3 5 8 + 2 8 9 8 8 3

Try these.

A
$$\begin{array}{r} 399 \\ 730 \\ +854 \\ \hline 1983 \end{array}$$

B
$$\begin{array}{r} 425 \\ 693 \\ +747 \\ \hline 1865 \end{array}$$

3. Here is a way you might find sums mentally. Study the steps carefully.

Step 1	Step 2
5 8 + 3 4 6 2	5 8 + 3 4 6 2 9 2 ← sum
$58 + 4 = 62$	$62 + 30 = 92$

Try these.

A
$$\begin{array}{r} 27 \\ +35 \\ \hline 62 \end{array}$$

B
$$\begin{array}{r} 56 \\ +35 \\ \hline 91 \end{array}$$

C
$$\begin{array}{r} 47 \\ +69 \\ \hline 116 \end{array}$$

D
$$\begin{array}{r} 68 \\ +57 \\ \hline 125 \end{array}$$

E
$$\begin{array}{r} 466 \\ +58 \\ \hline 524 \end{array}$$

F
$$\begin{array}{r} 946 \\ +75 \\ \hline 1021 \end{array}$$



72

Discussion

Study the displayed algorithm in discussion exercise 1 with the children. If some of your children use the algorithm with partial sums, have them demonstrate how this method can be applied in the displayed example; that is:

$$\begin{array}{r} 564 \\ +389 \\ \hline 13 \\ 140 \\ +800 \\ \hline 953 \end{array}$$

Depending on the abilities and needs of the children, present other examples of the algorithm. Some children may still benefit from using the partial-sums method, but encourage most to learn the algorithm. Let the children work through parts A and B of exercise 1 on their own; if possible have several children work at the chalkboard. Let some of them explain the step-by-step procedures they use.

Then study with the children the column addition display in exercise 2 in the same manner as described above. Expand exercise 3 to include other examples, depending upon the children's needs.

With additional practice material you yourself prepare, you might want to treat this lesson as more than a single day's work.

Using the Ideas

1. Find the sums.

A 637 + 245 = 882	B 152 + 439 = 591	C 627 + 368 = 995	D 384 + 463 = 847	E 490 + 268 = 758
F 307 + 892 = 1199	G 626 + 184 = 810	H 939 + 384 = 1323	I 785 + 368 = 1153	J 897 + 106 = 1003
K 378 + 956 = 1334	L 625 + 785 = 1410	M 413 + 589 = 1002	N 652 + 73 = 725	O 92 + 658 = 750

2. Find the sums.

A 1789 + 7047 = 8836	B 3346 + 2478 = 5824	C 1567 + 9687 = 11 254	D 2027 + 8596 = 10 623	E 21 234 + 36 897 = 58 131

3. Find the sums.

A 243 + 572 + 146 = 961	B 568 + 57 + 223 = 848	C 68 + 322 + 417 = 807	D 635 + 444 + 511 = 1590	E 428 + 35 + 6 = 469
F 465 + 874 + 956 = 2295	G 526 + 204 + 789 = 1519	H 456 + 383 + 49 = 888	I 7764 + 6752 + 8568 = 23 084	J 3993 + 7306 + 8541 = 19 840

4. Solve the equations.

A $632 + 75 + 8 = n$ 715
B $68 + 232 + 40 = n$ 340
C $n = 9 + 6 + 34 + 346$ 395
D $n = 876 + 345 + 23$ 1244
E $693 + 27 + 8 = n$ 728
F $87 + 9 + 658 = n$ 754
G $n = 658 + 26 + 37 + 9$ 730
H $759 + 88 + 54 + 8 = n$ 909

think

Place 10 coins (or other discs) in a triangular shape.

Move only 3 coins and get



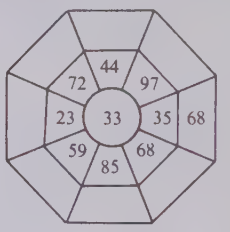
More practice, page A-8, Set 13

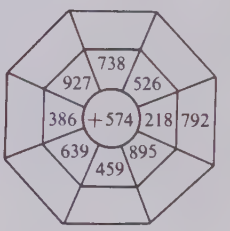


Follow-up

Prepare a duplicating master with six or eight of the following octagonal shapes. Write addends on six of them, leaving the outer ring blank, as in the illustration. Leave some of them completely blank so that the children can complete some practagons of their own design.

Find the sums.



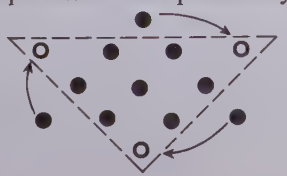


Duplicator Masters, pages 13–14
Workbook, page 21
Skill Masters, pages 13-14

Using the Exercises

Assign exercises 1 and 2 on page 73 as independent work. Exercise 3 might be developed in discussion, and other examples presented with it. However, if children understand regrouping with two 3-digit addends, the transition to column addition should not present much difficulty.

You may wish to give each of the children discs of paper or tagboard to manipulate to try to solve the *Think* problem. One possibility:



Assignments (page 73)
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

PAGES 74–75

Objective

Given a calorie chart and questions related to it, the child will be able to answer the questions by using data on the chart and finding the sums and differences.

Preparation

Because of the nature of the investigation in this lesson, it would be appropriate to proceed with the investigation without preparatory activity.

Investigation

Read the text at the top of page 74 with the children, encouraging discussion of these ideas. Then refer to the chart on page 75. Although this is simply a hypothetical menu for one day, children will find on it many items that they themselves eat. You might add to the chart in the text by writing the following data on the chalkboard or on a prepared duplicated worksheet.

Calorie Chart	
Hot dog	254
Hamburger	360
Peanut-butter and jelly sandwich	350
Hero or submarine sandwich	805
Pizza, one piece	245
Potato chips (8-10)	110
Orange	70
Banana	85
Chocolate bar	115
Brownie	120
Cola drink	107
Ice cream	167
Chocolate malted	450
Chocolate cake	356
Jelly doughnut	175
Apple pie	331
Hot fudge sundae	440
Banana split	1165

(If you desire a more complete chart, refer to "Nutrition," *World Book Encyclopedia*.) Note that the investigation question is aimed at individuals, so the children need not work in groups. However, free sharing of ideas might stimulate children to make up problems of their own.



How many calories are in the food you eat?

Investigating the Ideas

The food we eat gives us energy to work and play and keep our bodies warm. Some foods give us more energy than others.

Foods which supply more calories give us more energy. Nine- or ten-year-old girls and boys need about 2000 calories each day.



All the pizza you can eat



Your breakfast



All your meals in one day

? Can you find the number of calories you would get from each of the examples above or from a meal you choose?

Answers will vary. See Investigation.

Discussing the Ideas

1. Which activity do you think uses more calories? Running



running



reading

2. Who should eat more?
Football player



business man



football player

3. It takes 7700 "extra" (above what you need) calories to gain one kilogram. How many "extra" milkshakes (500 calories each) would you need to gain one kilogram? About 15

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Discussion

The children should try to guess the answers for discussion exercises 1 and 2 before you give them the following information:

Activity	*Calories used per kilogram per hour (approx.)*
running	7½
reading	1
playing football	8
businessman (desk job)	2

Exercise 3 may be worked in a variety of ways, and it would be good to let the children use their own methods in approaching the problem. For example, some may add 500 repeatedly until they reach 7700; others may subtract 500 repeatedly from 7700; still others may recall enough about division or missing factors from their work in Book 3 to apply these concepts here. All should benefit from a discussion of the various approaches that are used.

Using the Ideas

Jack made a chart to show his supply of calories for one day. Use his chart to answer these questions.

Caloric Chart

	Number of Calories
BREAKFAST	
Juice	85
Cereal	95
Strawberries	43
½ cup milk	85
Sugar	15
LUNCH	
Ham and Cheese sandwich	395
Lettuce Salad	48
Baked Beans	147
Apple	75
Milk, 1 glass	166
DINNER	
Round Steak	235
Hash Brown Potatoes	197
Peas and Carrots	60
Bread and Butter	157
Pumpkin Pie	198
Milk, 1 glass	166
SNACK	
Milk, 1 glass	166
2 Soda Crackers	45

1. How many calories did Jack get from
 - A his breakfast? 323
 - B his lunch? 831
 - C his dinner? 1013
2. How many calories did Jack get in all? 2378
3. How many calories did Jack get from milk and juice? 668
4. How many calories did Jack get from the sandwich and the pie? 593
5. Jack made this chart of his calories for one week.

SUN.	2607
MON.	2454
TUES.	2213
WEDS.	2378
THURS.	2185
FRI.	2732
SAT.	2578

 - A How many calories did Jack get during the 5 days of the school week? 11 962
 - B How many calories did he get on the weekend? 5185
 - C What was Jack's total calorie supply for the week? 17 147

★6. Jack needs 2000 calories a day to maintain his weight. If he gets about the same number of calories each week as in exercise 5, how many weeks does it take Jack to gain 1 kilogram? (Remember 7700 "extra" calories are needed to gain 1 kilogram.)
About 2½ weeks



Follow-up

As a follow-up for this lesson, ask the children to keep records of their own caloric intake for a given meal, day, or week. Or, ask them to choose a lunch or a day's meals and then write a set of problems based on the data they selected. The children might like to trade papers and do one another's exercises.

Duplicator Masters, page 15

Using the Exercises

You might choose to use page 75 as a basis for discussion. Read and study the calorie chart and the table directly below it.

Because of their exposure to public interest in dieting and calories, some children may have interesting comments to contribute during a discussion of the chart.

Following this discussion, give the children an opportunity to work the exercises. Although exercise 6 is starred, you should include all children in its discussion. When all the children have completed exercises 1 through 5, allow time for checking papers and for discussion.

Assignments (page 75)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–6.

PAGES 76–77

Objective

Given word problems involving money, the child will be able to solve the problems by adding and subtracting the numbers represented by the dollar and cent notation.

Preparation

Write the dollar symbol ($) on the chalkboard and ask children to explain how and when they use it. Have a few children write on the chalkboard an amount of money you tell them, and have the class read the amounts.

Investigation

Read the directions with the class. Suggest that they carefully record the pairs that they choose. Their possible choices are:

model and doll
baseball and doll
baseball and book
doll and record
doll and book

During investigations such as this, allow children the freedom to share ideas among themselves but stress the importance of *individual* achievement. Since some may finish more quickly than others, you might write this additional question on the chalkboard:

What pairs of gifts could you buy if you had $5.50? (besides the pairs listed above: model and book, model and baseball)

Is "adding" money like adding whole numbers?

Investigating the Ideas

Suppose you have five dollars to spend on two of these gifts.



? Can you list the different pairs of gifts you could buy?

See Investigation.

Discussing the Ideas

Explain how you can use these sums to give answers to these short money problems.

See Discussion.

324	922
+599	324
923	+607
	1853
514	
+869	
1383	

1. Had $3.24.
 Collected $5.99.
 Have how much now? $9.23

2. Electricity . . . $9.22
 Water $3.24
 Telephone . . . $6.07
 How much for all three? $18.53

3. Meat $5.14
 Other groceries . . $8.69
 How much in all? $13.83

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Discussion

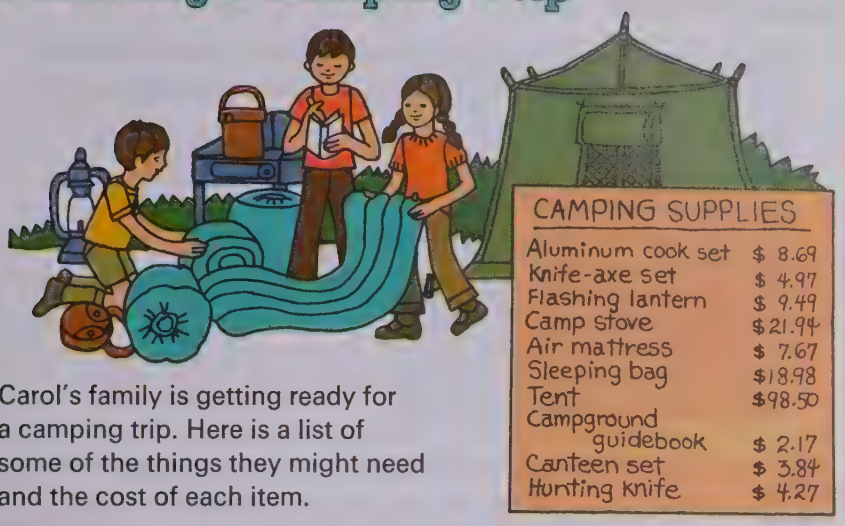
The main point of this lesson is to help children realize that adding numerals written in dollar and cent notation is analogous to adding whole numbers. Although the dollar and cent notation utilizes the concept of decimal fractions, the concept need not be discussed at this level. The exercises in this section illustrate how a number such as $2.75 may be thought of as 275¢. Thus, children should have little or no difficulty in adding numerals written in dollar and cent notation. Point out that a column addition may be written as illustrated at the right.

$9.22
3.24
6.07
$18.53

Note that the addition may be performed on it directly; it should be understood that each dollar and cent amount may be thought of as an amount of hundreds or thousands of cents. Call attention to the notation used with column addition; point out that the dollar sign is usually written only at the top of the column and in the answer.

Planning a Camping Trip

Using the Ideas



Carol's family is getting ready for a camping trip. Here is a list of some of the things they might need and the cost of each item.

1. Carol used part of her money to buy an air mattress and a campground guide book. How much did she spend? $9.84

2. Kent wants to earn enough money to buy a sleeping bag and a hunting knife. How much must he earn? $23.25

3. What is the total cost of the tent and the camp stove? $120.44

4. Alan has $11. How much more does he need to buy a flashing lantern and a knife-axe set? $3.46

5. A Each of the five members of the family bought a canteen set. What was the total cost? $19.20
 B Would they have paid more or less if they had bought two aluminum cook sets instead of the five canteen sets? Less
 How much more or less? $1.82 less

★ 6. Find the total cost of all the equipment listed. $180.52



Using the Exercises

Read with the children the material which introduces the word problems on page 77. Give them considerable freedom in talking about the various prices given and the kinds of supplies needed for a camping trip. If any of the children have had camping experiences, they might be able to add a great deal of interest to this discussion.

Following a discussion of the camping supplies, have the children work the exercises. Note that exercise 4 is a two-step problem. Exercise 5A may be solved by repeated addition, but some children may use multiplication. The solution to exercise 5B also requires a combination of steps.

When the children have finished the exercises, allow additional time for them to discuss camping and the problems presented here, as well as other problems which they might suggest.

Assignments (page 77)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–6.

Mathematics

One could maintain that the use of the decimal point in exercises in finding total amounts of money involves fractional-number concepts and a certain amount of study of fractional numbers. However, until fractional numbers are covered carefully, the children should be told simply that this decimal point is a dot separating the numerals which tell the number of dollars from the numerals which tell the number of cents. Of course, the observation that in the second, or tens', place we have the number of dimes and that ten pennies make a dime, and that in the third place we have the number of dollars and that ten dimes make a dollar draws an analogy between this situation and regular 3-digit problems. Hence, the regular addition algorithm is used to find the total amounts of money in this standard notation for dollars and cents, and nothing need be said to the children regarding the mathematical concepts involved in decimal notation.

Follow-up

Encourage children to make up problems of their own, using either the equipment suggested in the text or a list of their own. If possible, have available magazines which contain camping, fishing, or hunting ads, for the children to use while writing their problems.

Duplicator Masters, page 16
Workbook, page 22
Skill Masters, page 16

PAGES 78–79

Objective

Given the first 4 or 5 terms of a sequence which has a simple pattern, the child will be able to find the pattern and give the next four numbers of the sequence.

Preparation

If you wish to prepare for this lesson with an oral activity, give the children some skip-counting activities. For example, have them count by twos, threes, fours, and fives; or start with a number such as 17 and have them count by twos or threes. You might point out to the children that in these activities they are following a *pattern*.

Investigation

Read the directions with the children. Explain that they must first study the sequences in the text to see if they can find a pattern. Then allow ample time for the children to make up their own sequences before asking classmates to find the pattern. You might suggest to the mathematically creative children that they try to make up several sequences. If some children have difficulty finding the pattern in the sequences in the text, show them a simple sequence such as 3, 6, 9, 12, 15, 18 . . . and help them realize that each new number is the sum of the previous number and 3. Often children need considerable practice with patterns before they are able to recognize one on their own.



Let's explore sequences with patterns.

Investigating the Ideas

A sequence is a list of numbers. Some sequences have a pattern. Can you find a pattern and give the next four numbers for each of these sequences?

A 1, 7, 13, 19, 25, 31, 37, 43, . . .

B 2, 4, 8, 16, 32, 64, 128, 256, . . .

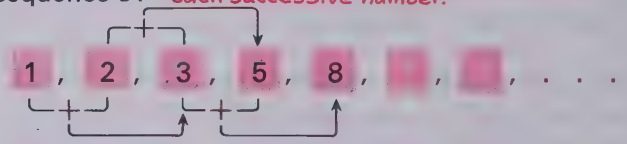
C 1, 2, 3, 5, 8, 13, 21, 34, 55, . . .

? Can you make up a sequence of your own and see if one of your classmates can find your pattern?

See Investigation.

Discussing the Ideas

1. Explain the pattern for sequence A.
 Add 6 to the preceding number.
2. Do you think the twentieth number in sequence B will be as large as 10 000? List as many numbers as you need to find out.
 See Discussion.
3. Can you explain how this diagram explains the pattern for sequence C? Add the two preceding numbers to get each successive number.



A Guess how many more numbers you would need to list in this sequence before you reached a number larger than 1000. 11 more (Guesses will vary.)

B List the numbers in the sequence until you reach a number larger than 1000. See Discussion.

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Discussion

Work through the discussion exercises carefully. For exercise 2, children will have to develop sequence B as follows: 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16 384.

They may be surprised to see that it only takes fourteen numbers to find a number greater than 10 000.

Similarly, in exercise 3, they may be surprised to find that they need list only sixteen numbers to surpass 1000: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597.

Encourage several children to present the sequences they made up in the investigation and have others try to find the pattern. Also, discuss the patterns displayed in the investigation section of the text. The rules for these may be stated simply: for part A, add 6; for part B, double the number; for part C, add the two preceding numbers.

Using the Ideas

1. Give the next four numbers in each sequence.

A 5, 10, 15, 20, . . . 25, 30, 35, 40

B 2, 4, 6, 8, . . . 10, 12, 14, 16

C 1, 0, 2, 0, 3, 0, . . . 4, 0, 5, 0

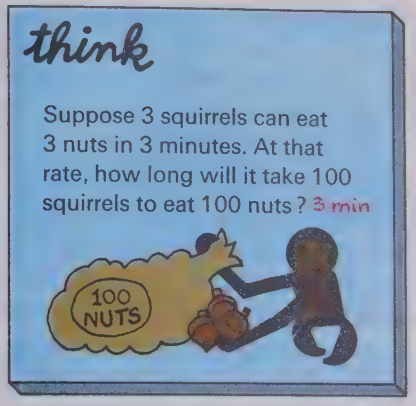
D 1, 3, 5, 7, . . . 9, 11, 13, 15

E 10, 100, 1000, 10 000; 100 000; 1 000 000; 10 000 000

F 12, 23, 34, 45, 56, 67, 78, 89

G 1, 34, 67, 100, . . . 133, 166, 199, 232

H 3, 1, 1, 6, 1, 1, 9, 1, 1, . . . 12, 1, 1, 15



2. Here are the first 10 numbers in the sequence of odd numbers.

1 3 5 7 9 11 13 15 17 19

Pick any four of the numbers in order (like 3, 5, 7, 9) and add them as shown in this diagram.

outer sum, 3 + 9 = 12

3, 5, 7, 9 (+ between 3 and 5, + between 7 and 9, + between 5 and 7)

inner sum, 5 + 7 = 12

Are the inner and outer sums the same for any four odd numbers in a row? Yes
Try three more examples.
Examples will vary.

★**3.** Sometimes the pattern of a sequence is hard to find. Can you find these patterns?

A 4, 3, 2, 8, 7, 6, 12, 11, . . . 10, 16, 15, 14

B 5, 5, 10, 15, 25, . . . 40, 65, 105, 170

C 1, 3, 6, 10, 15, 21, . . . 28, 36, 45, 55

D 1, 2, 6, 16, 44, . . . 120, 328, 896, 2448



Using the Exercises

Give the children an opportunity to try the exercises on page 79 on their own. The starred exercise is intended to challenge most children. The pattern in part D may be difficult for the class to find quickly, but do not tell them the rule. Let the more capable children work at it for short periods of time over a number of days before you give them the rule: Add each pair of adjacent numbers and multiply by 2 to get the following number.

When the children have finished exercises 1 and 2, carefully discuss the patterns for each sequence, particularly the interesting pattern for sums in odd-number sequences developed in exercise 2.

Assignments (page 79)*

Minimum: 1. Average: 1–2.
Maximum: 1–3.

Follow-up

Write some patterns on the chalkboard and see whether children can give the next four numbers and discover the rule used to build the sequence. Samples (rules are provided for your convenience):

\+ 3 + 3
1, 4, 7, 10, 13, . . .

\+ 2 + 3 + 2 + 3 + 2 . . .
1, 3, 6, 8, 11, 13, 16, . . .

\+ 1 + 2 + 1 + 2 + 1
1, 2, 4, 5, 7, 8, . . .

Resources for Active Learning

Teaching Aids for Elementary Mathematics, "Fibonacci numbers," pp. 36–37, Holt, Rinehart and Winston. [Botany and mathematics]

[The resources below deal with patterns and sequences.]

Developmental Math Cards, H²5, Addison-Wesley.

Freedom to Learn, p. 103, Addison-Wesley.

Math Activity Cards, D39, Macmillan.

Mathex: Operations and Problem Solving No. 8, pupil pages 7–9, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Green Set, No. 13, Wiley.

PAGES 80–81

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

If you choose to have a short warm-up activity to prepare for this lesson, write the numerals for several large numbers on the chalkboard. Then ask children to circle or underline certain digits; for example, say, "Circle the digit which represents 500" or "I'm thinking of the digit 4 in the thousands' place." You might also use pairs of these numerals in comparisons.

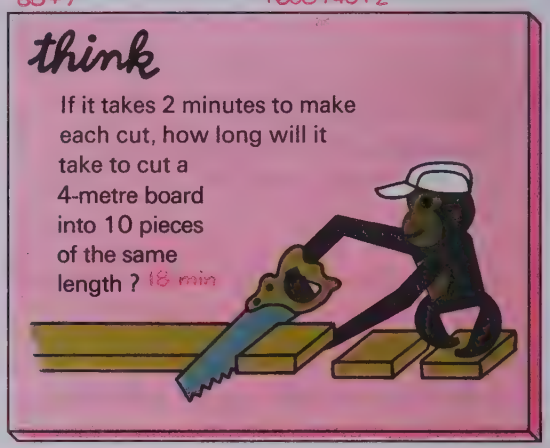
Keeping in Touch with Addition, Place value, Inequalities

1. Solve the equations.
 - A $287 = 200 + 80 + n$ 7
 - B $456 = 400 + n + 6$ 50
 - C $981 = n + 80 + 1$ 900
 - D $342 = 300 + n + 2$ 40
 - E $780 = 700 + 80 + n$ 0
 - F $604 = 600 + n + 4$ 0

2. Write the numeral for
 - A three thousand two hundred thirty-five. 3235
 - B fifty-six thousand, fifty-four. 56 054
 - C fifty-six thousand, two hundred fifty-four. 56 254
 - D nine hundred thirty-six thousand, one hundred one. 936 101
 - E nine hundred fifty thousand, twelve. 950 012
 - F four million, twenty-three thousand, forty-two. 4 023 042
 - G thirty-nine million, one hundred seventy-eight. 39 000 178

3. Write each of the following numbers as in the example.
 Example: 4386 = 4000 + 300 + 80 + 6
 - A 56 = 50 + 6
 - B 342 = 300 + 40 + 2
 - C 783 = 700 + 80 + 3
 - D 5287 = 5000 + 200 + 80 + 7
 - E 6043 = 6000 + 40 + 3
 - F 28 642 = 20 000 + 8000 + 600 + 40 + 2
 - G 75 480 = 70 000 + 5000 + 400 + 80
 - H 628 492 = 600 000 + 20 000 + 8000 + 400 + 90 + 2

4. In the numeral 43 682 the 3 stands for 3000. Give the number for each colored digit below.
 - A 34 682 4000
 - B 34 682 600
 - C 34 682 30 000
 - D 34 682 80
 - E 6 284 563 4000
 - F 6 284 563 200 000
 - G 6 284 563 6 000 000
 - H 6 284 563 500



Discussion

Have the children work the exercises on page 80. If you decide to discuss these exercises before having the children go on to page 81, you will want to pay particular attention to exercise 2. Ask several children to put their answers on the chalkboard. Then give the other children a chance to approve the answers or to discuss any difficulties.

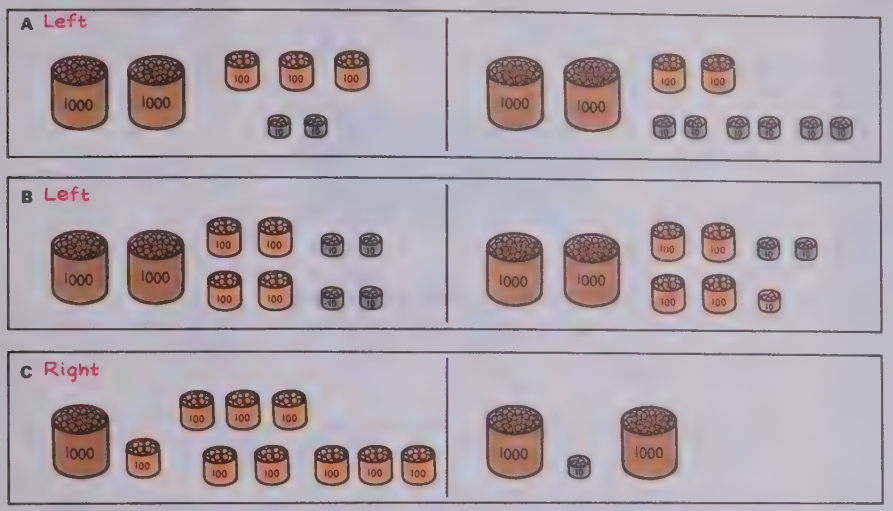
Exercise 4 should help you evaluate the children's progress in understanding place value.

Encourage the children to make the comparison of sets in exercise 5 without counting. Suggest that they think of place value, comparing the largest quantities first—much as they do when comparing large numerals.

An efficient way to use exercise 6 would be to have the children do the first step in each part as a written assignment; then do the second part orally, as a class activity, asking the children to take turns reading the inequalities. This should help you judge whether they complete inequalities mechanically or whether they really do understand the meaning of the symbols.

You might suggest to the children that they sketch a picture as an aid to understanding the *Think* problem. In this way they should be able to see that 9 cuts will make ten 40-cm

5. The number of beads in each can is labelled on the can. Which set, left or right, contains more beads ?



6. In each exercise, give the sign < or > for the ◍. Then give the words (greater than or less than) for the blank.

A 286 ◍ 276 >
286 is greater than 276.

B 304 ◍ 296 >
304 is greater than 296.

C 5279 ◍ 5280 <
5279 is less than 5280.

D 6848 ◍ 6792 >
6848 is greater than 6792.

E 8267 ◍ 4983 >
8267 is greater than 4983.

F 8663 ◍ 8636 >
8663 is greater than 8636.

G 9528 ◍ 7643 >
9528 is greater than 7643.

H 25 367 ◍ 25 267 >
25 367 is greater than 25 267.

I 83 000 ◍ 82 999 >
83 000 is greater than 82 999.



81

pieces and that 9×2 or 18 minutes is the time required to cut the board into 10 pieces.

Follow-up

Here is a game for groups of six or seven children which reviews place value and order for larger numbers. Make a set of seven 8-by-25-cm cards for each group, using a different-colored felt pen or a different-colored card for each set. On one card for each set write a 7-digit numeral such as 8 371 465. On the next card write the same seven digits, transposing only the last two digits. Continue writing the seven digits on the remaining cards, each time changing the value of the original ones' digit by moving it one place to the left. Make sure the sets have been shuffled, and then give the leader of each group a length of masking tape (or pins) and the cards.

At your signal the group leader gives each member of his group one of the cards. The children are to arrange the cards into one sequence from smallest to largest and then tape them on the chalkboard (or pin them on a bulletin board). The first team to finish wins a point. Then the entire class determines which sequences are arranged correctly and awards each team up to 7 points for the actual number of cards in the proper order. (If they are all correct, the sequences should be exactly alike.)

If you prefer, the Place-Value Card Game described on page 49 would also provide children practice with the concepts reviewed in this lesson.

Resources for Active Learning

[These resources review fundamental operations.]

Developmental Math Cards, F[1]6, Addison-Wesley.

Teaching Aids for Elementary Mathematics, "Signs-of-Operations," pp. 78–79, Holt, Rinehart and Winston.

PAGES 82–83

Objective

Given a subtraction exercise which requires regrouping, the child will be able to find the difference by using the subtraction algorithm.

Preparation

To prepare for this lesson, review regrouping tens and hundreds. For example, ask, "If I have 7 tens and 13 ones, how many do I have?" (83). If I have eight tens and 15 ones, how many do I have?" (95). "If I have 5 hundreds and 17 tens, how many do I have?" (670). "If I have 2 hundreds and 10 tens, how many do I have?" (300).

Vary the regrouping review by changing the form of the questions: "75 may be thought of as 60 tens and how many ones?" (15) "843 may be thought of as 7 hundreds, how many tens, and how many ones?" (14 tens and 3 ones) If the children have difficulty responding, you might show some of these in writing, such as:

$83 = 70 + ?$ $\quad 75 = 60 + ?$

$? = 80 + 15$ $\quad 843 = 700 + ? + ?$

$? = 500 + 150$ $\quad 632 = 500 + ? + ?$



How does regrouping help you subtract?

Discussing the Ideas

Regrouping → $\begin{array}{r} 75 \\ -29 \\ \hline \end{array}$ → $\begin{array}{rr} 70 & 5 \\ -20 & -9 \\ \hline & ? \end{array}$ → $\begin{array}{rr} 60 & 15 \\ -20 & -9 \\ \hline 40 & 6 \end{array}$ → $\begin{array}{r} 75 \\ -29 \\ \hline 46 \end{array}$

Shortcut → $\begin{array}{r} 75 \\ -29 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 75 \\ -29 \\ \hline 46 \end{array}$

1. Find this difference by using both the regrouping method and the shortcut. $\begin{array}{r} 65 \\ -38 \\ \hline 27 \end{array}$

See Discussion.

2. Explain each step for finding this difference. → $\begin{array}{r} 532 \\ -163 \\ \hline \end{array}$

$\begin{array}{rrr} 500 & 30 & 2 \\ -100 & -60 & -3 \\ \hline \end{array}$ → $\begin{array}{rrr} 500 & 20 & 12 \\ -100 & -60 & -3 \\ \hline \end{array}$ → $\begin{array}{rrr} 400 & 120 & 12 \\ -100 & -60 & -3 \\ \hline \end{array}$

Can you use this method to find this difference? → $\begin{array}{r} 961 \\ -478 \\ \hline 483 \end{array}$

See Discussion.

3. Study the example below.

$\begin{array}{r} 634 \\ -378 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} {\scriptstyle 2\ 14} \\ 6\,\not{3}\,\not{4} \\ -378 \\ \hline 6 \end{array}$ $\quad$ $\begin{array}{r} {\scriptstyle 5\ 12\ 14} \\ \not{6}\,\not{3}\,\not{4} \\ -378 \\ \hline 56 \end{array}$ $\quad$ $\begin{array}{r} {\scriptstyle 5\ 12\ 14} \\ \not{6}\,\not{3}\,\not{4} \\ -378 \\ \hline 256 \end{array}$

Can you find this difference by using the shortcut above? $\begin{array}{r} 726 \\ -259 \\ \hline 467 \end{array}$

See Discussion.

Discussion

As you work through the regrouping example displayed at the top of page 82, ask children to explain each step. Be sure that they see why 75 is rewritten as 60 and 15. That is, since we cannot subtract nine from five, we must think about 75 as 60 and 15. The same concepts are involved in the shortcut; the only difference is in notation. Have the children explain the relationship between the steps in the shortcut and those in the first example.

Let the children try the problem in exercise 1, and then develop it as in the first example.

$65 \rightarrow \begin{array}{r} 65 \\ -38 \\ \hline \end{array}$ $\quad$ $\begin{array}{rr} 60 & 5 \\ -30 & -8 \\ \hline & ? \end{array}$ → $\begin{array}{rr} 50 & 15 \\ -30 & -8 \\ \hline 20 & 7 \end{array}$ → $\begin{array}{r} 65 \\ -38 \\ \hline 27 \end{array}$

Then present the shortcut.

$\begin{array}{r} 65 \\ -38 \\ \hline ? \end{array}$ $\quad$ $\begin{array}{r} {\scriptstyle 5\ 15} \\ \not{6}\not{5} \\ -38 \\ \hline 27 \end{array}$

After discussing the steps shown for the first example in exercise 2, guide the children in developing the second example like this:

$961 \rightarrow \begin{array}{r} 961 \\ -478 \\ \hline \end{array}$ $\quad$ $\begin{array}{rrr} 900 & 60 & 1 \\ -400 & -70 & -8 \\ \hline \end{array}$ →

$\begin{array}{rrr} 900 & 50 & 11 \\ -400 & -70 & -8 \\ \hline \end{array}$ → $\begin{array}{rrr} 800 & 150 & 11 \\ -400 & -70 & -8 \\ \hline 400 & 80 & 3 \end{array}$

When you work through the shortcut notation in exercise 3, point out to the children that they must always begin to subtract first in the ones' column and regroup a ten into ten ones, if necessary; then they must consider the tens' column and regroup the hundreds, if necessary. Help the children develop the last example in shortcut notation as follows:

$726 \rightarrow \begin{array}{r} 726 \\ -259 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} {\scriptstyle 1\ 16} \\ 7\not{2}\not{6} \\ -259 \\ \hline 7 \end{array}$ → $\begin{array}{r} {\scriptstyle 6\ 11\ 16} \\ \not{7}\not{2}\not{6} \\ -259 \\ \hline 67 \end{array}$ → $\begin{array}{r} {\scriptstyle 6\ 11\ 16} \\ \not{7}\not{2}\not{6} \\ -259 \\ \hline 467 \end{array}$

Using the Ideas

1. Find the differences.

A 76 − 68 = 8	B 135 − 57 = 78	C 154 − 96 = 58	D 183 − 28 = 155	E 174 − 96 = 78	F 135 − 48 = 87
G 124 − 97 = 27	H 157 − 68 = 89	I 194 − 99 = 95	J 173 − 69 = 104	K 176 − 89 = 87	L 191 − 97 = 94

2. Find the differences.

A 762 − 425 = 337	B 847 − 363 = 484	C 934 − 568 = 366	D 654 − 257 = 397	E 820 − 316 = 504	F 717 − 209 = 508
G 438 − 59 = 379	H 564 − 209 = 355	I 713 − 307 = 406	J 876 − 497 = 379	K 521 − 475 = 46	L 923 − 854 = 69
M 863 − 429 = 434	N 562 − 367 = 195	O 674 − 289 = 385	P 1246 − 753 = 493	Q 1342 − 658 = 684	R 1437 − 619 = 818

3. Make up subtraction problems so that when you cover the ones' digits, they look like these.

A

8
−3
5

B

6
−2
3

See Using the Exercises.

4. One digit is covered. Is the difference

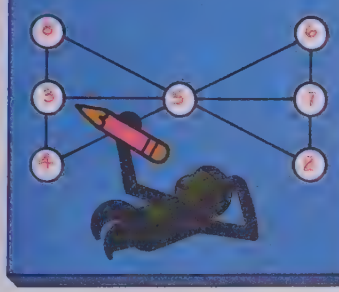
5■
−29

A less than 20? No
B 30 or less? Yes
C between 30 and 40? No

More practice, page A-9, Set 14

think

Draw a figure like the one below. Put the digits 2, 3, 4, 5, 6, 7, and 8 in the circles so the sum along any line is 15.



Using the Exercises

Direct the children to try several of the exercises on page 83.

When they have finished, put representative exercises on the chalkboard and have children explain them. Make sure the children see that in exercise 3, though various digit combinations are possible, their combinations must fit the fact that regrouping was needed in part B but not in part A.

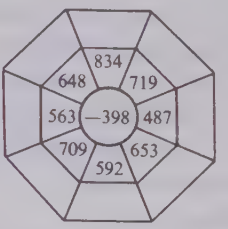
The strategy for solving the *Think* problem is similar to the one suggested for the *Think* on page 69. Some children may recall that class discussion and be able to solve this problem independently. Consider 5 as the "middle" of each set of three numbers whose sum must be 15; so 5 belongs in the middle of the figure. The difference 15 − 5 is 10, so the two remaining numbers must total 10. Thus, one pair to write in opposing circles would be 2 and 8.

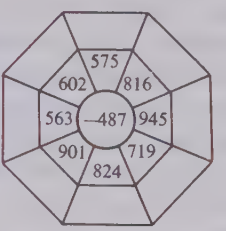
Assignments (page 83)

Minimum: 1–2. Average: 1–4.
Maximum: 1–4.

Follow-up

To provide children with more practice in regrouping with addition and subtraction, prepare a duplicator worksheet with the following practagons.





Alternatively, these might also be written on the chalkboard with letters inside the empty sections so that children can label their answers.

Resources for Active Learning

Mathematics in Modules, WN12, Addison-Wesley.

Duplicator Masters, pages 17–19
Skill Masters, pages 17-19

PAGES 84–85

Objectives

Given subtraction exercises with numerals of no more than 5 digits, the child will be able to find the differences.

Given data in a chart and word problems requiring addition or subtraction, the child will be able to answer the questions by finding the data from the chart and performing the required operations.

Preparation

Any short review activity used to reinforce the children's facility with basic facts would be suitable preparation for this lesson. For example, you might have the children be ready with paper and pencil and give them some chain games such as the following:

"Start with 47 . . . Subtract 25 . . . Add 37 . . . Subtract 46 . . . Add 349 . . . Subtract 178." (184)

"Start with 56 . . . Add 75 . . . Subtract 29 . . . Add 48 . . . Subtract 100." (50)

"Start with 163 . . . Add 87 . . . Subtract 184 . . . Add 34 . . . Subtract 100." (0)

Improving subtraction skills

1. Find the differences.

A 56 − 19 = 37	B 85 − 23 = 62	C 43 − 19 = 24	D 74 − 24 = 50	E 75 − 58 = 17	F 66 − 49 = 17
G 524 − 519 = 5	H 425 − 208 = 217	I 230 − 150 = 80	J 964 − 787 = 177	K 787 − 499 = 288	L 455 − 277 = 178
M 631 − 356 = 275	N 715 − 538 = 177	O 376 − 189 = 187	P 1428 − 619 = 809	Q 1536 − 657 = 879	R 1847 − 588 = 1259

2. Find the differences.

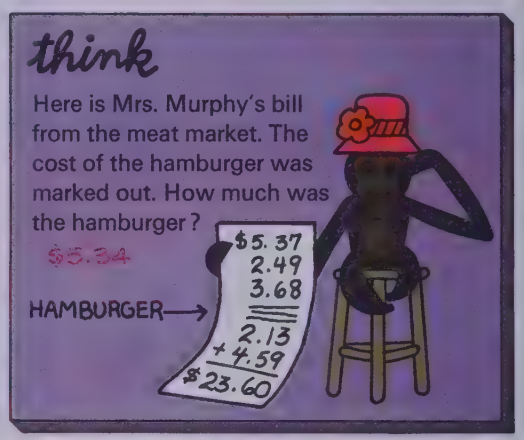
A 1543 − 1369 = 174	B 2173 − 1478 = 695	C 1467 − 873 = 594	D 3424 − 1945 = 1479	E 6531 − 3874 = 2657
F 98 752 − 75 176 = 23 576	G 86 345 − 38 751 = 47 594	H 97 683 − 69 794 = 27 889	I 83 745 − 51 876 = 31 869	

3. Solve the equations.

A $34 - 29 = n$ 5	D $565 - 27 = n$ 538	G $732 - 727 = n$ 5
B $156 - 48 = n$ 108	E $654 - 646 = n$ 8	H $854 - 8 = n$ 846
C $347 - 9 = n$ 338	F $1548 - 538 = n$ 1010	I $923 - 19 = n$ 904

★**4.** Copy the problems. Give the missing digit for each ▥.

A ▥▥▥ 538 − 264 = 274	B 8▥35 − 3▥48 = 50▥5	C 4▥73 − 2▥▥5 = 348	D 727 − ▥▥▥467 = 260
E 9▥39 − 488 = 5▥0▥5	F 58▥8 − 2▥60 = 3▥82		



84

Discussion

Have the children work the exercises on page 84. If you prefer, you may use exercise 4 as optional material. However, most children can work several parts of it successfully and, if not, can certainly benefit by attempting some of them.

You may choose to have several children write some problems from exercises 1, 2, and 3 on the chalkboard and explain them to the class.

Before assigning page 85, it would be helpful to study the chart with the children, helping them to read the words properly. A special benefit provided by a page such as this is the opportunity to relate the study of mathematics to other interests of the children. Encourage discussion of ideas and take advantage of any enthusiasm the children show for further research concerning the human skeleton. In the course of such research, some children might notice that many references give 33, rather than 26, as the number of bones in the spinal column. (The discrepant figures arise from the fact that some authorities count the 5 fused sacral vertebrae and the 4 fused coccygeal vertebrae as 2 single bones rather than as 9 separate bones.) Some children may be motivated to create more problems on the bones of the body. (See the follow-up section.)

THE HUMAN SKELETON

The head	Number of bones
Cranium	8
Face	14
Ears	6
Hyoid in neck	1
The trunk	
Spinal column	26
Ribs	24
Breastbone	1
Collar bones	2
Shoulder bones	2
Pelvic bones	2
The limbs	
Arms	60
Legs	60



1. How many bones are in the head? 29
2. How many bones are in the trunk? 57
3. How many more bones does the trunk have than the head? 28
4. Each arm has 30 bones. The hand and wrist contain all but three of these. How many bones are in the hand and wrist? 27
5. Each leg has 30 bones. The ankle and foot have 26 of these. How many bones are in the rest of the leg? 4
6. Each of 12 bones in the spinal column has two ribs attached to it. How many bones in the spinal column do not have ribs attached? 14
7. How many bones are in the human body? 206

85

Follow-up

Children might enjoy comparing the number of bones in the human body to the number of bones in the skeleton of a dog, horse, or bird. They might make charts to show these comparisons, or illustrations of the various skeletons with the number of bones labelled.

Workbook, page 23

Assignments (page 84)

Minimum: 1. Average: 1–3.
Maximum: 1–4.

Assignments (page 85)

Minimum: 1–7, oral. Average: 1–7.
Maximum: 1–7.

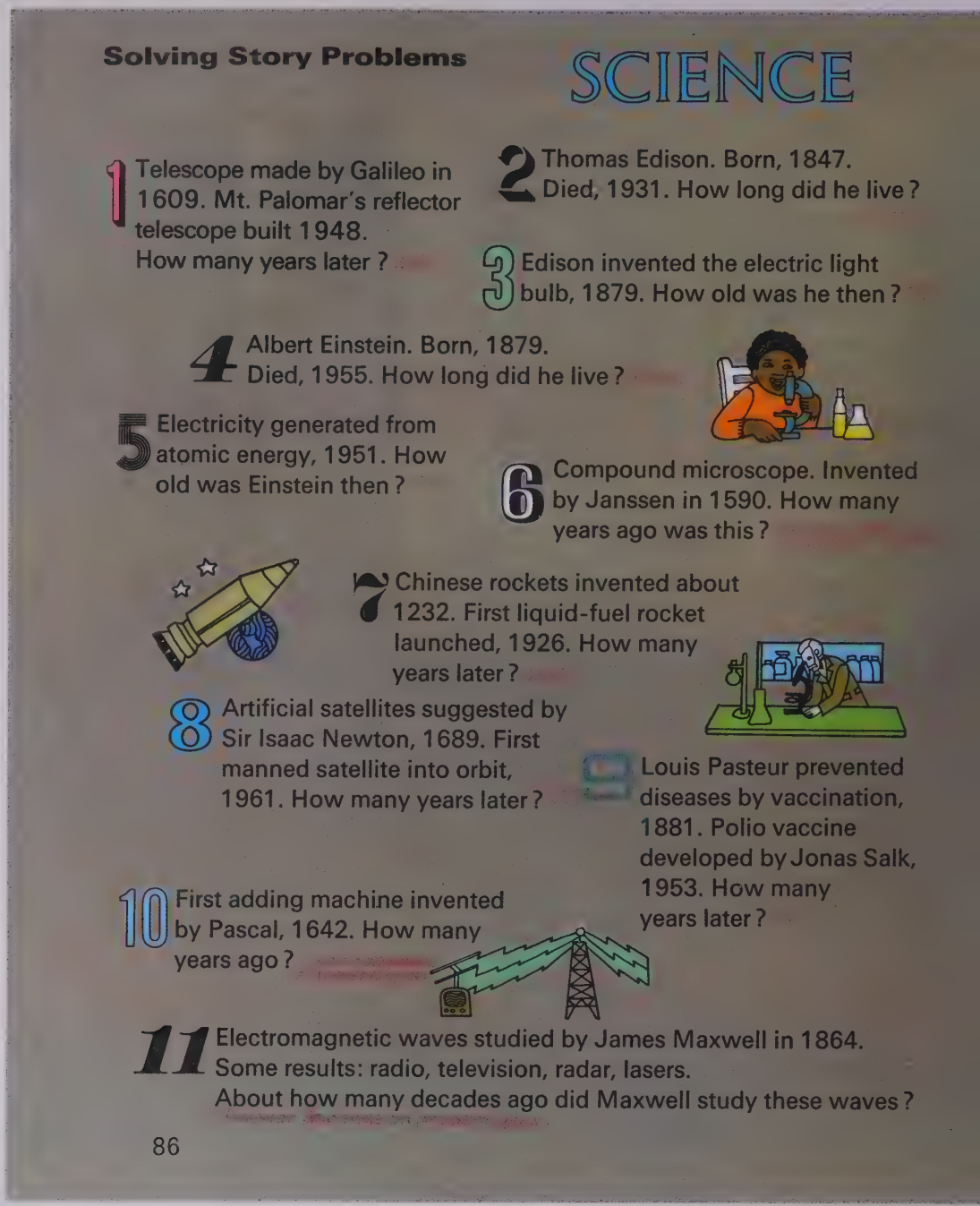
PAGES 86–87

Objective

Given word problems which require subtraction with regrouping, the child will be able to solve the problems.

Preparation

To prepare for this lesson, provide children with an opportunity to perform subtraction using dates. For example, ask children to imagine that someone asks them how old they are. Instead of responding with their age in years, suppose they tell the person the year of their birth. Ask them how the person would use this information to find out their age. You might ask a few children to tell you the year of their birth; then you respond with their age. Help them see that you subtract the year of their birth from the date of the present year.



Discussion

Before assigning these problems, encourage children to discuss a few of the pictures on each page. One of the chief benefits of problem sets of this type in addition to the arithmetic involved is the interest generated in various topics. It would be helpful to work through one or two problems on each page with the children. Stress the importance of recognizing the inequality of the dates of the years and of subtracting the lesser number (that is, the earlier year) from the greater number.

Afterward, children who seem especially interested should be encouraged to do further research on other kinds of inventions and their dates or on other important historical dates.

Assignments (page 86)

Minimum: Odd-numbered problems.
Average: 1–11. Maximum: 1–11.

Solving Story Problems

HISTORY

* **1** America. Discovered in 1492.
How many years ago?

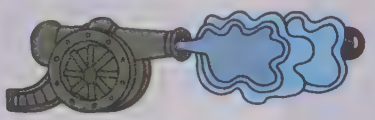
2 Quebec. Founded by Champlain in 1608. How many years ago?

3 Pilgrims. Landed at Plymouth, Massachusetts, in 1620.
* A How many years ago?
B How many years after the founding of Quebec? 12

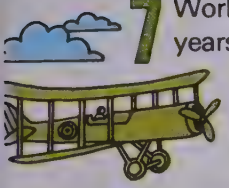


* **4** British North America Act.
Signed, 1867. How many years ago?

5 Sir John A. Macdonald, Prime Minister of Canada, 1867-1873, 1878-1891. How many years? 17



* **6** Quebec captured, 1759. How many years ago?



7 World War I, 1914. How many years after the capture of Quebec? 155

* **8** World War II, 1939.
How many years ago?

9 Fortress of Louisbourg.
Built by French in 1720.
Captured by New Englanders in 1745.
How many years did the French hold the fortress? 25



10 Lester Pearson. Born, 1897.
Died, 1972.
How long did he live? 75 yr

* **11** Gold discovered in Klondike, 1896. How many years ago?

12 Albert Schweitzer. Born, 1875.
Nobel Peace Prize, 1952.
How old was he then? 77 yr

* Answers to these questions depend on year solved.

13 MacKenzie reached Pacific, 1793.
How many years was this after the discovery of America? 301

More practice, page A-9, Set 15

Follow-up

Encourage children to make up other problems using dates from history and science. If these are printed on index cards and arranged by letter or number, children may make a file of problems for the class to use. The solutions may be printed on separate cards but labelled to match the problem cards so that the children can check their own work.

Assignments (page 87)

Minimum: Odd-numbered problems.
Average: 1–13. Maximum: 1–13.

PAGES 88-89

Objective

Given numerals in subtraction problems in which there are no tens and/or no hundreds, the child will be able to regroup these numerals and perform the subtraction.

Preparation

To prepare for this lesson, run through a short review of thinking of numerals in expanded form. For example, say: "I'm thinking of a number which is 7 tens and twelve. What is my number?" (82) Include examples in regrouping across zeros, such as: "I'm thinking of the number that is 49 tens and 17. What's my number?" (507) Some other examples are as follows:

72 tens and 13	(733)
48 tens and 16	(496)
10 tens and 12	(112)
9 tens and 15	(105)
13 tens and 11	(141)
59 tens and 14	(604)



Let's explore regrouping for subtraction again.

Discussing the Ideas

1. Explain the following statements.
 - A For 805, we can think 8 hundreds, 0 tens, and 5.
 - B For 805, we also can think 80 tens and 5.

 See Discussion.

2. Complete each sentence.
 - A For 206, we can think 20 tens and 6.
 For 206, we also can think 19 tens and 16.
 - B For 507, we can think 50 tens and 7.
 For 507, we also can think 49 tens and 17.
 - C For 7004, we can think 700 tens and 4.
 For 7004, we also can think 699 tens and 14.
 - D For 9002, we can think 900 tens and 2.
 For 9002, we also can think 899 tens and 12.

3. In Step 2 below, we first think of 507 as 50 tens and 7. Explain the regrouping. See Discussion.

Step 1	Step 2	Step 3
5 0 7 − 2 4 9	4 9 17 5 0 7 − 2 4 9	4 9 17 5 0 7 − 2 4 9 2 5 8 (c) (b) (a)
There is no whole-number answer.	507: 49 tens and 17	(a) 17 − 9 = 8 (b) 90 − 40 = 50 (c) 400 − 200 = 200

4. Explain each part of Step 3 above. See Discussion.

5. Can you find these differences?

 A
 $$\begin{array}{r} 506 \\ -\ 358 \\ \hline 148 \end{array}$$

 B
 $$\begin{array}{r} 403 \\ -\ 156 \\ \hline 247 \end{array}$$

6. Copy and complete this problem.

 $$\begin{array}{r} 7\ 9\ 9\ 14 \\ \cancel{8}\ \cancel{0}\ \cancel{0}\ \cancel{4} \\ -\ 4\ 3\ 7\ 8 \\ \hline 3\ 6\ 2\ 6 \end{array}$$

88

Discussion

Lead from the oral regrouping to a chart such as the following for numbers such as 85, 805, 8005, and 8025.

tens		tens		tens		hundreds	
8	5	0	5	0	5	0	25
		80	5	800	5	80	25
7	15	79	15	799	15	79	125

Use exercise 1 to show the children how they might think of the numbers. For example, for 805, they could think 8 hundreds, 0 tens, and 5; or 80 hundreds and 5; or 79 hundreds and 15.

Use exercise 2 to discuss the regrouping the children will use in subtraction. Emphasize in exercises 3, 4, and 5 that for the regrouping of numerals such as 506 or 403, it is helpful to think in terms of how many tens rather than how many hundreds and how many tens. Similarly, in exercise 6 it is best to think how many tens rather than how many thousands, how many hundreds, and how many tens. It would be helpful to show several other examples of this kind of regrouping.

Using the Ideas

1. Give the missing numbers.
 - A For 306, we can think ||| tens and 6. 30
 - B For 306, we can think 29 tens and |||. 16
 - C For 704, we can think ||| tens and 4. 70
 - D For 704, we can think 69 tens and |||. 14
 - E For 6007, we can think ||| tens and 7. 600
 - F For 6007, we can think 599 tens and |||. 17
 - G For 6025, we can think ||| hundreds and 25. 60
 - H For 6025, we can think 59 hundreds and |||. 125

2. Copy and complete each subtraction exercise.

A	B	C	D	E
1 9 14 ~~204~~ −126 78	4 9 12 ~~502~~ −247 255	3 9 9 12 ~~4002~~ −1275 2727	7 9 14 ~~8047~~ −1562 6485	6 4 9 13 ~~6503~~ −1345 5158

3. Find the differences. Check your answers by adding.

A	B	C	D	E
602 −38 564	706 −87 619	406 −298 108	1507 −348 1159	700 −389 311

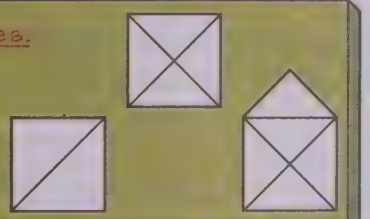
F	G	H	I	J
534 −125 409	607 −437 170	832 −564 268	6004 −2386 3618	4300 −2643 1657

★4. Copy the problems. Give the missing digit for each |||.

A	B	C	D												
679 −			305 374	600 			 −595 5	5 7 1	0	 −8	5 0 702	1 0 0 			5 −57 948

think See Using the Exercises.

Try to draw each figure without lifting your pencil from the paper and without retracing. Two of them **can** be done. One of them **cannot**.

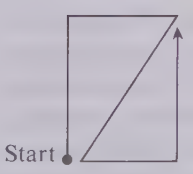


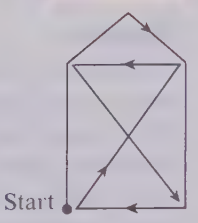
More practice, page A-10, Set 16

Using the Exercises

On page 89 work through parts of exercises 1 and 2 before assigning the exercises as independent work. Exercise 4 may be considered enrichment, though all the children would benefit from discussion of the solutions. When the children finish the exercises, help them check their work carefully. You may find that an explanation of many of the exercises is necessary.

The children can reproduce figures 1 and 3 in the *Think* problem without lifting their pencils from the paper and without retracing. Two of several possible solutions are shown at the right.





Assignments (page 89)*

Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Duplicator Masters, page 89
Workbook, page 24

PAGES 90–91

Objective

Given an assortment of addition and subtraction exercises and problems, the child will be able to solve the exercises and problems by choosing and performing the correct operation.

Preparation

To prepare for this lesson, use a game such as "What's My Rule" (see page 65) or, to help the children develop speed, use a few mental chain games such as the following:

"Start with 24 . . . Subtract 9 . . . Add 6 . . . Subtract 5 . . . Subtract 8." (8)

"Start with 17 . . . Add 8 . . . Subtract 9 . . . Subtract 7 . . . Add 3 . . . Subtract 7." (5)

"Start with 33 . . . Subtract 5 . . . Add 1 . . . Subtract 10 . . . Add 2 . . . Subtract 7 . . . Subtract 7 . . . Subtract 7." (0)

Let's practice adding and subtracting.

1. Find the sums and differences.

A 567 +839	B 843 −456	C 705 −239	D 618 +496	E 583 +819	F 900 −347
G 608 −439	H 874 −567	I 767 +869	J 409 +786	K 676 −186	L 501 −403
M 871 623 489 376 +554	N 304 122 304 169 +785	O 652 374 418 920 +513	P 364 905 845 367 +978	Q 768 858 979 659 +675	R 896 784 599 867 +657
S 5076 −3489	T 6785 +8432	U 70 043 −46 528	V 8403 −7685		
W 17 954 +98 767	X 8006 −4875	Y 16 934 +8 657	Z 35 007 −3 879		

2. Find the totals as in the example.

Example:
5 h 40 min
3 h 30 min
or

A 6 h 35 min
4 h 35 min

B 2 wk 5 days
3 wk 6 days

C 20 min 53 s
30 min 12 s

D 3 h 50 min
4 h 30 min

3. Find the differences as in the example.

Example:
3 h 10 min
1 h 50 min →

A 6 days 7 h
1 day 13 h

B 5 h 30 min
2 h 45 min

C 7 min 14 s
6 min 52 s

D 10 wk 2 days
3 wk 5 days

90

Discussion

The exercises on page 90 should be assigned as independent work, and then discussed as you have children check their papers. It would be helpful to discuss the bar graph on the top of page 91 before assigning the exercises. Help the children recognize that the height of each bar corresponds to the number line at the left of the graph.

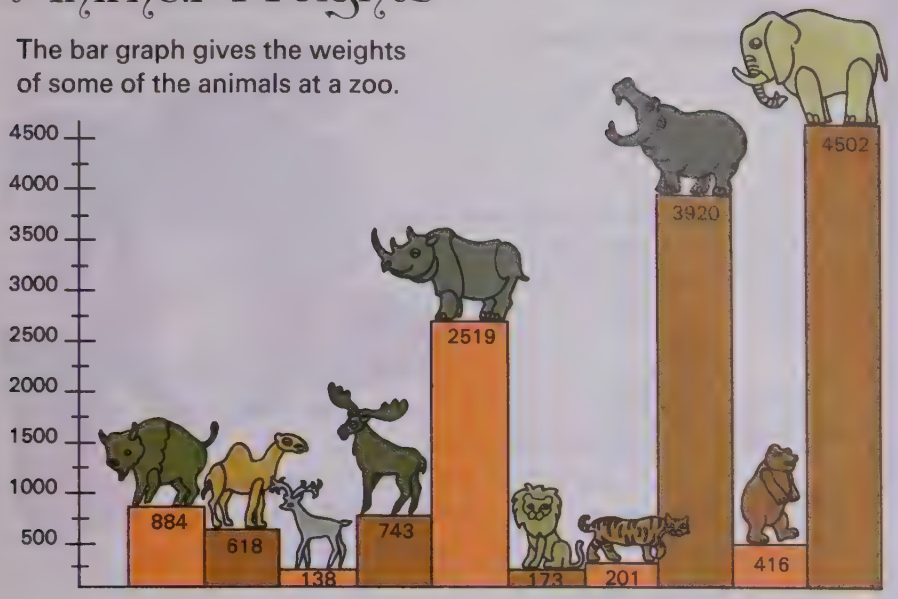
The starred exercises on both pages are intended for those competent to handle them, but all children would benefit from a discussion of each. For exercises 8 and 9 on page 91, have the children who solve them explain their solutions to the class.

Assignments (page 90)

Minimum: 1A–L, 1S–Z.
Average: 1–2. Maximum: 1–3.

Animal Weights

The bar graph gives the weights of some of the animals at a zoo.



1. How many kilograms must the lion gain to weigh as much as the tiger ? 28
2. The rhinoceros lost 350 kilograms. How much did it weigh then ? 2169 kg
3. Find the total weight of the two heaviest animals. 8422 kg
4. Find the total weight of the three lightest animals. 512 kg
5. Find the difference in the weights of the two heaviest animals. 582 kg
6. If a buffalo and a camel were weighed on the scales together what would the scales read then ? 1502 kg
7. How much more does the buffalo weigh than the bear ? 468 kg
- ★ 8. Some animals were on the scales. The scales read 2585 kilograms. The moose got on the scales and the bear got off. What did the scales then read ? 2912 kg
- ★ 9. A medium-sized car weighs 2164 kilograms. What is the difference in the weight of the elephant and the weight of two cars ? 174 kg

Resources for Active Learning

[These resources offer other bar graph ideas.]

Applied Mathematics Cards, Group 1/23–26, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Math Activity Cards C6, Macmillan.

Assignments (page 91)

Minimum: 1–7, oral. Average: 1–7. Maximum: 1–9.

PAGES 92–93

Objective

Given exercises and problems involving amounts of money and requiring simple addition and subtraction, the child will be able to find the sums or differences.

Preparation

Materials

department store newspaper ads or mail order catalogues

Because of the nature of this lesson's investigation, you may choose to omit any specific preparation. However, depending on the needs of your class, you might review dollar-and-cent notation by writing several suitable examples on the chalkboard and having children read them.

Investigation

You will want to plan your treatment of this investigation according to the materials you have available for the children to use. You might have the children work in groups of two or three so that they will be able to check each other's work. If appropriate ads or catalogues are not available, write a suitable list on the chalkboard. For example:

basketball	$ 6.49
doll	4.37
fishing rod	7.25
game	1.98
gloves	3.80
shoes	10.55

Give the children sufficient time to work out the investigation problem. If any finish quickly, encourage them to make up money problems of their own to solve.

Can you add and subtract amounts of money?

Investigating the Ideas

Suppose you have $20 to spend. Choose three items you could buy from a newspaper advertisement.



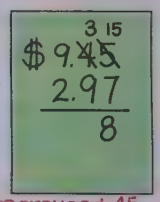
See Investigation and Discussion.

? Can you find the total cost and the change you would get from $20?

Answers will vary.

Discussing the Ideas

1. Susan and Sara went shopping with their mother and father. Sara had $9.45. She wanted to buy a book which cost $2.97. She started to figure how much she would have left.



 A Explain what Sara has done so far.

She has regrouped 45 as 3 tens and 15.

 B Tell how Sara should finish the problem.

Regroup 930 as 8 hundreds and 13 tens and finish subtracting.

 C Susan had $10. How much change would she get if she bought the book? $7.03

2. Eric bought a game for 63 cents. As the clerk returned change from $1.00, he said, "63 cents."

"Here is 64, 65, 70, 75, one dollar."

Can you tell what coins Eric got as change?

2 pennies, 2 nickels, and 1 quarter

See Discussion.

92

Discussion

As you have the children discuss the investigation and the exercises on page 92, help them to realize that regrouping in subtraction with amounts of money is not really different from subtraction of numerals without the dollar-and-cent notation. Have the children present examples from their work in the investigation, writing the numerals on the chalkboard and explaining the steps they took to find the sums or differences.

In exercise 2, you might wish to note that the example given is just one of several ways the clerk could have made the proper change. If he had used the minimum number of coins (2 pennies, 1 dime, and 1 quarter), he would have said, "Here is 64, 65, 75, one dollar."

If necessary, present other examples similar to exercise 2. You might use items from the investigation activities and present other situations of receiving change.

Using the Ideas

1. Find the total amounts.

A	B	C	D	E
\$5.25	\$7.98	\$15.95	\$2.49	\$.89
3.90	6.50	6.75	3.20	1.56
\$9.15	\$14.48	\$ 22.70	6.98	.28
			\$12.67	\$2.73

2. Find the differences in the amounts.

A	B	C	D	E
\$3.98	\$1.00	\$5.00	\$10.00	\$20.00
1.25	.79	3.95	6.50	15.50
\$2.73	\$.21	\$1.05	\$ 3.50	\$ 4.50

3. Susan saw these roller skates and ice skates. Which cost more? How much more?
Ice skates cost \$2.67 more.

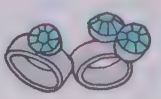


4. Maria is saving her money to buy a record player. She has saved \$7.89. How much more must she save? \$12.06



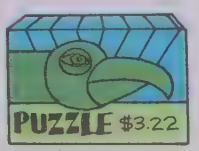
5. Eric bought a book about flowers for \$2.97, a book about horses for \$1.47, and a children's dictionary for \$3.73. What was the total cost? \$8.17

6. Jean saw a birthstone ring that cost \$6.49. She saw a double birthstone ring that cost \$8.95. How much more did the double birthstone ring cost than the single birthstone ring? \$2.46



7. Bob and Alan each bought a game to give to friends.
 A. Which game cost more? Puzzle
 B. How much more? \$1.27





8. Linda's father gave the clerk \$40 for a bicycle that cost \$37.97. How much change did he receive? \$2.03

More practice, page A-10, Set 17

93



Using the Exercises

Assign the exercises on page 93 as independent work. When the children finish, have them carefully check their work. Discuss any items which caused difficulty. It would be helpful to work through parts of exercise 2 where regrouping with zero tens and hundreds is required, as shown below.

$$\begin{array}{r} {}^{4}\cancel{5}\,{}^{9}\cancel{0}\,{}^{10}\cancel{0} \\ -3.95 \\ \hline \$1.05 \end{array}$$

Assignments (page 93)*
Minimum: 1–2. Average: 1–8.
Maximum: 1–8.

Follow-up

To give children further experience with money notation, suggest that they find items they would like to purchase and compare the prices. Or, pretend to give each child a certain amount of money and have the children choose from catalogues and advertisements various items they might buy, and again compare the prices.

Resources for Active Learning

Discovery, Section II, Unit 8/3, Encyclopaedia Britannica Educational Corp.

Mathematics in Modules, WN16, Addison-Wesley.

Nuffield Project: *Computation and Structure* 2, "Money," pp. 91–102, Wiley.

Duplicator Masters, pages 21–22
Workbook, page 25
Skill Masters, page 21

PAGES 94–95

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

If you wish to introduce this lesson with a preparatory activity, present a brief review of any concept which has been troublesome for the children, or spend a few minutes conducting a brisk drill.

Reviewing the Ideas

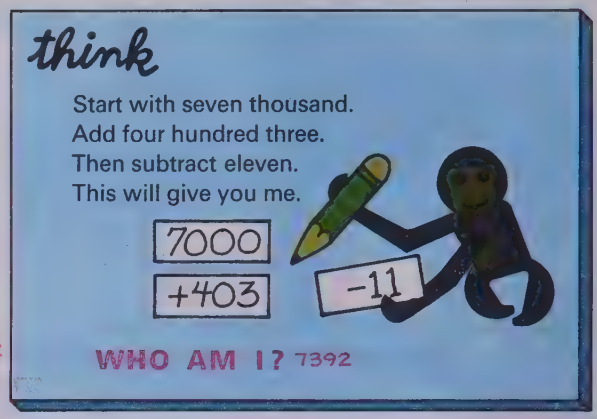
1. Find the differences by finding the missing addends.

A $n + 49 = 52$ 3
$52 - 49 = n$ 3

B $n + 28 = 32$ 4
$32 - 28 = n$ 4

C $n + 357 = 362$ 5
$362 - 357 = n$ 5

D $n + 4349 = 4361$ 12
$4361 - 4349 = n$ 12



2. Find the sums and differences.

A 36 +25 = 61	B 53 −26 = 27	C 60 −37 = 23	D 97 +68 = 165	E 79 +87 = 166	F 87 −79 = 8
G 326 +497 = 823	H 547 −293 = 254	I 603 −329 = 274	J 700 −287 = 413	K 754 +869 = 1623	L 1565 +978 = 2543
M 8046 −3779 = 4267	N 5008 +6053 = 11 061	O 9002 −5437 = 3565	P 5037 −2649 = 2388	Q 8703 −5827 = 2876	
R 67 85 +49 = 201	S 385 277 +969 = 1631	T 243 166 +437 = 846	U 5471 2364 +6575 = 14 410	V 56 732 85 968 +97 879 = 240 579	

3. Copy each problem on your paper. Then find the sum.

A 356 + 49 + 8 + 6 = 419
B 57 + 476 + 387 + 8 = 928
C 5439 + 367 + 28 = 5834
D 395 + 2007 + 66 = 2468
E 6975 + 3428 + 26
F 875 + 15 693 + 78
G 9 + 892 + 3467 + 12
H 54 937 + 6843 + 58

(E) 10 429 (G) 4380
(F) 16 646 (H) 61 838

94

Discussion

You might work through several of these exercises with the children as a review, or you might have them work the exercises independently before any discussion. In either case, plan to provide sufficient time to discuss exercises which caused difficulty.

In connection with exercise 4, it might be interesting to have the boys explain to the girls some of the baseball terminology (though some of the girls, too, may already know quite a bit about the game). For exercise 4C, make sure that all the children understand that an "extra-base" hit is a hit that allows the batter to run more than one base. Thus, the answer to this exercise is the sum of all Babe Ruth's home runs, doubles, and triples.

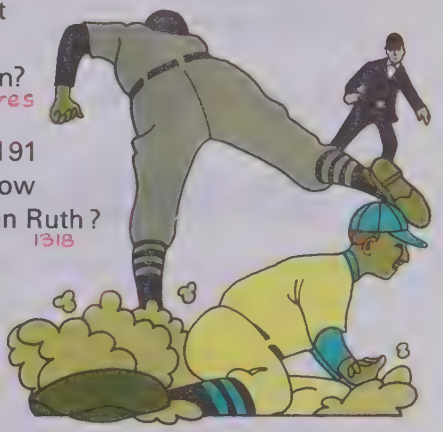
You may also point out, prior to assigning page 95, that there are some tricks that will save time in working exercises 5 and 6. For example in exercise 5A, the children are to compare the sum $57 + 34$ with the sum $54 + 37$. Some children can easily observe that by interchanging the ones' digit on either side they have identical numbers. This prompts the observation that the ones' digits can be interchanged, but the sum remains the same. Note that for exercise 5H the trick is to exchange various digits. For exam-

4. Solve these problems about baseball batting records.

A One of the longest home runs on record was hit by Babe Ruth. The ball went 177 metres. If the longest baseball throw was 134 metres, how much longer was the home run?
43 metres

B Ty Cobb had a lifetime record of 4191 hits. Babe Ruth made 2873 hits. How many more hits did Cobb make than Ruth?
1318

C Babe Ruth hit 714 home runs during his major-league play. He also hit 506 doubles and 136 triples. How many extra-base hits did Babe Ruth make? 1356



5. Give the sign (<, >, =) that should go in each ◍.

A $57 + 34$ ◍ $54 + 37$ =
B $532 + 256$ ◍ $256 + 532$ =
C $633 + 45$ ◍ $533 + 55$ >
D $857 + 384$ ◍ $867 + 374$ =
E $8324 + 637$ ◍ $8224 + 647$ >
F $8007 - 678$ ◍ $8007 - 578$ <
G $3967 - 1888$ ◍ $3867 - 1888$ >
H $6595 + 2387$ ◍ $6387 + 2595$ =

★**6.** For each exercise, tell how much more the first sum or difference is than the second sum or difference.

A $376 + 423$, $276 + 123$ 400
B $658 + 49$, $638 + 59$ 10
C $865 + 378$, $875 + 358$ 10
D $593 + 476$, $493 + 366$ 210
E $847 + 599$, $647 + 699$ 100
F $603 - 37$, $503 - 37$ 100
G $7004 - 295$, $7004 - 395$ 100
H $876 - 498$, $776 - 598$ 200
I $738 - 299$, $538 - 199$ 100
J $993 - 489$, $793 - 499$ 210

ple, we are comparing the sum $6595 + 2387$ with $6387 + 2595$. By carefully rearranging the digits we can show that the sum of each pair of numbers is the same. That is, for each sum, the two ones' digits are 5 and 7, the two tens' digits are 9 and 8, the two hundreds' digits are 5 and 3, the two thousands' digits are 2 and 6. Do not explain these things to the children until they have had a chance to look for the tricks to solving the problems without actually doing the adding. It could be that one of the best ways to find the tricks is actually to do the addition. Certainly, some children will notice a certain similarity between adding one pair of numbers and adding the other pair of numbers.

Follow-up

To give more capable children experience in analyzing multi-step problems, create a worksheet similar to the one below. Adjust the names and situations to your own class, if possible.

The 16 girls and 14 boys in Mrs. Powell's class are having a Halloween party.

1. Doughnuts cost 8¢ each. The children are allowing 3 doughnuts per boy and 2 doughnuts per girl. How many doughnuts will they buy? How much will they cost per boy? per girl? Estimate the total cost of doughnuts.
2. Jody's father donated a basket of apples. If there were 68 apples in the basket, how many apples could each child have? How many children might get an extra apple?
3. Cider costs 25¢ a litre. Polly figures they can get 20 glasses of cider from a litre. How many litres should they buy to give everyone 2 glasses of cider? How much will that amount of cider cost?
4. About how much money would the class spend for cider and doughnuts? Will they have enough money if each child brings 30 cents for the party?

Resources for Active Learning

Developmental Math Cards, F[4]12, G[1]7, Addison-Wesley. [Reinforcement activities and games]

Workbook, page 26

PAGES 96–97

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Materials

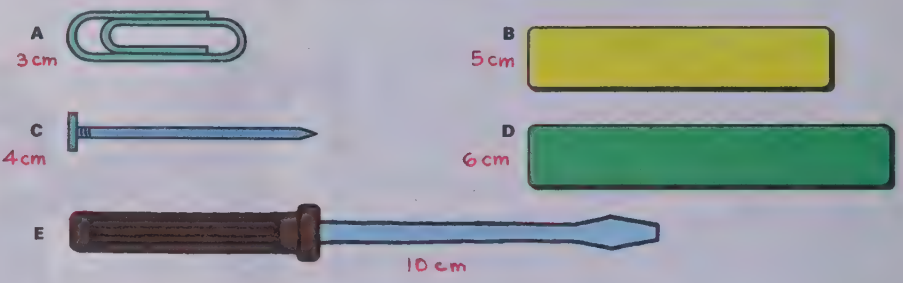
centimetre ruler

Review any topic treated on these pages with which the children had previous difficulty. For example, you might wish to remind them that to measure to the nearest centimetre they should use the half-centimetre mark on their ruler as a guide.

Keeping in Touch with

Measurement
Place value
Inequalities
Addition
Subtraction

1. Find the length of each object to the nearest centimetre.

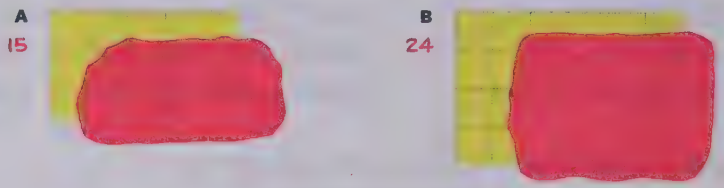


2. Give the area of each region. Each small square is a square centimetre.



3. Find the perimeter of the region in 2A in centimetres. 10 cm

4. Find the area of each rectangle. Each small square is the unit.



5. Give the volume of each figure.



96

Discussion

Assign these exercises as independent work unless you choose to work through them as a review. When children finish, discuss the topics of difficulty and review the meaning of perimeter, area, and volume. You might also ask children to explain their answers for exercise 9. Point out how the difference in one position of place value greatly changes the value of the number.

Encourage many children to try the *Think* problem. They might want to use graph paper to help them find the square.



6. Find the sums and differences.

A $4 + 7 = n$ 11 C $16 - 9 = n$ 7 E $15 - 6 = n$ 9 G $6 + 8 = n$ 14

B $17 - 8 = n$ 9 D $7 + 8 = n$ 15 F $7 + 9 = n$ 16 H $14 - 9 = n$ 5

7. Write the numeral for

A 6 tens and 9. 69

B 2 hundreds, 5 tens, and 4. 254

C 7 thousands and 2. 7002

D 9 thousands, 7 hundreds, 8 tens, and 6. 9786

8. Solve the equations.

A $437 = 400 + 30 + n$ 7

B $786 = 700 + n + 6$ 80

C $952 = n + 50 + 2$ 900

D $168 = n + 60 + 8$ 100

9. Write the sign (> or <) that should go in each ◍.

A 17 482 ◍ 17 842 <

B 20 317 ◍ 20 371 <

C 439 901 ◍ 439 001 >

D 746 811 ◍ 746 810 >

10. Give the number that is 100 000 more than

A 56 432. C 300 000.

B 7183. D 1 000 000.

A 156 432 C. 400 000

B 107 183 D 1 100 000

think



I am a special square because the number for my area is the same as my perimeter number.

WHAT SQUARE AM I? 4 by 4



You are invited to explore

ACTIVITY CARD 3 Page 348

Follow-up

Suggest to the children that they try to find out if they are 'square'. To do this they should measure their height and the distance from one finger tip to the other when their arms are outstretched. Comparison of these two measurements should prove interesting. If the height and width are the same the child can say he is "square."

Resources for Active Learning

[The following activities are worthwhile for review and assessment.]

Developmental Math Cards, F[5]21, Addison-Wesley.

Discovery, Section II, Unit 3/4, Encyclopaedia Britannica Educational Corp.

Pages 98–115

General Objectives

To provide experience in working with cubes, in constructing cubes, and in counting the number of edges, faces, and vertices

To review the geometric concepts of point, line, line segment, ray, and angle

To develop understanding of parallel lines

To provide experience in constructing parallelograms, rectangles, rhombuses, trapezoids, and triangular pyramids

To develop understanding of simple closed curves

To develop understanding of symmetric figures.

The primary purpose of this chapter is to develop the children's intuitive notion of basic geometric concepts. These concepts are not developed formally, but children are given an opportunity to explore these concepts by working with models which represent them in the physical world. They will construct cubes and pyramids, and draw parallel lines and various kinds of quadrilaterals. Pertinent questions will help them relate the terms *point*, *line*, *angle*, and *ray* to appropriate parts of the figures they construct. Polygonal figures are studied by a development of the concept of simple closed curves. Finally, children explore the concept of symmetry by folding symmetric figures. Throughout the entire chapter, your enthusiasm should set the mood not only for the investigations suggested in the text but also for any related explorations which the children might find appealing.

Mathematics

The first few lessons deal with the construction of a cube and pyramid from a two-dimensional pattern. Thus, spatial relations are developed as the child works from the pattern to the three-dimensional cube. A cube is defined as a rectangular prism (box) such that all its faces are squares.

The basic geometric concepts of point and line are usually treated as undefined terms in a formal study of geometry. In this text's development these basic concepts are introduced by considering objects from the physical world that are familiar to the student. Thus, edges of a cube suggest line segments, vertices of a cube suggest points, and two opposite edges suggest parallel line segments. Simple closed curves are presented through a comparison of models that are simple closed curves with models that are not simple closed curves. A simple curve is a curve whose path does not cross itself. A piece of string is a reasonable physical model of a curve. Mathematically, curves are defined in such a way as to include straight lines. A closed curve is a curve that is traceable from one point back to the same point. Simple closed curves in a plane have the property of dividing the plane into three sets: the set of points in the interior, the set of points in the exterior, and the set of points on the curve itself. A closed curve may be thought of intuitively as a piece of string connected into a loop capable of being formed into a figure whose sides may even be line segments.

Symmetrical figures give children an intuitive grasp of the concept of symmetry.

Teaching the Chapter

Materials

Cardboard square, 5 by 5 centimetres (1 per child)
Colored strips
Construction paper or heavy paper suitable for construction of cubes and pyramids
Demonstration cube, approximately 25 cm each side
Demonstration square, approximately 25 cm square
Flannelboard
Glue or transparent tape
Paper for tracing
Ruler
Scissors
Yarn (or string)

Vocabulary

angle	parallelogram
construct	pentagon
cube	point
diagonal	polygon
edge	quadrilateral
face	ray
hexagon	rectangle
inside	regular polygon
line	rhombus
line segment	right angle
line of symmetry	simple closed curve
model	triangle
octagon	triangular pyramid
outside	vertex
parallel	

This chapter should provide fun and excitement for both teacher and children. As stated previously, the purpose is mainly to develop an intuitive notion of basic geometric concepts. The opportunity to construct three-dimensional objects such as the cube and pyramid might stimulate the children to investigate patterns for other shapes. Try to maintain a classroom mood of creative enjoyment in constructing different shapes and in using different patterns to construct the same shape. Try to provide stimulating activities for all the children, and special challenges for the more able. If the children respond remarkably well to any particular lesson, plan a class project to develop it further. The children will have a better understanding of the terms and concepts if they use them while working together on a class project.

Lesson Schedule

How much time you spend on this chapter will depend on how many lessons you enrich with class or

individual projects. Plan to spend a maximum of two weeks, unless one of the lessons develops into an extended valid learning experience.

Evaluation of Progress

As you observe the children's work, keep in mind that the purpose of this chapter is to develop *intuitive* concepts and to provide children with an opportunity to enjoy constructing various models. Neatness in making the model should be encouraged but not stressed. Allow for individual capabilities: you might praise one child for work you would find unacceptable from another. Be patient with the children's use of the vocabulary terms; correct usage will develop naturally if children are encouraged to follow your example. Try to correct any gross misconceptions of the basic ideas here, but do not expect the same degree of understanding from all the children.

Resources for Active Learning

GENERAL ACTIVITIES

Activities in Geometry for Primary Pupils, Activities 4, 11, 13, 16, 21, 22, 24, 26, 30, 32-34, 36-44, 47, 49, 50, Addison-Wesley

Freedom to Learn, "Patterns in Shapes," pp. 94–99; "Geometry—Properties of Shapes," pp. 149–152, Addison-Wesley

Geometry in the Classroom: New Concepts and Methods, pp. 23-27; 39-61; 76-85, Holt, Rinehart and Winston

Mathex: Geometry No. 9, "From 3D to 2D," pp. 10–11, Wiley

Nuffield Project: *Problems*—Green Set, Nos. 33, 45, Wiley

Nuffield Project: *Shape and Size* 2 , pp. 1–14; 25–61, Wiley

Nuffield Project: *Environmental Geometry*, Wiley. [Studying shape and size in the environment]

MANIPULATIVE DEVICES

Attribute Games and Problems (A Blocks) [Now, while the children's attention is focussed on properties of shapes, is the time to make this and the entire unit part of the classroom activities.] (Selective Educational Equipment; Webster, McGraw-Hill)

Geo Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Geoboards (Addison-Wesley)

Geometric Figures and Solids (La Pine Scientific; school supplier)

Geo Strips (Math Media; Selective Educational Equipment)

Pattern Blocks—for symmetry, ordering, and tessellating (Selective Educational Equipment; Webster, McGraw-Hill)

Polyhedra construction kits (Childcraft; Edmund Scientific; Creative Publications)

Soma Cubes and Wooden Block Puzzles (Edmund Scientific; Selective Educational Equipment)

COMMERCIAL GAMES

Polyhedron-Rummy (Scott Foresman)

Pyramid Puzzle (or Tower of Hanoi) (Creative Publications; World Wide Games)

PAGES 98–99

Objective

Given the pattern of a cube, the child will be able to construct a cube and identify its edges, faces, and vertices.

Preparation

Materials

5-cm cardboard squares (1 per child); glue or transparent tape; heavy paper (1 sheet per child)

To arouse enthusiasm for this lesson, you might show the children a flat piece of paper and a cube and ask them if they think they would be able to use a flat piece of paper to make a cube.

Investigation

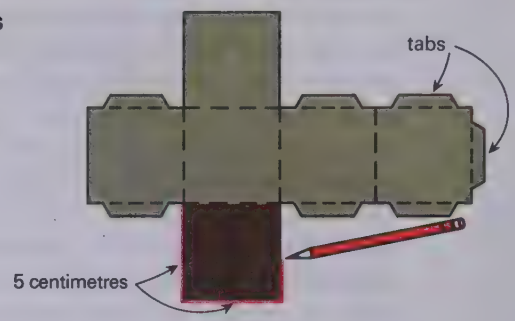
Read the directions with the children. Direct them to trace their first 5-cm square at least 5 centimetres from the top and bottom of the paper; otherwise, they are apt to begin tracing without leaving enough space for the side squares. Also, remind them about the tabs, noting that they should draw the tabs onto their pattern before they begin cutting. (If you prefer, distribute duplicated patterns of the cube similar to that in the text. See *Duplicator Masters*, page 69.) Give the children ample time to construct the cube; some may find the construction awkward. If possible, have the children glue or tape the tabs on the inside of the cube so that they will not be seen. However, if this is too awkward for some to do, let them tape some of the tabs on the outside. *Caution the children to handle the cubes carefully, for they will be used again in later lessons.*

4 Geometry

Let's construct a cube.

Investigating the Ideas

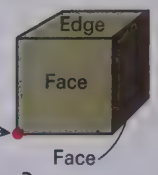
Use a 5-cm cardboard square to help you draw a pattern like this on heavy paper.



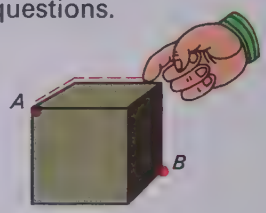
? Can you cut out your pattern and fold it to make a cube? Use paste or tape to stick the tabs in place.
See Investigation.

Discussing the Ideas

1. How many **faces** does your cube have? 6



2. How many **edges** does your cube have? 12
3. How many **vertices** (corners) does your cube have? 8
4. Use your cube to help you answer these questions.
 - A What is the smallest number of edges you can move your finger along to get from *A* to *B*? 3
 - B What is the greatest number of edges you can move along to get from *A* to *B* if you never go along the same edge twice? 7
 - C Can you get from *A* to *B* by going along 4 different edges? No
 - D Can you get from *A* to *B* by going along 5 different edges? Yes



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Discussion

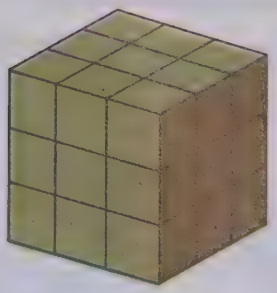
After the experience of constructing a cube, the children are provided with a review of the terms used to describe parts of a cube. The words *edge*, *face*, and *vertex* will be familiar to the children if they used the preceding book in this series, but the meaning of the terms should be reviewed, with specific application to the appropriate parts of a cube.

As you discuss exercises 1, 2, and 3, it would be helpful to use an actual demonstration cube and describe the terms *faces*, *edges*, and *vertices*, as illustrated on student book page 98.

In discussing exercise 4, give the children an opportunity to show the various paths they can trace along the edges of a cube to get from point *A* to point *B*. You might suggest that they lightly mark points *A* and *B* on their cubes to help them investigate these questions.

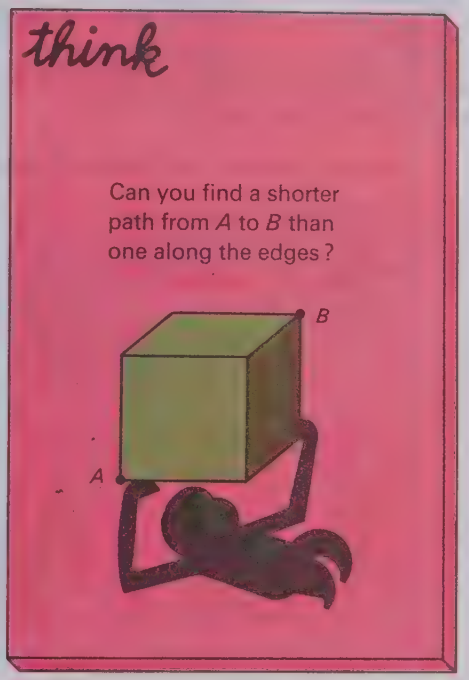
Using the Ideas

1. Each student in a class made a cube. Then they stacked their cubes together like this → to form one large cube.
 - A How many cubes in each layer? 9
 - B How many layers? 3
 - C How many students in the class? 27



2. If you could not pick up the stack, how many of the cubes could you **not** see at all? 2

3. If you could pick up the stack and look at all sides, how many of the cubes could you **not** see at all? 1



★4. A What is the area of each face of your cube in square centimetres? 25 cm²
 B What is the total area of **all** the faces? 150 cm²

★5. What is the volume of your cube in cubic centimetres? 125 cm³



Using the Exercises

If possible, provide 27 cubes that can be stacked together. (If necessary, the cubes which the children constructed may be used.)

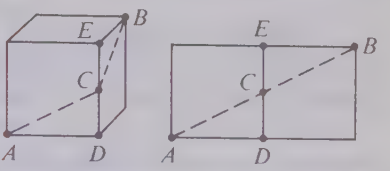
You might use starred exercises 4 and 5 as a basis for discussion. Lead the children to realize that since the length of each square face is 5 centimetres, there are 25 square centimetres on each face. Since there are 6 faces, there are 150 square centimetres in all. Remind the children how to measure volume. Have them imagine a cubic centimetre fitting inside the box. Since there are 25 square centimetres on the base and 5 centimetres in height, they should picture 125 cubic centimetres all together.

Assignments (page 99)

Minimum: 1–3, oral. Average: 1–3. Maximum: 1–5.

Solution, Think, page 99

The *Think* problem may challenge even the more imaginative children. The simplest solution is to think of the segments AC and CB as shown below, where C is the midpoint of DE.



If we imagine the two sides of the square laid flat, it is easy to see why the path from A to C to B is shorter than any other path that could be chosen. If children do not find this path, you might suggest it to them and have them compare its measure to that of alternative paths.

Resources for Active Learning

Activities in Geometry for Primary Pupils, Activities 4, 13, 22, Addison-Wesley.

Franklin Series: *Pencil and Paper Geometry*, pp. 85–88, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Geometry in the Classroom, pp. 23-27; 76-80, Holt, Rinehart and Winston.

Math Activity Cards, C16, D13, 17, Macmillan.

Nuffield Project: *Problems*—Green Set, No. 27, Wiley.

PAGES 100–101

Objective

Given various patterns of a cube, the child will be able to select one and make a cube.

Preparation

Materials

5-cm cardboard squares (at least 1 per child); tape or glue; heavy paper (at least 2 sheets per child)

This is an optional lesson. Its main purpose is to provide an opportunity for the children to enjoy some recreational mathematics. It should only be used if the children enjoyed the activity in the previous lesson. No specific preparation should be necessary.

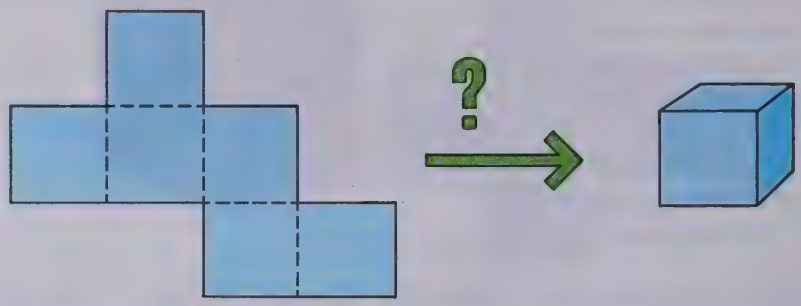
Investigation

Have the children use 5-cm cardboard squares as templates to copy the pattern at the top of the page. If they copy the pattern correctly, they should be able to fold it into a cube. Remind them to put tabs on a few outer sides before they cut it out.

Are there other patterns for cubes?

Investigating the Ideas

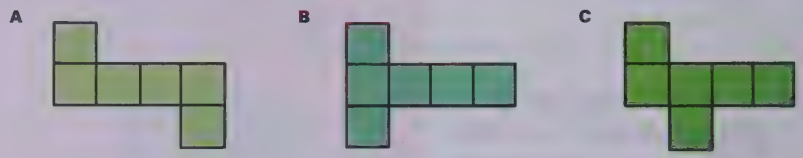
Make a large pattern of this figure and cut it out.



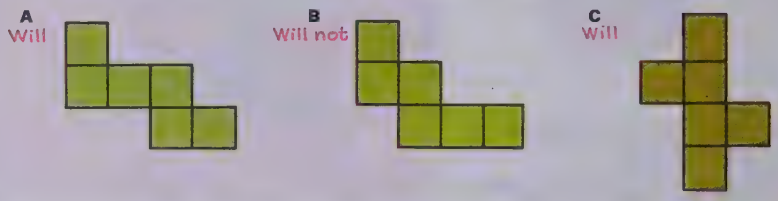
Can you fold this pattern to make a cube?
See Investigation.

Discussing the Ideas

1. Select at least one of these patterns and use it to make a cube. See Discussion.



2. Can you find which one of these patterns will not form a cube?





100

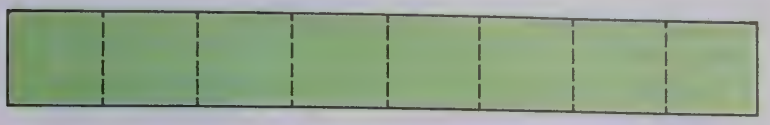
Discussion

Since one of the main purposes of this lesson is to give children an opportunity to enjoy constructing cubes, there is no need for a lengthy discussion. Exercises 1 and 2 may be treated as an extension of the investigation. Some children may construct cubes from several patterns in exercise 1, but you might suggest that they do exercise 2 before they spend all of their time with the patterns in exercise 1. Also, some might investigate making patterns of other shapes, such as a rectangular prism.

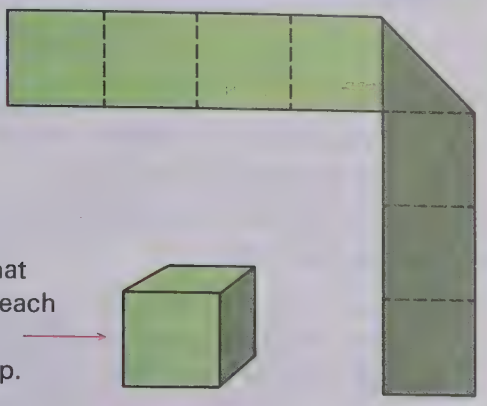
See Using the Exercises.

Using the Ideas

1. A Make a pattern like the one below and cut it out.

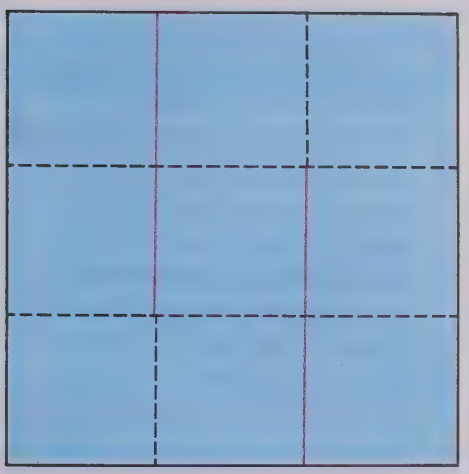


B Now fold the strip as in the figure. →



C Continue folding so that you have a cube with each face the size of one of the squares on the strip. →

★ **2.** Trace the square. Color the interior of the square. Cut along the solid red lines.



A Folding only along dashed lines, make a cube.

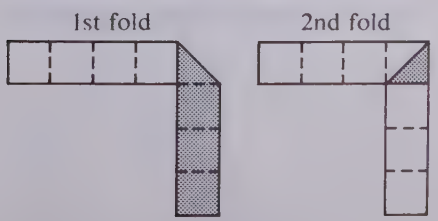
B Folding only along dashed lines, make a cube that has all its faces colored.



101

Using the Exercises

Although there are a variety of ways by which to fold the strip pictured in exercise 1 to form a cube, the simplest is the following: After the initial fold (illustrated in the text), the face on which the diagonal fold has been made may be folded back upon the face to the left of it.



Then the sides may be wrapped around without the need for any other folds on the diagonal. It would be best to let the children try to discover this or other possible solutions on their own.

Assignments (page 101)

Minimum: 1. Average: 1.
Maximum: 1–2.

Follow-up

Children might enjoy coloring various faces on the pattern of a cube and seeing where these faces appear when they fold the cube. Many variations are possible and they would make an attractive room display.

Resources for Active Learning

Activities in Geometry for Primary Pupils, Activities 42, 43, 44, Addison-Wesley.

Developmental Math Cards, G³14, Addison-Wesley.

Experiments in Mathematics, Stage 1, pp. 26–27, Houghton Mifflin. (Available from Thomas Nelson & Sons)

Geometry in the Classroom, pp. 76-77. Holt, Rinehart and Winston.

Math Activity Cards, C15, D12, Macmillan.

PAGES 102–103

Objectives

Given a cube, the child will be able to find parts of it which suggest points, angles, and segments.

Given the terms point, line, line segment, ray, *and* angle, *the child will be able to find objects which suggest these ideas.*

Preparation

Materials

cubes made in the first lesson of this chapter; demonstration square (optional)

It would be best to incorporate the preparation time with the investigation, so begin immediately with the text.

Investigation

Use the square illustrated in the text to review the concepts of point, angle, and segment. Make sure the children see how the square suggests four of each. For this purpose, you might use a large demonstration square and have children point out the 4 points, 4 segments, and 4 angles.

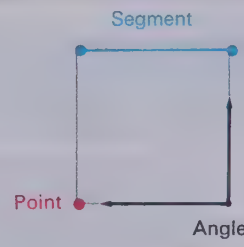
Then direct the children to study the investigation question, using the cube they constructed previously. They should discover 8 points corresponding to the 8 corners and, if they count carefully, 12 segments corresponding to the 12 edges. However, they may have difficulty finding all the angles. After they have had sufficient time to do their own thinking, you might suggest some easy ways to "count" the angles. For example, the cube has 6 faces or squares and for each of these there are 4 angles, so altogether there must be 24 angles.

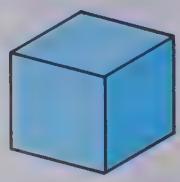


What are some simple figures on a cube?

Investigating the Ideas

A **square** suggests
4 points
4 angles
4 segments





See Investigation.

How many points, angles, and segments can you locate on your cube?
8 points, 24 angles, 12 segments

Discussing the Ideas

1. You can think of a **point** as a location. What are some things around your room that remind you of points?



Sample answers: dot on the chalkboard, cities' locations on a wall map, thumbtack on the bulletin board, etc.

2. You can think of a **segment** as a certain "part of a line." (The line does not end.) Name some things in your room that remind you of segments.



Sample answers: edges of chalkboard tray, picture frame, window frame, etc.

3. You can think of a **ray** as a certain "part of a line." An **angle** is 2 rays from one point. The angles of the square and cube are **right angles.** Are there things in your room that remind you of right angles?



Sample answers: corner of a book or piece of paper, corner of the room or table, etc.

4. Can you find some angles that are not right angles?

Sample answers: various angles formed by scissors' blades or compass legs, hands of clock, etc.

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Discussion

One of the main points of this lesson is to review some basic geometric concepts. If the children studied from Book 3 of this series they have been introduced to these concepts and terms previously. As you discuss exercises 2 and 3, help children to understand the idea that a line continues on and on, that a line segment is a part of a line that has two endpoints, and that a ray is a part of a line that has one endpoint.

Some things which the children might mention for each concept are as follows.

point: pencil tip, corner of a book, centre of the clock

line segment: window frame, edge of box or book, metre stick

ray: flagpole, each of the hands of the clock

right angle: corner where floor and wall meet, corners of bulletin boards, chalkboard

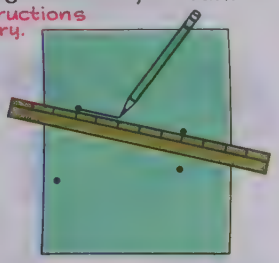
angles that are not right angles: angles formed by the hands of the clock, a book partially opened

See Using the Exercises.

Using the Ideas

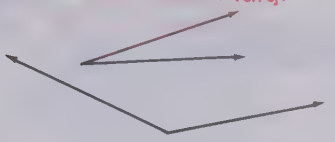
1. Mark 4 points on your paper. Use them to draw as many segments as you can.

Constructions will vary.

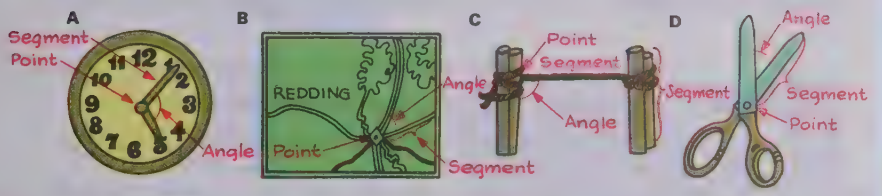


2. Some angles are more "open" than others. Draw three "different-looking" angles.

Constructions will vary.



3. Tell what part of each picture reminds you of points, segments, or angles.



★ 4. Study the chart. Then draw and name a figure of each type.

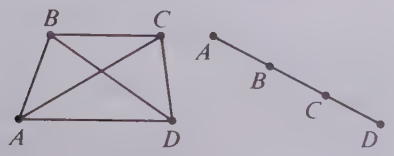
Constructions will vary.

We see	We label some points	We write	We say
	R S	$\overleftrightarrow{RS}$	"line *RS*"
	X Y	$\overrightarrow{XY}$	"ray *XY*"
	A B	$\overline{AB}$	"segment *AB*"
	E, F, G	$\angle EFG$	"angle *EFG*"



Using the Exercises

Direct the children to work independently on the exercises on page 103. In exercise 1, it is not necessary for the points to be noncollinear; that is, if a child happens to mark the points on one line segment, it will not matter. In either case, there are 6 line segments which may be drawn.



$\overline{AB}$, $\overline{BC}$, $\overline{CD}$ $\overline{AD}$, $\overline{AC}$, $\overline{BD}$

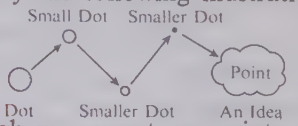
$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AC}$, $\overline{AD}$, $\overline{BD}$

Although exercise 4 is starred, it would be helpful to use the chart as a basis for discussion. If time permits, you might have the children label the points and line segments they drew in exercise 1 and the angles they drew in exercise 2. Stress that the middle letter of the abbreviation for an angle is the letter at the vertex of the angle.

Assignments (page 103)
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Mathematics

The study of geometry is the study of sets of points and the relationships between special sets of points. In abstracting the idea of a geometric point, we might proceed as suggested by the following illustration.



Although we cannot see points, we say, "draw a point" when we mean *draw a dot to represent a point.*

Ideas of point, line, and plane are the basic notions with which the study of geometry starts. All geometric figures can be considered as sets of points, often as subsets of lines or planes, or as the union of some of these subsets. Some of these sets of points (called geometric figures) which are introduced in this lesson are described below.

Some Geometric Figures (Sets of Points)		
Description	Picture	Name and Symbol
Two points and all the points between them	A B	Segment $\overline{AB}$
A half-line with the endpoint	S R	Ray $\overrightarrow{RS}$
The union of two rays with a common endpoint	M N Q	Angle $\angle MNQ$

Follow-up

The children might enjoy making posters to illustrate objects which suggest the geometric ideas discussed in this lesson. Others might draw basic geometric symbols, such as point, line, line segment, ray, and angle, and label them as shown in the chart in exercise 4.

Resources for Active Learning

Franklin Series: *Pencil and Paper Geometry*, pp. 26–37, 63–70, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Geometry in the Classroom, pp. 82-85, Holt, Rinehart and Winston.

Mathematics in Modules, SK6, Addison-Wesley.

Workbook, page 27

PAGES 104–105

Objectives

Given parallel lines or segments, the child will be able to identify them as parallel lines or segments.

Given suitable materials, the child will be able to draw parallel line segments, a parallelogram, a rectangle, and a rhombus.

Preparation

Materials

cubes constructed in the first lesson of this chapter; demonstration cube (optional)

Since the investigation in this lesson requires a discussion of the items illustrated, it would be appropriate to begin immediately with the investigation.

Investigation

In order for the children to identify the parallel edges of their cube, they must have a clear understanding of parallel lines. Use the illustrations to discuss parallel lines. Help the children notice that parallel lines are always the same "straight" distance apart.

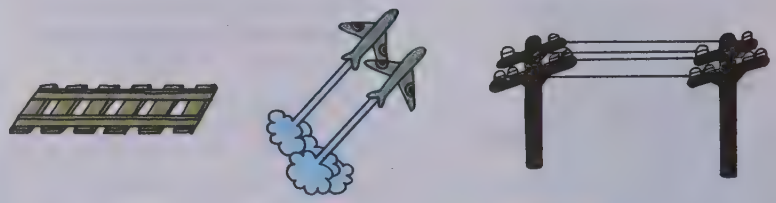
Direct the children to color one edge of their cube. It would be helpful for them to label each corner of the cube, if you wish to have them make a record of the edges they find that are parallel. However, such labelling would not be useful in discussion unless everyone labelled the corners in the same order. If possible, use a large demonstration cube and have children use it to show the parallel edges and the edges that are not parallel.



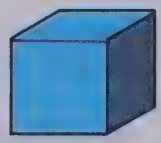
When are two lines or segments parallel?

Investigating the Ideas

These pictures suggest **parallel lines.**



Your cube has some parallel edges (segments). Color one edge of your cube.

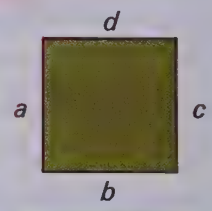


? How many segments on your cube are parallel to the colored segment? How many are not? 8
See Investigation. 3

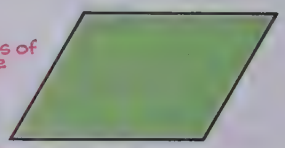
Discussing the Ideas

1. Can you find some things in your room that remind you of parallel lines or segments? See Discussion.

2. The sides of the square are marked *a, b, c,* and *d*. Which pairs of sides are parallel to each other? a ‖ c, b ‖ d



3. Why do you think this figure is called a parallelogram? Both pairs of sides are parallel. Do you think a square should be called a parallelogram? Yes, a "special kind" of parallelogram.



Discussion

One of the main objects of this lesson is to develop the concept of parallel lines so that children can recognize parallel lines in figures such as a square, rectangle, parallelogram, and rhombus.

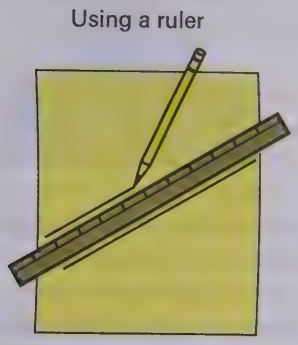
In exercise 1 children might mention the opposite borders of a bulletin board or the chalkboard, lines on a tablet or binder paper, and opposite sides of window frames, among other items. Accept any object in which the child points out some pair of parallel lines.

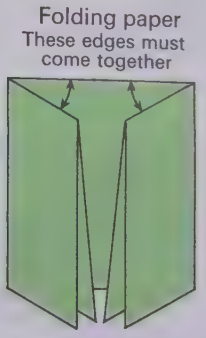
As you discuss exercise 3, help the children recognize that a parallelogram is any four-sided figure in which opposite sides are parallel and that a square satisfies these conditions. You might also discuss the statement, "Every square is a parallelogram, but not every parallelogram is a square."

See Using the Exercises.

Using the Ideas

1. The two pictures suggest ways you might make parallel lines. Draw some parallel lines by using each method.





Can you find a method of your own for drawing parallel lines?

2. Use parallel lines to help you draw a parallelogram and a rectangle.

Constructions will vary.



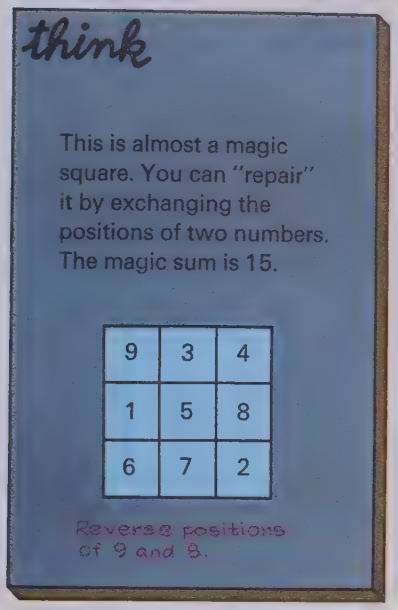
Parallelogram



Rectangle

3. A rhombus is a parallelogram with all sides the same length. Can you use your ruler to draw a rhombus?

Yes. Constructions will vary.



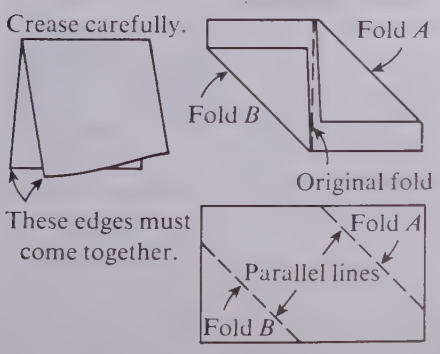


105

Using the Exercises

You might choose to have the class work through several of the exercises on page 105 together. Help children interpret the paper-folding diagram.

Here is another way to make parallel lines by folding paper.



In exercise 2 you might suggest using a combination of the methods illustrated in exercise 1. Thus, if the children first fold their paper and then use their ruler to form the second pair of parallel lines, they will form a parallelogram that is not a rhombus. Similarly, if they line up their ruler at right angles to the lines of folding they will form a rectangle that is not a square.

Assignments (page 105)

Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Mathematics

A key point in this lesson is raised by the following definition of parallel lines.

Two lines are parallel if they are in the same plane and they do not intersect.

If asked to draw parallel lines, the children might draw a pair of line segments which we would not want to call parallel, even though they do not visibly intersect on the particular sheet of paper on which they have been drawn. The children should be led to an understanding of parallel lines which will enable them to make such observations as, "Parallel lines do not intersect no matter how far we 'extend' them."

Definitions of the various quadrilaterals are given in the Glossary. Some of the distinguishing characteristics of these quadrilaterals are given below. You may wish to incorporate them in a discussion of the various parts of this lesson.

Square—all sides equal in length, four right angles

Rectangle—opposite sides equal, four right angles

Rhombus—all sides equal in length (Note that a rhombus may be a square.)

Parallelogram—opposite sides are parallel

Follow-up

Children might enjoy coloring the figures they have drawn, or they might make others using colored construction paper.

Resources for Active Learning

Activities in Geometry for Primary Pupils, Activity 34, Addison-Wesley.

Franklin Series: *Learn to Fold . . .*, "Parallel Folds and Segments," pp. 86–88, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Geometry No. 4, "Parallel Lines," pp. 25–26, Encyclopaedia Britannica Publications Ltd.

Mathex: Geometry No. 9, "Lines in Space," pp. 7–9, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Green Set, No. 20, Wiley.

Nuffield Project: *Shape and Size* 2, "Parallel Lines," p. 89, Wiley.

Workbook, page 28

PAGES 106–107

Objective

Given suitable materials, the child will be able to construct a model of a triangular pyramid and identify its edges, faces, and vertices.

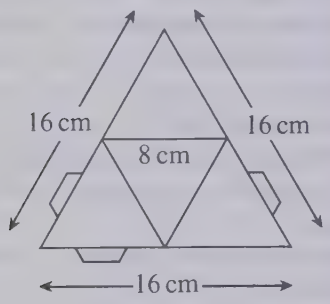
Preparation

Materials

cube from previous lesson; paper for tracing and cutting; heavier paper for construction of a pyramid; tape or glue

Or, patterns duplicated on paper suitable for the construction of a pyramid (*Duplicator Masters*, page 70)

The usual preparation time would be well spent in organizing the materials for the investigation. If possible, have a pattern duplicated on suitable paper so that the children need only cut and fold. Otherwise, supply the children with paper on which they can trace the triangle to be used in drawing the pattern.



Suitable dimensions for a pattern

Investigation

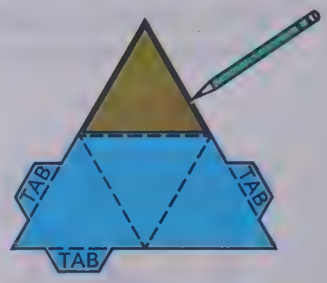
Give the children considerable guidance in constructing the pyramid. If they are drawing the pattern themselves, direct them to place the first triangle far enough from the edge of the paper to allow room for the other triangles. Once the children have cut out the pattern, remind them to refer to the illustration for guidance in folding the pyramid. Then move around the room helping children who are having difficulty.



Let's construct a triangular pyramid.

Investigating the Ideas





Use your triangle "cut-out" four times to draw this pattern on heavy paper. Cut out your pattern on the solid lines. You may want to add the tabs shown.

Triangular Pyramid



? Can you fold your pattern to make a triangular pyramid? See Investigation.

Discussing the Ideas

1. Give the missing words.
 - A Each **face** of your pyramid has the shape of a __?__. triangle
 - B Each **edge** of your pyramid reminds you of a __?__. segment
 - C Each **vertex** of your pyramid reminds you of a __?__. point
2. Which do you think has the greater volume, your cube or your pyramid? Cube. See Discussion.

106

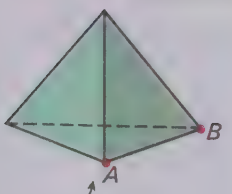
Discussion

When the children have constructed their pyramid, use the statements in exercise 1 as a basis for discussion. Exercise 2 compares the volumes of the cube and pyramid. Challenge the children to think of a way of finding the volumes of the cube and pyramid, such as by placing both in a container of water and comparing the difference in the water displacement; or by opening a face of the cube and the base of the pyramid and filling them with sand for comparison. Commercial or very sturdy models might be more suitable for such explorations. It would be appropriate also to include in your discussion the questions in exercise 1 on page 107. Then discuss comparisons of various properties of the cube and pyramid, such as the number of vertices, number of edges, and number of faces. These might be written in chart form on the chalkboard as shown.

	Cube	Triangular Pyramid
Number of edges	12	6
Number of vertices	8	4
Number of faces	6	4

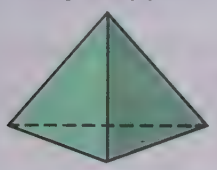
Using the Ideas

1. A How many faces does the triangular pyramid have? 4
 B How many edges does it have? 6
 C How many vertices does it have? 4

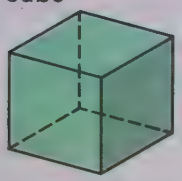


2. Use your triangular pyramid to answer these questions.
 A What is the smallest number of edges you can move along to get from *A* to *B*? 1
 B What is the greatest number of edges you can move along to get from *A* to *B* if you never go along the same edge twice? 4

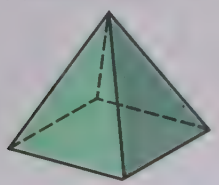
3. For each figure below, add the number of vertices to the number of faces. Then subtract the number of edges. What is your answer each time? 2
 A triangular pyramid

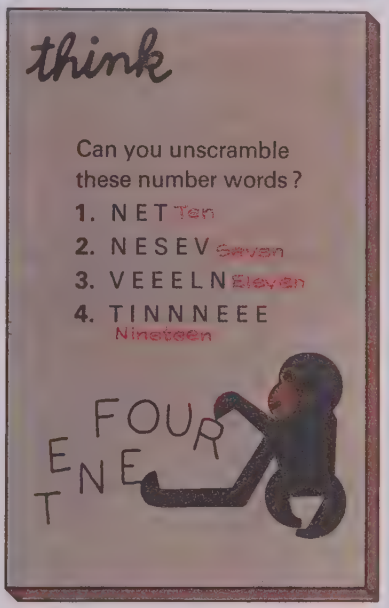


 B cube



 C pyramid with one square face







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Using the Exercises

You might choose to work through the exercises on page 107 together with the class. Note that exercise 1 was suggested for possible use in the discussion section. In exercise 2, have the children mark two vertices, *A* and *B*, on their pyramid so that they can use it to study the questions. Children should be able to use the pictorial representations to answer exercise 3.

Assignments (page 107)
Minimum: 1–3, oral. Average: 1–3. Maximum: 1–3.

Mathematics

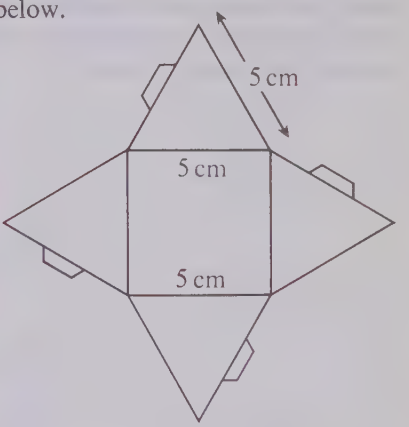
In addition to providing children with an opportunity to investigate and work with a solid geometric figure other than the cube, this lesson could be used to provide some experiences that lead to an intuitive grasp of the Euler formula concerning the relation between the number of vertices, faces, and edges of polyhedrons.

If v is the number of vertices, f is the number of faces, and e is the number of edges, then $(v + f) - e = 2$ for space figures such as those described above.

If children show interest in this relationship, provide models of other polyhedrons and let the children check to see whether the relation given by the formula above holds for other geometric solids.

Follow-up

As a follow-up activity, or for those who construct the triangular pyramid very quickly, you might distribute a pattern of another figure, such as a pyramid which has a square as one face. A pattern with appropriate dimensions is given below.



Resources for Active Learning

Activities in Geometry for Primary Pupils, Activities 11, 13, Addison-Wesley.

Experiments in Mathematics, Stage 1, "The Pyramid," pp. 28–29, Houghton Mifflin. (Available from Thomas Nelson & Sons)

Franklin Series; *Pencil and Paper Geometry,* pp. 89–96, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Geometry in the Classroom, pp. 76-80, Holt, Rinehart and Winston.

Nuffield Project: *Shape and Size* 3, "Polyhedra," pp. 55–59, Wiley.

PAGES 108–109

Objective

Given pictures of closed curves, the child will be able to identify which of the curves are simple closed curves and to state whether a given point is in the interior or the exterior of the simple closed curve.

Preparation

Materials

flannelboard; yarn and thin strips of paper for demonstration (optional)

Although this investigation includes no construction of figures and is based on a study of the text, it would be best to omit specific preparation. The main goal here is for children to discover what simple closed curves are.

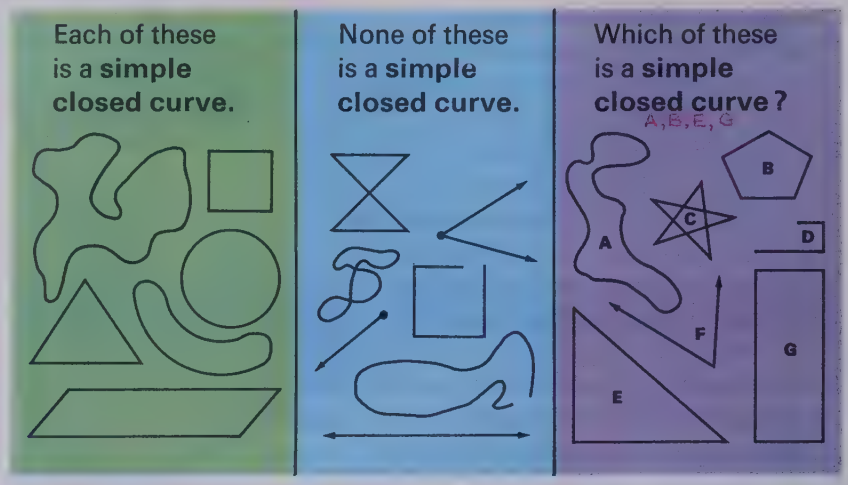
Investigation

Direct the children to the illustrations at the top of the page. Explain that they are to study each of the three groups of figures and see whether they can discover what a simple closed curve is. When they think they know what a simple closed curve is, they are to draw some of their own. Circulate through the room and check to see whether or not the figures the children are drawing are simple closed curves.



What is a simple closed curve?

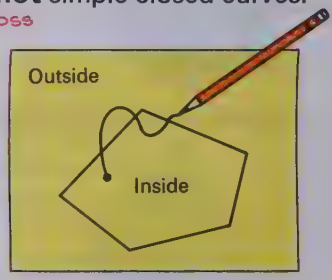
Investigating the Ideas



Can you draw some simple closed curves of your own?
Constructions will vary. See Investigation.

Discussing the Ideas

1. You can think of a simple closed curve as a loop of string that is on a flat surface and does not cross itself. Explain why some of the figures above are **not** simple closed curves.
 Sample answers: not closed, lines cross
2. Draw a simple closed curve and put a dot inside.
 - A Starting at the dot, draw a path that crosses the curve 6 times. Where will the end of your path be? Inside
 - B Where will the end of the path be if you cross the curve 9 times? Outside



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Discussion

The goal of this lesson is not to develop a formal definition of simple closed curves but to help the children develop an intuitive idea of simple closed curves so that they can recognize them in comparison with curves that are not simple closed curves. As you discuss exercise 1, you might expect children's explanations of why some of the figures are not simple closed curves to include such observations as: "Lines in the figure cross each other," "The figure is open," "The curve loops back on itself," "A line can't be a closed curve," and so on. Be sure the children realize that a simple closed "curve" may be a figure formed of straight line segments. It would be helpful to use a flannelboard with yarn and straight line segments and have the children illustrate some simple closed curves.

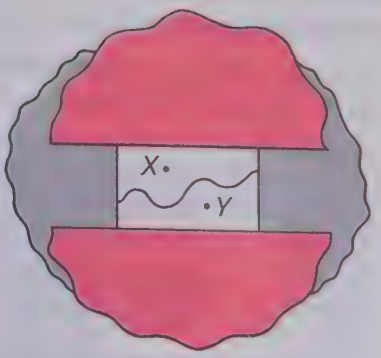
If the children follow the development of the questions in discussion exercise 2, they should come to the conclusion that a path which crosses the curve an *even* number of times always *ends on the same side* of the curve on which it began; a path which crosses the curve an *odd* number of times always *ends on the other side* of the curve.

Using the Ideas

1. The red and gray screens are hiding all but a small part of a simple closed curve.

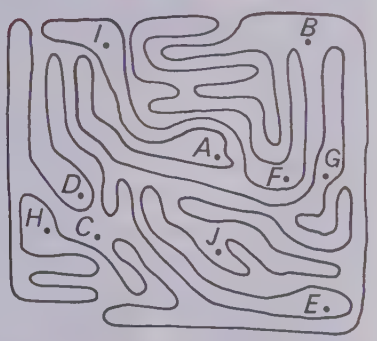
A If X is inside, where is Y? Outside

B If X is outside, where is Y? Inside



2. If A is outside the simple closed curve, give the location (inside or outside) of each of the other points.

B Inside
C Inside
D Outside
E Outside
F Outside
G Outside
H Outside
I Inside
J Outside



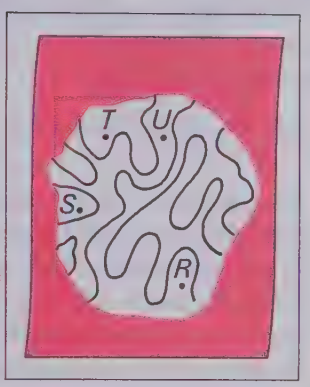
3. The hole torn in the red paper shows part of a simple closed curve.

A If R is inside, where is S? Inside

B Can both T and U be inside? No

C If S is outside, where is T? Outside

D If S is outside, where is U? Inside





Using the Exercises

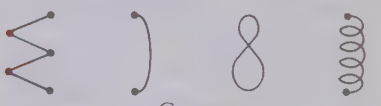
The exercises on page 109 extend the ideas introduced in discussion exercise 2. You may choose to work through exercise 1 with the children, but assign exercise 2 as independent work. Suggest to the children that drawing a straight line with a ruler from point A to each other point will enable them to count the number of times that path crosses the curve. After the children have finished the exercises, allow sufficient time for discussion.

Assignments (page 109)

Minimum: 1–3, oral. Average: 1–3. Maximum: 1–3.

Mathematics

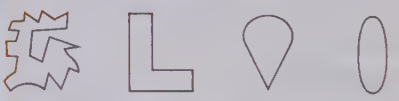
The children can draw plane curves without lifting their pencils from the paper. Such a curve becomes a closed curve if a child brings his pencil back to the same point he started from. If a closed curve does not intersect itself at any point, it is a simple closed curve. Some examples are shown below as well as in the student text. Line segments are plane curves because plane curves may or may not have portions that are straight.



Curves



Closed Curves



Simple Closed Curves

Follow-up

To encourage the children to think further about the inside and the outside of a simple closed curve, suggest considering these situations:

1. Think about the walls of the room forming a simple closed curve. If Suzy starts in the room and crosses the threshold into the hall 7 times, does she stop inside the room or in the hall? Where is she after 9 times? After 4 times?
2. A squirrel is gathering nuts for his winter food supply. Each morning he makes several trips in and out of his hole. One morning, after crossing the entrance to his hole 17 times, he sits back to rest. Is he resting inside his tree house or outside? Where would he be if he crossed the entrance 5 more times? 30 more times?

Resources for Active Learning

Experiments in Mathematics, Stage 1, "Topology," pp. 46–47, Houghton Mifflin. (Available from Thomas Nelson & Sons)

Geometry in the Classroom, pp. 39-40; 59-61, Holt, Rinehart and Winston.

Mathex: Geometry No. 9, "Open and Closed Curves . . . ," pp. 20–21, Encyclopaedia Britannica publications Ltd.

Objectives

Given the set of strips listed in the student text, the child will be able to construct the following quadrilaterals: parallelogram, trapezoid, rhombus, rectangle.

The child will be able to draw pictures of each of the following polygons: triangle, quadrilateral, pentagon, hexagon, octagon.

Preparation

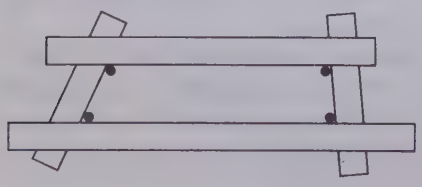
Materials

colored strips, as listed on student book page 110

Review the term *simple closed curve* by asking a few children to draw an example of a simple closed curve on the chalkboard. If none of them draws a straight-line figure include one yourself. Also, ask for examples of figures that are not simple closed curves.

Investigation

Before the children begin their constructions of the quadrilaterals, it would be helpful to guide them in using the strips to make a triangle. Direct them to take one red, one light green, and one purple strip. Use some demonstration strips and the flannelboard, or small strips and the overhead projector, to show them how to place the strips, mark a dot at each vertex, remove the strips, and with pencil connect the dots. All of the figures may be constructed with the strips, but some children may find this method awkward. If so, you might wish to have the children use alternative methods to construct the same figures. For example, you might have them draw the figures on graph paper or construct them on geoboards. Larger strips of paper could also be used. In this case, the corners need not meet exactly, but the child still should mark the dots on the inside of every intersecting pair of strips and then use ruler and pencil to connect the dots.



Let's explore some special simple closed curves.

Investigating the Ideas

You can use your strips to help you "construct" some special simple closed curves.

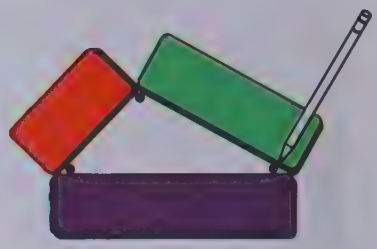
You will need

2 of these →

4 of these →

1 of these →

1 of these →



Place the corners of the strips together. Mark a dot at each corner and then connect the dots.

? Which of the special figures below can you "construct" with the strips listed above?

See Investigation.

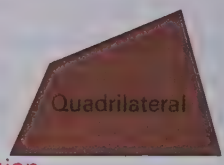


Discussing the Ideas

1. Tell all you can about the special figures you constructed.

See Discussion.

2. Simple closed curves whose sides are segments are called **polygons**. A polygon that has four sides is a **quadrilateral**. Can you use the strips to draw a quadrilateral with all of its sides different lengths? Yes. See Discussion.



110



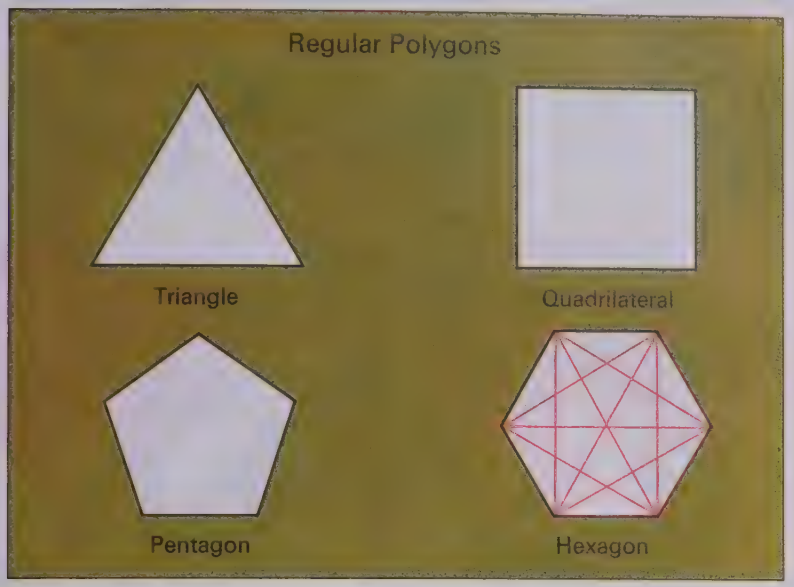
Discussion

Have children use whatever materials they worked with in the investigation to demonstrate the various figures they made. Then, to elicit meaningful responses to discussion exercise 1, you might pose a specific question such as: "How does a rectangle differ from a square?" "How many sides of a trapezoid are parallel?" "In what way are the square and the rhombus alike?" "In what way are they different?" and so on. (See descriptions of the figures in the mathematics section on page 105.)

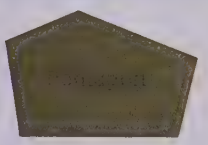
Stress that each of these figures is a *quadrilateral* (*quadri*=four, *lateral*=side). Help the children read the various figure names correctly, but do not place undue emphasis on vocabulary.

Using the Ideas

1. A **regular** polygon has all of its sides the same length and all of its angles the same size. What is another name for a regular quadrilateral? Square

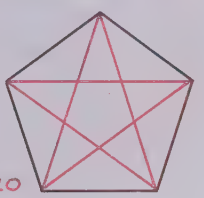


2. Draw one of each kind of polygon shown that is **not** regular. Example:
Constructions will vary.



3. An **octagon** is an 8-sided polygon. Draw an octagon.
Constructions will vary.

4. This pentagon has all its diagonals shown in color. Can you draw all the diagonals of a hexagon? See above.



★ 5. How many diagonals does an octagon have? 20

111

Follow-up

Encourage children to see how many kinds of polygons they can construct on the geoboard. Challenge them also to find how many of the regular polygons they can construct on a 5-by-5 geoboard. (The only *regular* polygon that can be constructed thus is the square.)

Resources for Active Learning

Activities in Geometry for Primary Pupils, Activities 26, 30, 33, 38, 39, 40, 47, Addison-Wesley.

Developmental Math Cards, F[3]14, G[3]4, Addison-Wesley.

Experiments in Mathematics, Stage 2, "Game of Hex," p. 53, Houghton Mifflin. (Available from Thomas Nelson & Sons)

Geometry in the Classroom, pp. 40-59, Holt, Rinehart and Winston.

Math Activity Cards, "Tessellations," C13, 19; "Regular and Irregular Polygons," C21, Macmillan.

Mathex: Geometry No. 9, "Constructing Plane Figures," p. 12 (pupil pages 13–19), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Green Set, Nos. 7, 40, Wiley.

Nuffield Project: *Shape and Size* 2, "Classification of 2-D Shapes," pp. 91–101, Wiley.

Nuffield Project: *Shape and Size* 3, "Rigid or non-rigid 2-D Frameworks," pp. 2–10 (pictures, pp. 37–40), Wiley.

Duplicator Masters, page 23

Using the Exercises

Much of the material on page 111 may serve as a basis for further discussion. You might ask for an oral response to the question in exercise 1, and with the children read the name of each polygon pictured and explain the meaning of the terms *polygon*, *regular*, and *diagonal*. The children might then draw the irregular polygons, as suggested in exercise 2, and continue with exercises 3 and 4 on their own. Allow a child who successfully answers starred exercise 5 to demonstrate his answer for the class.

Assignments (page 111)

Minimum: 1–2. Average: 1–4.
Maximum: 1–5.

PAGES 112–113

Objective

Given a variety of different-shaped figures, the child will be able to determine whether the figure is symmetric by folding it in half.

Preparation

Materials

paper for tracing, cutting, and folding

It would be helpful, though not essential, for each child to have a rectangle to fold as pictured at the top of page 112. However, you may judge that a demonstration rectangle, as suggested in the section that follows, is sufficient.

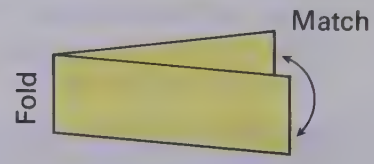
Investigation

Read with the children the explanation of a symmetric rectangle at the top of the page. It would be helpful to use a demonstration rectangle to be sure the children understand what *symmetric* means. Then direct them to trace, cut, and fold the other figures to see which are symmetric. Suggest that those who finish quickly try to draw a figure which they think would be symmetric and then cut it out and fold it to see whether they were right.

When is a figure symmetric?

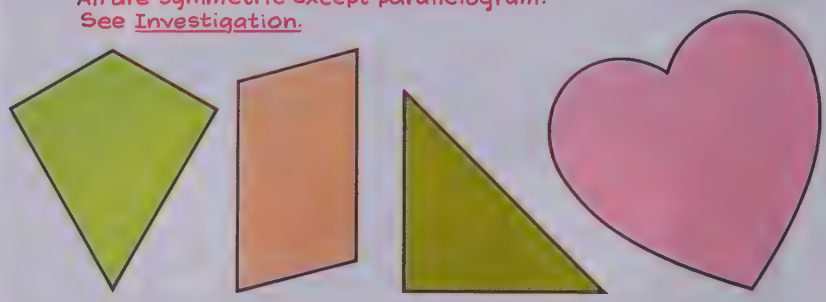
Investigating the Ideas

A rectangle is **symmetric** because you can fold it so that one half exactly matches the other half.



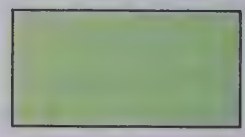
? Can you trace, cut out, and fold to find out which of these figures are symmetric?

All are symmetric except parallelogram.
See Investigation.

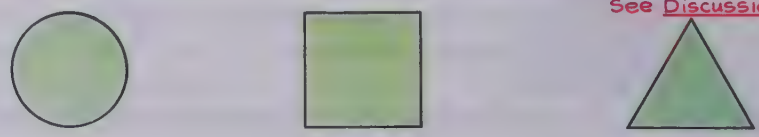


Discussing the Ideas

1. A figure is symmetric when it has a **line of symmetry** (the fold). Does a rectangle like this have more than one line of symmetry? Explain.
Yes. See Discussion.



2. Which of these figures have more than one line of symmetry? All.
See Discussion.

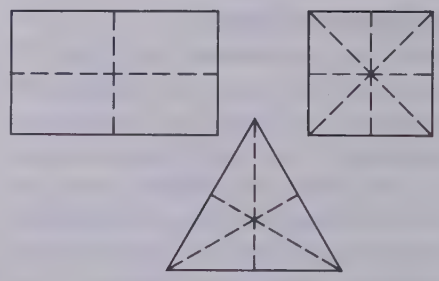




112

Discussion

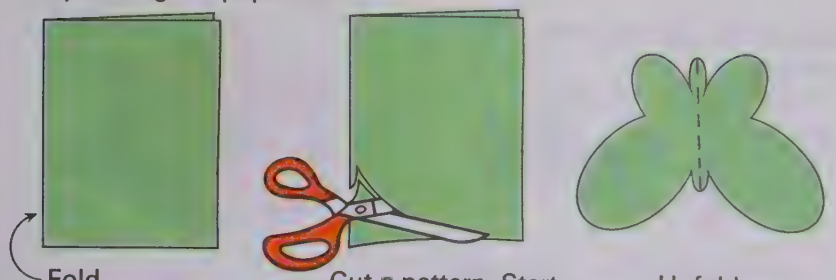
In order for the discussion questions to be understood, have the children trace, cut out, and fold the rectangle, circle, square, and triangle. If you prefer, prepare a duplicated worksheet of these figures so that the children need to do only the cutting and folding. Children will not have difficulty seeing that there are innumerable lines of symmetry in the circle—no matter how they fold it in half, the halves match. Similarly, a few foldings of the rectangle, square, and equilateral triangle will show the lines of symmetry for each, which are illustrated by dashed lines in the figures at the right.



You might ask the children if they think every triangle has a line of symmetry and point out that the triangle in the text is an equilateral triangle. A few experimentations with a scalene triangle, in which no sides have the same length, should convince them that not every triangle is symmetric.

Using the Ideas

1. You can make your own symmetric figures by folding the paper first.



Fold

Cut a pattern. Start and end on the fold.

Unfold

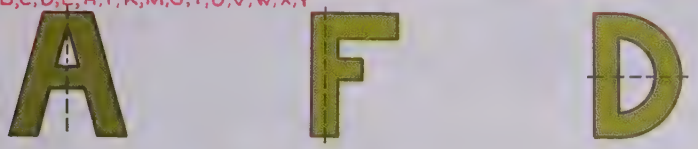
A Fold and cut out as many different quadrilaterals as you can.

B Cut out some special designs of your own.

Constructions will vary.

2. Some letters are symmetric and some are not. Can you find all the letters that are symmetric? (In some alphabet styles, K and U are less symmetric than in others)

A, B, C, D, E, H, I, K, M, O, T, U, V, W, X, Y



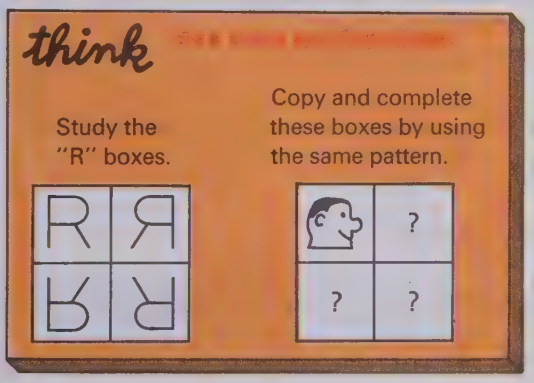
Yes No Yes

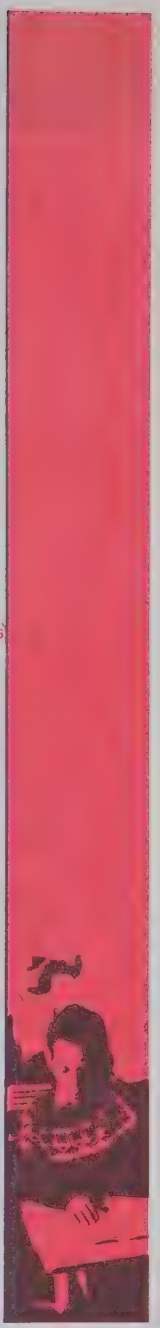
3. The word BIKE is a "symmetric word."



Can you find some other words that are symmetric?

Samples: BED, BOX, DEED, CHECK





Using the Exercises

Treat page 113 as an activity page. In exercise 2, the answers will depend on how the letters are printed. This exercise would be a good basis for discussion; be flexible in accepting a child's reason for why a letter is or is not symmetric. You might have them use letters from newspaper headlines, magazine titles, etc.

Encourage all the children to try the *Think* problem and then let a volunteer display the solution.



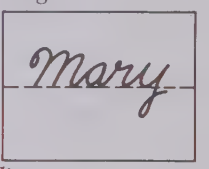
Assignments (page 113)

Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Follow-up

The children might enjoy studying patterns in leaves and snowflakes or creating more symmetrical designs of their own. Most leaves give the appearance of being symmetric, but if examined closely, the design made by the veins in the leaf is usually not symmetric. However, the outline of a leaf may be used to make a symmetrical figure. Another method for creating symmetrical designs is to have children fold a piece of drawing paper in half, then unfold the paper, and write their name along the fold line either in ink or heavy crayon. Fold the blank half over the name and press. The imprint on the blank side should be a mirror image dark enough for the children to use to finish the design.

Before folding:



After folding:



Resources for Active Learning

Activities in Geometry for Primary Pupils, Activities 16, 21, 24, 32, 36, 37, 41, 49, 50, Addison-Wesley.

Developmental Math Cards, G³14, Addison-Wesley.

Franklin Series: *Mirror Magic,* "Symmetry," pp. 58–107, Lyons and Carnahan. [Activities for small-group work in a math centre or for display] (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Geometry-Congruence and Symmetry," pp. 153–154, Addison-Wesley.

Geometry in the Classroom, pp. 57-59, Holt, Rinehart and Winston.

Nuffield Project: *Beginnings* 1, "Axes of Symmetry," pp. 91–92, Wiley.

Nuffield Project: *Shape and Size* 3, "Another Look at Symmetry," pp. 23–26, Wiley.

Workbook, page 29

PAGES 114–115

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

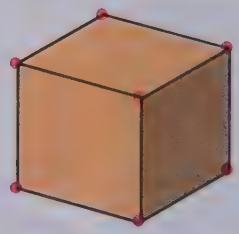
The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

You may plan a preparation suitable for either page 114 or 115. If you choose to use 114 as an evaluation page, you might use the preparation time for a brief review of major concepts of the chapter. If you use page 114 as a review lesson, you might plan a preparation for page 115, such as an oral game to review basic combinations or regrouping.

Reviewing the Ideas

1. The red dots suggest points (vertices).
 - A How many can you **see** in the picture? 7
 - B How many vertices does a cube have all together? 8

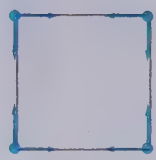


2. You can see three faces of the cube.
 - A How many faces can you **not** see in the picture? 3
 - B How many faces does a cube have in all? 6



3. How many edges (segments) can you count in the picture of a cube above? 9

4. How many right angles does a square have? 4



5. Draw a simple closed curve
 - A with segments only. Sample: △
 - B with no segments. Sample: ♡

 Constructions will vary.

6. Draw a quadrilateral (4-sided figure) that has a pair of parallel sides. Sample: ⏢ Constructions will vary.

7. Which of these figures is symmetric? B



114

Discussion

As children work through exercises 1, 2, and 3 on page 114, you might permit them to use the cubes they constructed in the first lesson of this chapter. Some children may need to be reminded that exercise 5A refers to a simple closed curve made of straight line segments and 5B refers to a simple closed curve made without any straight line segments. The term *quadrilateral* in exercise 6 may also require review.

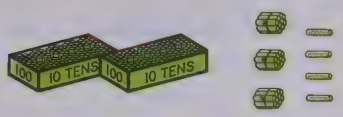
If you use page 114 as a review, discuss any concepts which have given difficulty during treatment of this chapter. If you use this page as evaluation, plan time for a discussion of each exercise to help children correct any erroneous concepts.

Keeping in Touch with

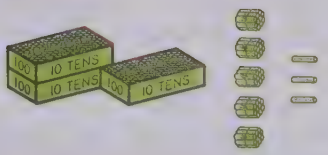
Place value
Addition
Subtraction
Measurement

1. How many?

A 234



B 353



2. Measure in centimetres.

A How long? 5 cm
B How wide? 2½ cm
C How far around all four sides? 15 cm



3. What is the area of the region? 18



4. Find the sums and differences.

A 36 +45 = 81	B 48 +55 = 103	C 96 +56 = 152	D 74 −21 = 53	E 63 −45 = 18	F 81 −19 = 62
G 714 −261 = 453	H 2337 +7665 = 10002	I 802 −358 = 444	J 7037 −2848 = 4189	K 1001 −747 = 254	
L 132 + 421 +214 = 767	M 278 + 517 +809 = 1604	N 843 + 126 +31 = 1000	O 425 + 189 +647 = 1261	P 23 427 + 19 366 +53 109 = 95 902	



You are invited to explore

ACTIVITY CARD 4
Page 349

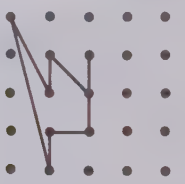
Using the Exercises

You might have the children work the exercises on page 115 independently or work through them with you. If your discussion accompanies the correction of the children's papers or the exercises themselves, present other examples to illustrate the concepts reviewed. For example, write several large numerals on the chalkboard and have children identify digits in various places. Review the term *perimeter* as you discuss exercise 2.

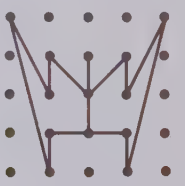
Follow-up

If children enjoy working with the geoboard, suggest that they work in pairs and challenge each other to complete a half of a symmetric figure each child constructs.

For example, one child might construct the following design.



Then his partner should complete it by constructing the half symmetric to it.



Workbook, page 30

General Objectives

To develop multiplication and division concepts

To associate multiplication with groups of equivalent sets, product sets, repeated addition, number lines, and skip counting

To relate division to repeated subtraction, multiplication, sets, and the number line

To develop mastery of multiplication and division facts

To develop basic principles of multiplication

To provide word problems involving multiplication and division

To provide additional work which requires logical thinking

The key ideas in this chapter include multiplication and division concepts, the relation between multiplication and division, and mastery of multiplication and division facts. Later chapters will deal with the multiplication and division algorithms for finding greater products and quotients.

Following a brief introduction in which we use rectangular arrays to investigate the multiplication and division operations, we concentrate on the relationships of multiplication to addition and of division to subtraction. Sets and number-line investigations are used to emphasize the relationship between multiplication and division. Then the basic principles for multiplication are explored and, with these basic principles and general reasoning, the multiplication-fact table is built step by step.

A number-line game is among the practice provided the children for finding products. Following this, the idea of finding quotients by thinking about missing factors is reviewed and division facts are emphasized.

We do not expect the children to memorize subtraction and division facts per se. Rather, we expect them to find differences and quotients by knowing the related addition and multiplication facts. The child is expected to know that $35 \div 5 = 7$ because he knows that $7 \times 5 = 35$.

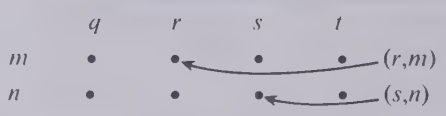
Following the work on division facts, the children analyze story problems designed to help them understand that division is used to solve problems involving one of two things: finding the number of sets of the same number or finding the number in each set. Material is then provided for a review of all operations and for a study of the set interpretation of multiplication.

Mathematics

We begin the study of multiplication by defining an operation on sets. Consider two sets, $C = \{q,r,s,t\}$ and $D = \{m,n\}$. To determine the set of all pairs of elements such that the first member of the pair is from C and the second is from D, pair each of the four elements of C with each of the two elements of D to give eight pairs. This set of pairs is known as the Cartesian product of sets C and D (written $C \times D$). The elements of $C \times D$ are

(q,m), (r,m), (s,m), (t,m),
(q,n), (r,n), (s,n), (t,n).

The diagram indicates these pairs in a rectangular array.



Careful examination of this array will reveal how it is related to the multiplication concept. The Cartesian product $C \times D$ has eight elements (pairs); eight is the product of four and two. The numbers 4 and 2 are the factors of 8, and 8 is a multiple of each number.

The product $D \times C$ is obtained by changing the order of the members in each pair of $C \times D$. Thus, $C \times D$ and $D \times C$ have the same number of pairs. This illustrates the order principle for multiplication. We demonstrate the order principle for children by having them consider rectangular arrays, first by rows, then by columns.

Our definition of division for whole numbers is related to multiplication.

If a, b, and c are whole numbers such that $b \neq 0$ and $a \times b = c$, then

$$a = c \div b.$$

Observe that $c \div b$ is the first factor in the equation $a \times b = c$. An example of this is

$$2 \times 3 = 6,$$
$$2 = 6 \div 3.$$

The relation between division and multiplication may be brought into focus more clearly by this type of example, which shows that finding the quotient is equivalent to finding the missing factor:

$n \times 4 = 12$ $\quad$ $12 \div 4 = n$
(These two numbers are the same.)

This idea parallels the addition-subtraction relationship, with the single exception that in division one of the factors (the second in our definition) cannot be zero. We rule out division by zero. The reasoning behind this choice is brought out in the mathematics section for page 136 of the student text.

Teaching the Chapter

Materials

Colored paper cutouts (skirts and blouses; sails and sailboats; etc.)
Counters (24 per child)
Crayons
Cups and saucers, three sets in different colors or patterns
Felt object and strips
Flannelboard
Flash cards showing multiplication and division facts
Graph paper,
Index cards, small (at least 10 cards per child)
Multiplication tables—duplicated copies to be filled in by the children
Overhead projector
Paper suitable for tracing
Scissors
Slips of paper, about 5 by 10 cm (10 per child)
Small boxes or containers (optional, for page 138)
Watch with a second hand

Yarn

Vocabulary

divide
division
divisor
factor
missing factor
multiplication-addition (distributive) principle
multiplication order (commutative) principle for multiplication
pairing
product
quotient
times

Although children explore the basic concepts of multiplication and division with materials in the investigations, some demonstrations will be helpful in summarizing and clarifying these concepts, either for the whole class or for small groups.

To demonstrate the concepts of this chapter, you can show rectangular arrays or product sets with set materials or with diagrams on the chalkboard. You may find it more helpful to use concrete set materials to demonstrate product sets in the initial stages. For example, you can illustrate the product 2×3 by showing the color combinations obtained by placing two cups of different color with three saucers of different color. Record the color combinations on the chalkboard to show that there are six different combinations in all and that $2 \times 3 = 6$.

Use of set materials for demonstrating concepts in division will include finding the number of sets of a given size within another set, as well as parcelling out a set of objects into a given number of equivalent sets. For example, you might exhibit a set of 24 objects and ask the children how many sets of 4 they can take from this set of 24; when they have found 6 sets of 4, exhibit the equation $24 \div 4 = 6$. This procedure shows finding the number of sets of a given size within a set. To show the "partitioning" that is associated with division, ask the children to group a set of 18 objects into 3 sets of the same number. Then draw from them the response that each set contains 6 objects, thus $18 \div 3 = 6$.

In developing the vocabulary for this chapter, pay particular attention to the word *factor*, which becomes a key word when the idea of a missing factor is used to demonstrate the general concept of division and its relationship to multiplication. All the words given in the vocabulary list are important. They are used so often that the children will have little difficulty mastering them.

Lesson Schedule

Although the time schedule should be adjusted according to the abilities and backgrounds of your children, plan to cover this chapter in $4\frac{1}{2}$ to $5\frac{1}{2}$ weeks.

Evaluation of Progress

For most children, this chapter will be a review of topics studied in the earlier grades. We expect the children to gain both a feeling for, and an understanding of, the general concepts of multiplication and division, the relationship between the two, and some of the basic principles of multiplication. The children are expected to master the multiplication and division facts.

In your evaluation, focus on the children's skill in handling multiplication facts, since the division facts are derived from a knowledge of multiplication facts. The best way to evaluate the children's understanding of concepts is to observe their daily participation in class discussions. As in any evaluation at this level, make sure the children can interpret a variety of word problems.

We recommend that you answer any questions about the meaning of the division operation by referring to the basic connection between multiplication and division. Set interpretations as a descriptive definition of division may also be helpful.

Resources for Active Learning

GENERAL ACTIVITIES

Developmental Math Cards, (games) G[4]2, G[4]12, Addison-Wesley

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs: Multiplication with Whole Numbers," pp. 71–72, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Franklin Series: *Patterns and Puzzles*, "Without a Word," p. 46; "Operations," p. 93, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Operations No. 3, "Games," pp. 35–38 (pupil pages 50–56), Encyclopaedia Britannica Publications Ltd.

Mathex: Operations and Problem Solving No. 8, "Hexagon drills," and "Finger Multiplication," pp. 18–19, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 3, "Multiplication," pp. 24–60; "Simple Sharing," pp. 61–64, Wiley

MANIPULATIVE DEVICES

Dienes Multibase Arithmetic Blocks (Herder and Herder)

Grid Kit (Sigma, Scott Scientific)

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

Map Measure (Edmund Scientific; Math Media)

Multiplying Machine (Creative Publications; Hammett)

SEE Calculator (Selective Educational Equipment)

Unifix Math Lab Kit (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

COMMERCIAL GAMES

Games to help master and maintain basic facts and sharpen computational skills:

Equations (Creative Publications)

Heads Up (Creative Publications, E. S. Lowe)

Krypto (Creative Publications; Edmund Scientific)

Quinto (Hammett; Selective Educational Equipment)

Twin Choice—a game similar to Hearts (Holt, Rinehart and Winston)

The Winning Touch (CCM School Materials: school supplier)

PAGES 116-117

Objective

Given a rectangular array of objects and a multiplication or division equation related to the array, the child will be able to solve the equation.

Preparation

Materials

Graph paper *(Duplicator Masters,* page 66); counters (24 per child)

To develop interest in this lesson's investigation, you might ask the children to mention some uses of graph paper. Ask them if they can think of ways to use graph paper to show multiplication and division, and explain that they will have an opportunity to do so in this lesson.

Investigation

If you do not have 24 counters for each child, you will need to direct the children to draw pictures on the graph paper without first arranging objects in rectangular arrays. However, many materials may be used as counters—beans, paper clips, or small squares of paper—and children would benefit from using the physical objects. Before the children begin with their investigation of arranging 24 counters in different arrays, discuss other ways to arrange 12 counters. Note that one of the illustrated arrays shows 3 rows of 4 columns; a different array might show 4 rows of 3 columns. Similarly, the array of 6 rows of 2 columns might be shown differently as an array of 2 rows of 6 columns.

Remind the children that they are to record each array they find by drawing a picture of it on graph paper. Encourage the children to work independently, but allow for free sharing of ideas.

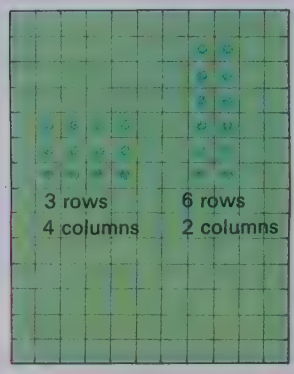
5 Multiplication and Division

● *Are multiplication and division related?*

Investigating the Ideas

The graph paper shows two different ways to arrange 12 counters in a rectangular **array.**

Row	Col
1	24
24	1
2	12
12	2
3	8
8	3
4	6
6	4



? How many different ways can you arrange 24 counters in a rectangular array? 8 (See above.)

Record your findings by drawing pictures on graph paper.



Discussing the Ideas

1. Did one of your arrays have 3 rows and 8 columns? If not, try it. Can you solve this multiplication equation?

Factor × Factor = Product
3 × 8 = *n* 24

2. Did one of your arrays have 4 rows? 6 How many were in each row? Can you solve this division equation?

Product ÷ Factor = Factor
24 ÷ 4 = *n* 6

The factors are sometimes called → Divisor Quotient

3. How many other multiplication and division equations can you write about your arrays? See Discussion.

Discussion

After you discuss exercise 1 with the children, discuss in a similar way the other arrays they made. Point out the terms *factor* and *product*. Exercise 2 relates the arrays to division. Emphasize that the total, 24, must be recognized in order to find how many were in each row. Use other examples to illustrate the use of the terms *divisor* and *quotient*. Sum up the discussion by having children write the possible multiplication and division equations for exercise 3 on the chalkboard.

$1 \times 24 = 24$	$24 \div 24 = 1$
$24 \times 1 = 24$	$24 \div 1 = 24$
$2 \times 12 = 24$	$24 \div 12 = 2$
$12 \times 2 = 24$	$24 \div 2 = 12$
$3 \times 8 = 24$	$24 \div 8 = 3$
$8 \times 3 = 24$	$24 \div 3 = 8$
$4 \times 6 = 24$	$24 \div 6 = 4$
$6 \times 4 = 24$	$24 \div 4 = 6$

Note that the equation 1×24 should be included only if the children agree to accept an arrangement of 24 single items as a rectangular array.

Using the Ideas

1. Draw 4 sets of dots with 5 dots in each set.

A How many dots in all? 20 ■ Solve: $4 \times 5 = n$ 20

2. Draw a set of 18 dots on your paper.
Ring as many sets of 3 as you can.

A How many sets did you ring? 6 B Solve: $18 \div 3 = n$ 6

3. Study the sets. Then solve the equation.

A 

$4 \times 2 = n$ 8

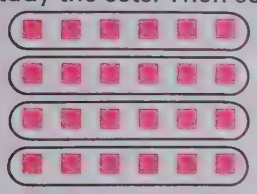
B 

$3 \times 7 = n$ 21

C 

$6 \times 3 = n$ 18

4. Study the sets. Then solve the equation.

A 

$24 \div 6 = n$ 4

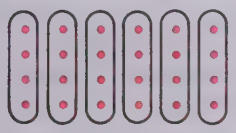
B 

$21 \div 7 = n$ 3

C 

$20 \div 4 = n$ 5

5. Study the sets. Then solve the equations.

A 

$6 \times 4 = n$ 24
$24 \div 4 = n$ 6

B 

$3 \times 6 = n$ 18
$18 \div 6 = n$ 3

C 

$7 \times 5 = n$ 35
$35 \div 5 = n$ 7

6. Give the missing numbers.

A 5 threes are ▮. 15
$5 \times 3 = n$ 15

B There are ▮ fives in 20. 4
$20 \div 5 = n$ 4



Using the Exercises

Assign the exercises on page 117 as independent work. If you think the children need further discussion, work through exercises 1 and 2 together. If some children have trouble with the exercises, you may want to work with them individually or in a small group, using counters or other physical objects. For example, have them separate a pile of beans into four cups so that there are the same number in each cup. Then ask them how many are in each cup.

Assignments (page 117)
Minimum: 1–6. Average: 1–6.
Maximum: 1–6.

Mathematics

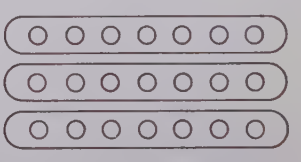
We relate multiplication to groups of equivalent sets and use this relationship as a basic definition of multiplication because it is an extremely simple idea which is easily grasped by the children. From this basic definition, the children can readily relate multiplication to repeated addition, skip counting, and number-line activities. Later in the chapter, multiplication is related to product sets, and then product sets are related to rectangular arrays, as in this lesson.

Division corresponds to finding the number of equivalent sets of a given size. To see how this idea applies to multiplication, we have only to look at the set interpretation of multiplication; for example, the equation $3 \times 4 = 12$ is associated with the idea of three sets of four. In division, when we attempt to find the number of sets of four in a set of 12, we are actually finding the factor three in the multiplication equation $3 \times 4 = 12$. Note that three is associated with the number of sets and four is associated with the number in each set. Later, of course, division will be shown to correspond to finding the number in each set and then related to multiplication in a more abstract sense. Of course, our basic working definition of division eventually will be in terms of multiplication.

Follow-up

To reinforce the children's understanding of the set interpretation of multiplication and division, write several multiplication and division equations on the chalkboard. Suggest that for each equation the children draw a set of objects and circle them in a manner which shows the meaning of the equation. For example:

$3 \times 7 = 21$ and $21 \div 7 = 3$



Resources for Active Learning

Mathex: Operations and Problem Solving No. 8, "Arrays," pp. 13–15, Encyclopaedia Britannica Publications Ltd.

PAGES 118–119

Objective

Given multiplication or division problems, the child will be able to find the products by repeated addition and to find the quotients by repeated subtraction.

Preparation

Unless you think the children need a review of the set interpretation of multiplication and division, begin immediately with the investigation. If you prefer to have a review, keep it brief and brisk. For example, show rectangular arrays which you have previously prepared and have the children give you the multiplication and division equations which the arrays suggest.

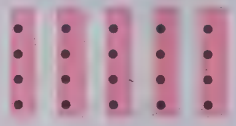
Investigation

This investigation may be handled in a variety of ways. You might choose to discuss the two examples at the top of the page with the children and then direct them to study the investigative questions on their own. However, a more stimulating approach might be to have the children work in groups of three or four. Explain that the two examples of repeated addition and repeated subtraction should help them understand the investigation questions. Encourage them to explain to one another the meaning of the two examples and how they would solve the problems. Stress the importance of adding and subtracting carefully.

Are there other ways to find products and quotients?

Investigating the Ideas

5 fours are how many?



$4 + 4 + 4 + 4 + 4 = 20$

$5 \times 4 = 20$

How many fours in 20?

$20 - 4 = 16$, $16 - 4 = 12$, $12 - 4 = 8$, $8 - 4 = 4$, $4 - 4 = 0$

$20 \div 4 = 5$

Can you **add** enough fours to find 17×4? 68
Can you **subtract** enough fours to find $52 \div 4$? 13 See Investigation.

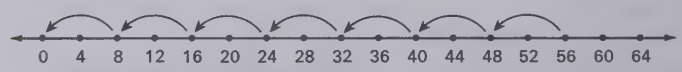
Discussing the Ideas

1. Explain how you could find this product by adding. 7×9
 See Discussion.
2. How could you find this quotient by subtracting? $54 \div 6$
 See Discussion.
3. Add together a "string of sevens" (as many as you want). Write a multiplication and a division equation about your work.
 Sample answer: 7+7+7+7=28 4x7=28 28÷7=4
4. Use the number line to help you solve the equations.



A $5 \times 8 = n$ 40 B $6 \times 8 = n$ 48 C $4 \times 8 = n$ 32

5. Use the number line to help you solve the equations.



A $48 \div 8 = n$ 6 B $32 \div 8 = n$ 4 C $56 \div 8 = n$ 7

Discussion

Let the children explain how they answered the questions in the investigation. Then work through the discussion exercises with the children. Ask a child to show the repeated addition and repeated subtraction for exercises 1 and 2.

$9 + 9 + 9 + 9 + 9 + 9 + 9 = 63$

54	48	42	36	30
−6	−6	−6	−6	−6
48	42	36	30	24

24	18	12	6
−6	−6	−6	−6
18	12	6	0

Use several such examples, if necessary, to help the children see how repeated subtraction relates to division. It would be helpful to use a number line on the chalkboard or a commercial number line while discussing exercises 4 and 5.

Using the Ideas

1. Solve the equations.

A $6 + 6 + 6 + 6 = n$ 24
$4 \times 6 = n$ 24

B $9 + 9 + 9 = n$ 27
$3 \times 9 = n$ 27

C $7 + 7 + 7 + 7 = n$ 28
$4 \times 7 = n$ 28

D $8 + 8 + 8 + 8 + 8 = n$ 40
$5 \times 8 = n$ 40

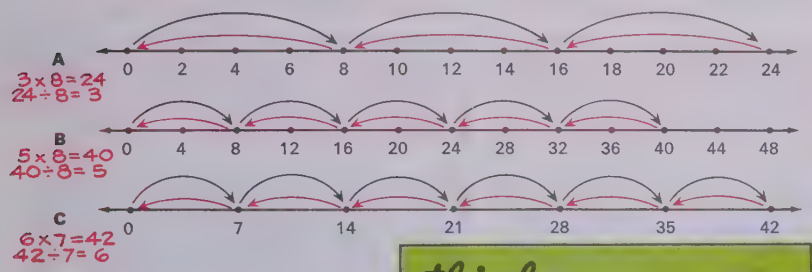
2. Copy the equations. Write the missing numbers instead of n.

A $24 - 8 = n$ 16
$16 - 8 = n$ 8
$8 - 8 = n$ 0
$24 \div 8 = n$ 3

B $45 - 9 = n$ 36
$36 - 9 = n$ 27
$27 - 9 = n$ 18
$18 - 9 = n$ 9
$9 - 9 = n$ 0
$45 \div 9 = n$ 5

C $28 - 7 = n$ 21
$21 - 7 = n$ 14
$14 - 7 = n$ 7
$7 - 7 = n$ 0
$28 \div 7 = n$ 4

3. Think about the black arrows and write a multiplication equation. Think about the red arrows and write a division equation.



★4. Use addition or subtraction to solve these equations.

A $2 \times 5280 = n$ 10 560
B $204 \div 68 = n$ 3
C $3 \times 526 = n$ 1578
D $332 \div 83 = n$ 4
E $3 \times 27\,800 = n$ 83 400
F $300 \div 75 = n$ 4
G $147 \div 49 = n$ 3

think

$587 \times 7285 = 4\,276\,295$

Use this equation and addition or subtraction to find these products.

$588 \times 7285 = n$ 4 283 580
$586 \times 7285 = n$ 4 269 010

More practice, page A-11, Set 18



Using the Exercises

Assign the exercises on page 119 as independent work. When the children have finished, allow time for a discussion. Encourage the children to explain how the pairs of equations in exercise 1 are related to each other.

Starred exercise 4 is intended for more able children. Some children may recall the multiplication algorithm for parts A, C, and E, but the problems may also easily be solved by repeated addition. Similarly, parts B, D, F, and G may be solved by repeated subtraction.

The children will actually apply the distributive principle in solving the *Think* problem. Because

$(587 \times 7285) + (1 \times 7285) = 588 \times 7285$

the children can add 7285 to the given fact 587×7285 to obtain the first answer (4 283 580).

Similarly, they can think

$(587 \times 7285) - (1 \times 7285) = 586 \times 7285$

to obtain the second answer (4 269 010).

Assignments (page 119)
Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Mathematics

Relating multiplication to repeated addition is quite simple once we take the idea of equivalent sets as the basic working definition of multiplication. To relate multiplication to these sets, we consider the total number of objects, or, in other words, we form the union of the sets involved. Of course, set union is related to addition, and this automatically leads to the interpretation of multiplication as repeated addition.

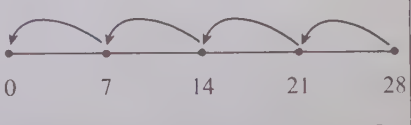
A problem arises in this interpretation when working with products involving zero and one. However, at this stage, the children are working primarily with the ideas of multiplication that do not involve factors of zero and one. Repeated addition will not be considered when there is a first factor of zero or one, since addition is an operation performed on two numbers and it makes no sense to speak of adding 0 threes or of adding 1 three.

Having related division to the idea of finding the number of given-sized sets within another set, we can easily show how division relates to repeated subtraction. The example on the right at the top of page 118 shows this clearly. By starting with 20 and subtracting 4 at a time, we find that we can subtract 4 five times; hence the division fact $20 \div 4 = 5$.

Follow-up

Many children would benefit from a worksheet of exercises like those shown in this sample.

Write and solve a multiplication or division equation for each.

1. $5 + 5 + 5 + 5 + 5 + 5$
2. $204 + 204 + 204$
3. $\begin{array}{ccc} 27 & 18 & 9 \\ -9 & -9 & -9 \\ \hline 18 & 9 & 0 \end{array}$
4. 

Resources for Active Learning

Developmental Math Cards, H14, Addison-Wesley.

Workbook, page 31

PAGES 120–121

Objective

Given short picture or word problems which require multiplication or division, the child will be able to solve the problems by using a power-skill method such as repeated addition or subtraction, the number line, or set interpretation with counters.

Preparation

Since this lesson has no specific investigation or discussion sections, you might develop the pattern of the If-Then problems before the children begin to work on page 120. For example, prepare them for these problems by giving them oral problems like these:

If 1 team has 5 players,
Then 3 teams have how many players?

If each girl has 4 books,
Then 7 girls have how many books in all?

If each boy can eat 2 hamburgers,
Then how many boys will it take to eat 12 hamburgers?

Short Picture Problems



120

Discussion

It is important for children to understand how to interpret these problems correctly. For this purpose, read through the first 3 or 4 problems with the children. You may or may not decide to include discussion of the answers as you do this. The first problem would be read in the following manner:

If 1 wagon has 4 wheels,
Then 3 wagons have how many wheels?

When the children understand how to read the If-Then problems, encourage them to solve the problems by using any power skill they choose (or any valid method they recall from Book 3). You might provide counters and a number line for those who need them. During your discussion of this page, you might have them use the pictures to make up stories about each problem. Notice that the starred exercise may be solved by repeated addition.

$25 + 25 + 25 + 25 + 25 + 25 = 150$

or, algorithmically:

$$\begin{array}{r} 25 \\ \times 6 \\ \hline 150 \end{array}$$

In either case, children should realize that 150 cents is $1.50, or one dollar and 50 cents.

Page 121 provides children with further practice in determining

Short Stories

Write a multiplication or division equation for each exercise. Then find the solution.



1. 4 boxes. 8 crayons in each box. How many crayons? $4 \times 8 = 32$

2. 24 marbles. 3 in each bag. How many bags? $24 \div 3 = 8$



3. 3 sets. 7 cards in each set. How many cards? $3 \times 7 = 21$



4. 36 boys. 9 on each team. How many teams? $36 \div 9 = 4$

5. 5 bags of candy. 7 pieces in each bag. How many pieces? $5 \times 7 = 35$

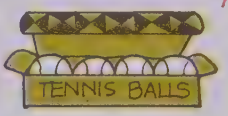
6. 30 band members. 5 in each row. How many rows? $30 \div 5 = 6$



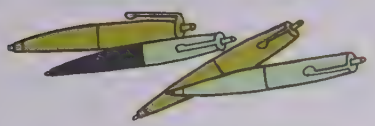
7. 6 cards. 6 pencils on each card. How many pencils? $6 \times 6 = 36$

8. 32 children. Divided into groups of 4. How many groups? $32 \div 4 = 8$

9. 7 boxes. 4 tennis balls in each box. How many tennis balls? $7 \times 4 = 28$



10. Candy: 5 cents each. Had 25 cents. Can buy how many? $25 \div 5 = 5$



★ 11. 84 children. 28 in each class. How many classes? $84 \div 28 = 3$



★ 12. Ballpoint pens: 39 cents each. Bought 4. How much in all? $4 \times 39¢ = 156¢$ or $\$1.56$

whether multiplication or division will solve a problem. The children are instructed to write equations for these exercises. For example, they might write $4 \times 8 = n$ for exercise 1, $n \times 3 = 24$ or $24 \div 3 = n$ for exercise 2, and so on.

Follow-up

Write several equations using the basic facts on the chalkboard and encourage children to make up short story problems of their own. You might use some of the following equations:

$3 \times 6 = n$	$5 \times 9 = n$
$15 \div 5 = n$	$64 \div 8 = n$
$42 \div 7 = n$	$28 \div 4 = n$
$7 \times 8 = n$	$6 \times 4 = n$

Assignments (page 121)

Minimum: Odd-numbered problems 1–9. Average: 1–10. Maximum: 1–12.

PAGES 122–123

Objectives

Given a multiplication fact and another equation with the same factors in commuted order, the child will be able to apply the order (commutative) principle to find the product.

Given 0 or 1 as factors, the child will be able to find the product.

Preparation

Although there is no investigation section in this lesson, it would be suitable to begin immediately with the text. However, if you prefer, use a short oral drill to review the basic multiplication facts.



Let's explore some basic principles.

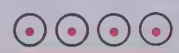
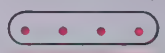
Discussing the Ideas

1. A There is an ORDER PRINCIPLE for addition. Sample answer: When we change the order of the addends, we still get the same sum.
 Here is an example:
 $6 + 7 = 13$ $7 + 6 = 13$
 Can you give this principle in your own words?
 B Is there an ORDER PRINCIPLE for multiplication?
 Here are two ways to think about a set of 20 dots.

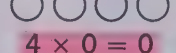
In columns	In rows
	
We see 5 sets of 4.	We see 4 sets of 5.
We write: $5 \times 4 = n$ 20	We write: $4 \times 5 = n$ 20

 Can you solve the equations and give this principle for multiplication in your own words? See Discussion.

2. Can you solve the equations and give a principle about multiplying by one? Sample answer: The product of one and another factor is the same as the other factor.

4 sets. 1 dot in each.	1 set of 4 dots.
	
$4 \times 1 = n$ 4	$1 \times 4 = n$ 4

3. Can you give a principle about multiplying by zero?
 Sample answer: The product of zero and another factor is zero.

4 sets. 0 dots in each.	0 sets of 4 dots.
	
$4 \times 0 = 0$	$0 \times 4 = 0$

122

Discussion

To introduce exercise 1, you might write on the chalkboard other examples of the additive order principle. Then encourage several children to describe the principle in their own words.

As you discuss exercise 1B with the children, have them observe that they can think of two multiplication equations for the rectangular array, by thinking first of columns and then of rows. You might expect them to state the principle somewhat like this: "When we change the *order* of the *factors*, we get the same *product*."

As you discuss exercise 2, help the children to see that multiplying by one is similar to adding zero.

$4 + 0 = 4$ $4 \times 1 = 4$

The sum or product is the same as the original number. You might have the children perform several multiplication operations in which 1 is a factor before you ask them to verbalize the principle. Similarly, in exercise 3, have them study several examples of multiplication by zero. For exercise 3, the children might give a statement such as, "Any number multiplied by zero is zero."

Using the Ideas

1. Find the missing numbers.

- A Since $4 \times 3 = 12$, we know that $3 \times 4 = n$. 12
- B Since $3 \times 6 = 18$, we know that $6 \times 3 = n$. 18
- C Since $5 \times 3 = 15$, we know that $3 \times 5 = n$. 15
- D Since $8 \times 7 = 56$, we know that $7 \times 8 = n$. 56
- E Since $9 \times 6 = 54$, we know that $6 \times 9 = n$. 54

2. Find the products. Use the table if you need help.

See Answers, T.E. p. 123.

A 14×8	E 15×19	$38 \times 56 = 2128$	$27 \times 16 = 432$
B 56×38	F 28×37	$49 \times 57 = 2793$	$28 \times 39 = 1092$
C 49×36	G 39×28	$37 \times 28 = 1036$	$19 \times 15 = 285$
D 16×27	H 57×49	$36 \times 49 = 1764$	$8 \times 14 = 112$

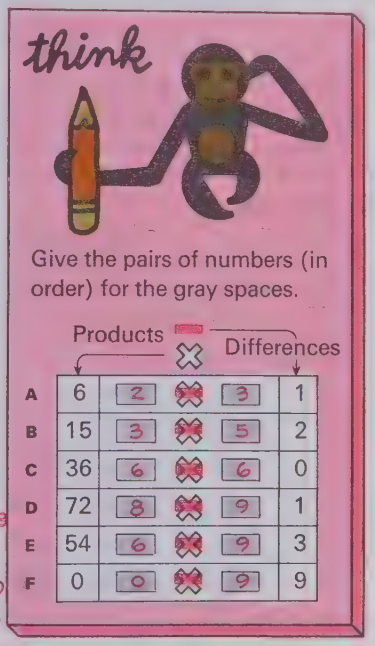
3. Find the missing numbers.

- A Since $9 \times 1 = 9$, we know that $1 \times 9 = n$. 9
- B Since $5 \times 0 = 0$, we know that $0 \times 5 = n$. 0
- C Since $1 \times 7 = 7$, we know that $7 \times 1 = n$. 7
- D Since $0 \times 8 = 0$, we know that $8 \times 0 = n$. 0

4. Find the equation that has no solution. Find the equation that has many solutions. Solve the other equations.

- A $8 \times 0 = n$ 0
- B $6 \times 1 = n$ 6
- C $1 \times n = 5$ 5
- D $n \times 7 = 0$ 0
- *E $n \times 0 = 24$
- F $9 \times n = 9$ 1
- G $n \times 1 = 78$ 78
- *H $0 \times n = 0$
- I $586 \times 0 = n$ 0
- J $0 \times 76 = n$ 0

*E has no solution.
H has many solutions.



A	6	2	3	1
B	15	3	5	2
C	36	6	6	0
D	72	8	9	1
E	54	6	9	3
F	0	0	9	9



Mathematics

Rectangular arrays are quite useful in investigating principles of multiplication. The material on page 122 shows that we can think about the rectangular array of dots in either columns or rows and thus illustrates the order (commutative) principle for multiplication. A formal statement of this principle follows:

If a and b are whole numbers, then

$$a \times b = b \times a.$$

The two generalizations concerning the special multiplicative properties of zero and one are as follows:

For each whole number a,

$$a \times 0 = 0.$$

For each whole number a,

$$a \times 1 = a.$$

Follow-up

A worksheet such as the following should strengthen the children's understanding of the order principle.

Study the equations. Then find the missing numbers.

$7 \times 9 = __ \times 7$
$8 \times 7 = 7 \times __$
$3 \times 8 = 8 \times __$
$4 \times 6 = 6 \times __$
$5 \times 8 = __ \times 5$
$6 \times 9 = 9 \times __$
$9 \times 3 = __ \times 9$
$47 \times 38 = __ \times 47$
$291 \times 635 = 635 \times __$
$850 \times 742 = 742 \times __$
$616 \times 166 = __ \times 616$
$378 \times 927 = 927 \times __$
$5174 \times 4175 = 4175 \times __$
$62\,830 \times 941 = __ \times 62\,830$

Workbook, page 32

Using the Exercises

Assign the exercises on page 123 as independent work. Point out that exercises 1 and 2 apply the order principle of multiplication and that exercises 3 and 4 deal with the special properties of zero and one. When the children have finished, allow time for checking the papers and discussing the exercises.

Answers, exercise 2, page 123

A 112	E 285
B 2128	F 1036
C 1764	G 1092
D 432	H 2793

Assignments (page 123)

Minimum: 1. Average: 1–3.
Maximum: 1–4.

PAGES 124–125

Objective

Given factors in a multiplication equation, the child will be able to apply the multiplication-addition principle, by "breaking apart" the first factor.

Preparation

Materials

graph paper; crayons; scissors

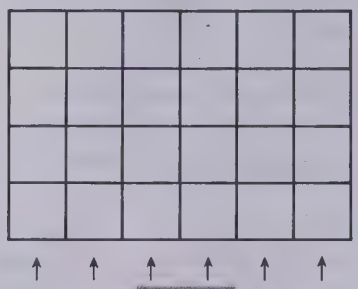
It would be suitable to begin this lesson immediately with the investigation. However, if you prefer, review a few basic multiplication facts shown in rectangular arrays.

Investigation

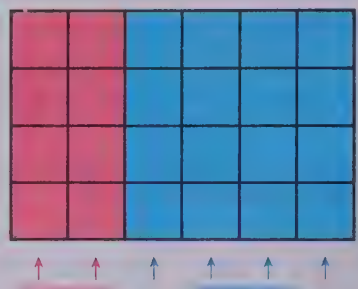
Read the material at the top of page 124 with the children. Before the children begin to color the columns of 4, you might suggest that they mark the squares with dots so that they do not inadvertently color more than 4 squares in a column. Remind them to record the combinations they color. Also suggest to those who finish quickly that they color the squares in different ways to show 7 fours or 8 threes, and so on. If a child wants to use three crayons to color, for example, 2 fours, 2 fours, and 2 fours, allow him to do so, but do not make this suggestion yourself, for the main purpose here is to help the child to begin breaking apart a factor into *two* groupings as he will do in studying basic facts and in multiplying with a 2-digit factor.

● *Let's explore the multiplication-addition principle.*

Investigating the Ideas



The graph paper shows 6 columns of 4 squares.



Here is another way to think of 6 fours.

? Using graph paper and two crayons, how many other ways can you color the rectangle to help you think of 6 fours?
5 fours + 1 four, 3 fours + 3 fours (See Investigation.)

Discussing the Ideas

1. Read the sentence and give the missing number.
 - A 6 fours are the same as 2 fours and __?__ fours. 4
 - B 6 fours are the same as 3 fours and __?__ fours. 3
 - C 6 fours are the same as 5 fours and __?__ four. 1

2. Complete this sentence in as many different ways as you can.
 7 fives are the same as __?__ fives and __?__ fives.
 6 and 1, or 1 and 6 ; 5 and 2, or 2 and 5 ; 4 and 3, or 3 and 4

3. You might think about the multiplication-addition principle like this:

 When we multiply, we can "break apart" a factor.

 What number did you "break apart" in exercise 2? 7

124

Discussion

As you discuss exercise 1, have several children show the ways in which they thought of 6 fours. In connection with the discussion of exercise 2, have the children explain ways of thinking about some other examples, such as 7 fours or 8 threes.

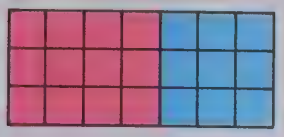
During this discussion, use such language as, "For 6 groups of four, we can think of 4 fours and 2 fours." Illustrate such an example with a multiplication equation, such as $6 \times 4 = (4 \times 4) + (2 \times 4)$. This language often helps children to understand the ideas behind the statement of the multiplication-addition (distributive) principle.

You might also display rectangular arrays on the chalkboard, flannelboard, or overhead projector. With yarn or chalk, circle sets in rows and columns, and ask the children to give you a multiplication equation for each array. Partition the set by rows or columns again, and ask the children to tell you the resulting factors. Explain that this is an application of the multiplication-addition principle, which allows us to find a product by breaking a factor into parts and adding the parts.

Using the Ideas

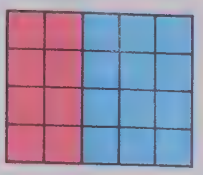
1. Give the missing numbers.

A 7 threes



4 threes and __?__ threes 3

B 5 fours

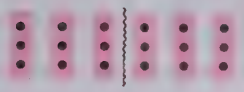


2 fours and __?__ fours 3

2. Find the missing number of threes.

A 

For 5 sets of 3, we can think
4 threes and ||| threes. 2

B 

For 6 sets of 3, we can think
3 threes and ||| threes. 3

C 

For 6 sets of 3, we can think
5 threes and ||| three. 1

3. Find the missing number.
Then solve the equation.

3A 7 twos → (4 twos and ||| twos)

3 $7 \times 2 = (4 \times 2) + (n \times 2)$

2B 7 twos → (5 twos and ||| twos)

2 $7 \times 2 = (5 \times 2) + (n \times 2)$

1C 7 twos → (6 twos and ||| two)

1 $7 \times 2 = (6 \times 2) + (n \times 2)$

think

1. If today is Wednesday, the 10th of December, what will the date be in a week? In 2 weeks? In 3 weeks?
17th, 24th, 31st
2. What day of the week is January 1 of the next year? December has 31 days.
Thursday



125

Using the Exercises

Unless you think your class needs further discussion, assign the exercises on page 125 as independent work. Exercise 3 might necessitate further discussion. It is important for children to be able to relate their verbal "breaking apart" of a factor with the mathematical equation.

The *Think* problem is intended for any children who wish to try it. Allow some to explain their solutions to the class.

Assignments (page 125)
Minimum: 1–3. Average: 1–3.
Maximum: 1–3.

Mathematics

A formal statement of the multiplication-addition (distributive) principle follows.

For all whole numbers a, b, and c,
$a \times (b + c) = (a \times b) + (a \times c)$.

The interpretation of multiplication as repeated addition is directly related to the distributive principle:

$5 \times 3 = 5 \times (1 + 1 + 1)$
$= 5 + 5 + 5$
$7 \times 4 = 7 \times (1 + 1 + 1 + 1)$
$= 7 + 7 + 7 + 7$

In each instance, we think of the second number as a sum of ones, and then multiply each of the ones by the first number.

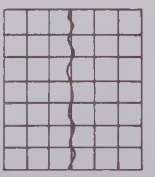
Rectangular arrays are useful in demonstrating the multiplication-addition principle. The illustration of colored graph paper at the top of page 124 shows the children that they can think about the multiplication-addition principle in multiplication as breaking a factor apart. The rectangular arrays shown in the follow-up section point out the fact that we can "break apart" either factor.

Follow-up

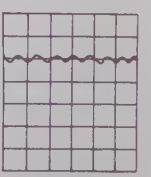
You might have the children write equations that correspond to the different ways in which they colored the squares for various factors.

You might prepare a worksheet to reinforce the concepts in this lesson. For example, draw (or transfer from graph paper) rectangles of various shapes. Show beneath each rectangle a partially completed equation. Instruct the children to complete the equation and color the rectangle with two crayons to match the equation.

Sample problems as they might appear on children's worksheets follow.



$7 \times \underline{6} = (7 \times \underline{3}) + (7 \times \underline{3})$



$7 \times 6 = (\underline{2} \times 6) + (\underline{5} \times 6)$

Resources for Active Learning

Discovery, Section II, Unit 17/5,6, Encyclopaedia Britannica Educational Corp.

Workbook, page 33

PAGES 126–127

Objective

Given the problem of finding products of single-digit factors in which one factor is 0, 1, 2, 3, 4, or 5, the child will be able to find the product.

Preparation

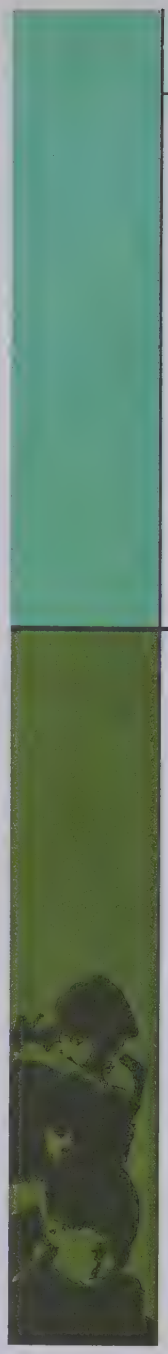
Materials

index cards (unlined, at least 10 per child); stopwatch or any watch or clock with a second hand; duplicated copies of a blank multiplication table *(Duplicator Masters,* page 62)

Although no specific preparation is necessary, you might ask the children what facts they think are the "easier" multiplication facts. After several children have given their opinion, explain that in this lesson they will review multiplication facts up to the five facts.

Investigation

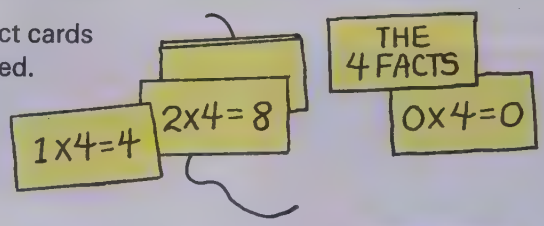
After you have distributed the materials and read the investigation question with the class, direct the children to choose a set of facts. Encourage the more capable children to choose a set that seems more difficult to them. If you prefer, assign specific sets to specific children, according to degree of difficulty of the set and ability of the child. As the children work on the cards, encourage them to be sure of the products they write. Remind them that they can figure out an unknown fact in a variety of ways. They might use repeated addition or a number line. Or they might use a known fact and apply the multiplication-addition principle. For instance, if they know that $5 \times 8 = 40$, then $5 \times 9 = (5 \times 8) + (5 \times 1) = 40 + 5 = 45$.



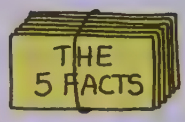
Do you know the "easier" multiplication facts?

Investigating the Ideas

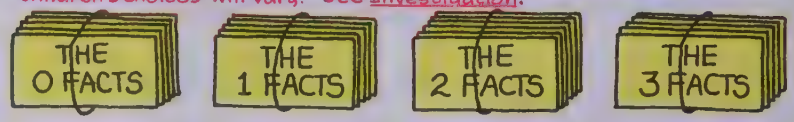
One pack of fact cards has been opened.



? Can you make all the cards in one of these sets of facts?



Children's choices will vary. See Investigation.



Discussing the Ideas

1. Do you know the facts when 0 or 1 is a factor?
 Can you find these products in 1 minute? Try it.

A	0×7^{0}	D	1×6^{6}	G	1×3^{3}	J	0×0^{0}	M	8×1^{8}	P	1×2^{2}						
B	7×1^{7}	E	0×4^{0}	H	1×1^{1}	K	5×1^{5}	N	7×0^{0}	Q	6×0^{0}						
C	5×0^{0}	F	9×0^{0}	I	0×8^{0}	L	1×0^{0}	O	0×3^{0}	R	9×1^{9}						

2. Do you know the facts when 2 or 3 is a factor?
 Can you find these products in 2 minutes? Try it.

A	2×5^{10}	D	5×3^{15}	G	2×2^{4}	J	2×6^{12}	M	3×7^{21}	P	2×7^{14}						
B	7×2^{14}	E	3×8^{24}	H	2×3^{6}	K	6×3^{18}	N	3×5^{15}	Q	7×3^{21}						
C	3×4^{12}	F	8×2^{16}	I	4×2^{8}	L	2×9^{18}	O	6×2^{12}	R	3×9^{27}						

3. Do you know the facts when 4 or 5 is a factor?
 Can you find these products in 2 minutes? Try it.

A	4×2^{8}	D	7×5^{35}	G	7×4^{28}	J	8×5^{40}	M	0×4^{0}	P	4×5^{20}						
B	5×6^{30}	E	4×8^{32}	H	3×5^{15}	K	4×6^{24}	N	5×9^{45}	Q	0×5^{0}						
C	1×4^{4}	F	4×3^{12}	I	5×2^{10}	L	4×4^{16}	O	5×5^{25}	R	9×4^{36}						

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Discussion

One of the chief purposes of this lesson is to begin a systematic review of basic multiplication facts. If the children recall the special properties of 0 and 1, they should easily finish exercise 1 within one minute. It would be helpful to have them letter their papers from A to R before they begin each exercise. Some children may need the full 2 minutes for exercises 2 and 3, in order to figure out a fact which they cannot recall. After you have checked the answers, ask volunteers to explain any method they used to find them.

Help the children realize the importance of knowing these facts by memory. The power skill of understanding how to figure out an unknown fact from a known fact or from understanding the basic multiplication concept is important, but this discussion section stresses the added importance of the speed skill. Encourage children to make cards for facts which give them difficulty and to try to memorize them.

Using the Ideas

1. "2" facts Find the products.

A 2×3 6	D 2×9 18	G 6×2 12	J 4×2 8
B 5×2 10	E 2×7 14	H 0×2 0	K 3×2 6
C 8×2 16	F 2×1 2	I 2×2 4	L 2×8 16

2. "3" facts Find the products.

A 3×3 9	D 3×9 27	G 6×3 18	J 4×3 12
B 5×3 15	E 3×7 21	H 0×3 0	K 3×3 9
C 8×3 24	F 3×1 3	I 3×2 6	L 3×8 24

3. "4" facts Find the products.

A 4×3 12	D 4×9 36	G 6×4 24	J 4×4 16
B 5×4 20	E 4×7 28	H 0×4 0	K 3×4 12
C 8×4 32	F 4×1 4	I 4×2 8	L 4×8 32

4. "5" facts Find the products.

A 5×3 15	D 5×9 45	G 6×5 30	J 4×5 20
B 5×5 25	E 5×7 35	H 0×5 0	K 3×5 15
C 8×5 40	F 5×1 5	I 5×2 10	L 5×8 40

5. "0, 1, 2, 3, 4, 5" facts

Find the products.

A	3×5 15	I	4×4 16	Q	3×2 6
B	2×6 12	J	5×1 5	R	6×0 0
C	6×5 30	K	4×2 8	S	5×5 25
D	5×0 0	L	2×8 16	T	1×3 3
E	4×6 24	M	3×6 18	U	4×5 20
F	1×7 7	N	0×8 0	V	0×7 0
G	5×8 40	O	9×3 27	W	8×4 32
H	4×3 12	P	9×1 9	X	5×2 10

6. Copy the table. Complete the colored part of the table.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30				
7	0	7	14	21	28	35				
8	0	8	16	24	32	40				
9	0	9	18	27	36	45				



Follow-up

To give the children further practice with the "easier" multiplication facts, divide the class into two or three teams for a relay competition. On the chalkboard make a chart for each team. Have the teams fill in the chart using a relay method. Have the children record which team finishes first, second, and so on. Then check the charts for correct answers. The first team finished with the highest number of correct answers wins.

Sample charts:

Team A

×	4	8	6
3			
4			
2			

Team B

×	3	7	5
5			
3			
2			

You might want to award three points for the first team to finish, two points for the second team, and one point for the third team; and award one point for each correct product. The winner is the team with the most total points.

Workbook, page 34

Using the Exercises

You may treat page 127 as a straightforward practice page, or you may use it as a diagnostic tool to determine what practice the children need on these easy facts. In either case, when you check the children's answers with them, point out that knowing the product of 5×6 enables them also to know the product 6×5. Help them see that such applications of the order or commutative principle leave them with only 16 facts to study. It would be helpful to prepare duplicated copies of the multiplication table for use in exercise 6 and in the following lesson.

Assignments (page 127)

Minimum: 1–5. Average: 1–5.
Maximum: 1–5.

PAGES 128–129

Objective

Given the problem of finding products of single-digit factors in which one factor is 6, 7, 8, or 9, the child will be able to give the products.

Preparation

Even after a clear presentation of the facts and adequate drill, some children will continue to have difficulty mastering all of the facts. Therefore, a brisk oral drill is often suitable to begin a lesson. For example, ask the children to give the products of the "easier" multiplication facts. You might include the "breaking apart" of some factors also, such as "7 × 5 may be thought of as 4 × 5 and 3 × 5."

Investigation

One of the important goals of this lesson is to help children understand how to apply known facts to figure out facts they cannot recall. Study and discuss with the children the three methods shown for finding 8 × 6. Then have the children work independently to discover and record ways of finding 7 × 6. For the faster children, suggest other examples such as 9 × 8 or 7 × 9.

Let's explore the "larger" multiplication facts.

Investigating the Ideas

Use one of these methods to help you find 8 × 6.

Maybe you know 4 × 6. 8 sixes are the same as 4 sixes and 4 sixes.	Maybe you are good at adding. 6+6+6+6+6+6+6+6	Maybe you already know 6 × 8. 8 × 6 = 6 × 8

Sample methods: 6+6+6+6+6+6+6
(6×6)+(1×6) or (5×6)+(2×6), etc.
6×7

?	How many ways can you find 7 × 6?	Record your different methods.

(Children's methods will vary.)
See Investigation.

Discussing the Ideas

1. "6" facts Solve the equations to find the products in the **6 row.**

36 A 6 × 6: (3 × 6) + (3 × 6) = *n*
42 B 6 × 7: (3 × 7) + (3 × 7) = *n*
48 C 6 × 8: (3 × 8) + (3 × 8) = *n*
54 D 6 × 9: (3 × 9) + (3 × 9) = *n*

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6							A	B	C	D
7							E	H	I	J
8							F	K		
9							G	L		

2. Use the order principle to find the products in the **6 column** (E, F, G in the table).
E 42 F 48 G 54

3. "7" facts Solve the equations to find the products in the **7 row.**

A 7 × 7: (6 × 7) + 7 = *n* 49
B 7 × 8: (6 × 8) + 8 = *n* 56
C 7 × 9: (6 × 9) + 9 = *n* 63

4. Find the products in the **7 column** (K, L in the table). K 56 L 63

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Discussion

Throughout this discussion, whenever you demonstrate "breaking apart" a factor, break apart the first factor (though there is no specific rule regarding this). For example:

$$7 \times 6 = (4 \times 6) + (3 \times 6)$$
$$9 \times 8 = (4 \times 8) + (5 \times 8)$$
$$7 \times 9 = (4 \times 9) + (3 \times 9)$$

When you refer to such equations, use both the mathematical reading of the equation ("Nine times eight equals four times eight plus five times eight") and a paraphrased sentence ("Nine eights may be thought of as four eights and five eights").

You may wish to give some children a chance to try to work through the discussion exercises independently before working through them with the class.

Using the Ideas

1. **"8" facts** Solve the equations to find products in the **8 row.**

A 8×8: $(4 \times 8) + (4 \times 8) = n$ 64

B 8×9: $(4 \times 9) + (4 \times 9) = n$ 72

C Use the order principle to find 9×8 (c in the table). 72

"9" facts Solve either of the equations to find 9×9.

D 9×9: $(8 \times 9) + 9 = n$ 81

9×9: $(10 \times 9) - 9 = n$ 81

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8									A	B
9									C	D

2. Find the products.

A 7×0 0 E 7×5 35 I 7×9 63 M 8×6 48 Q 9×2 18 U 9×6 54

B 7×1 7 F 7×6 42 J 8×3 24 N 8×7 56 R 9×3 27 V 9×7 63

C 7×2 14 G 7×7 49 K 8×4 32 O 8×8 64 S 9×4 36 W 9×8 72

D 7×3 21 H 7×8 56 L 8×5 40 P 8×9 72 T 9×5 45 X 9×9 81

3. Find the products.

A	B	C	D	E	F	G
5	0	4	4	6	8	9
×6	×7	×4	×9	×6	×6	×7
30	0	16	36	36	48	63

H	I	J	K	L	M	N
3	6	7	10	6	6	6
×9	×8	×9	×9	×9	×7	×8
27	48	63	90	54	42	48

4. Find the products.

A 6×0 0 C 6×6 36 E 5×9 45 G 6×7 42 I 5×8 40 K 0×0 0

B 9×1 9 D 6×8 48 F 7×8 56 H 5×5 25 J 0×9 0 L 8×9 72

★ **5.** Write two 1-digit numbers whose product is one less than the product of the numbers given.

Example: 5,5 → 4×6 is one less than 5×5.

A 4,4 3,5 B 6,6 5,7 C 8,8 7,9 D 2,2 1,3 E 7,7 6,8 F 3,3 2,4 (or 1,8)

More practice, page A-11, Set 19



Using the Exercises

Use exercise 1 on page 129 as a basis for further class discussion. Have the children discuss and solve the equations and then complete the table begun in the previous lesson. As you discuss the equations, continue to paraphrase them as mentioned in the preceding discussion section.

Exercises 2, 3, and 4 provide practice with the facts. If necessary, permit children to use their multiplication table. However, encourage children to try to learn the facts by memory rather than depend on the multiplication table.

Starred exercise 5 is intended for faster children, but all may benefit from observing the pattern which develops.

Assignments (page 129)

Minimum: 1–4. Average: 1–4.
Maximum: 1–5.

Follow-up/"Combo"

A game similar to bingo will help the children review multiplication facts. The object of the game is to cover five products in any row, column, or diagonal as the caller shows the facts on flash cards and calls them out. The caller should give only the factors, leaving to each player the task of finding the correct product. Winners call "Combo" and verify the products from a master sheet.

To make the game, cut 10-by-12 centimetre cards and rule 30 two-centimetre boxes on them. Fill in "Combo," "Free," and products at random (see illustration). The players should have plastic or paper discs to cover products. (Note: The caller should have a master sheet of facts, and the flash cards he uses should be large and easy to read.)

C	O	M	B	O
12	35	72	48	63
24	56	18	64	36
81	16	FREE	32	20
45	27	49	28	14
6	42	15	21	54

Resources for Active Learning

Mathex: Operations and Problem Solving, No. 8, "Tables," pp. 15–16, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 24
Workbook, page 35
Skill Masters, page 24

PAGES 130–131

Objective

Given the multiplication facts for 5, 8, or 9, the child will be able to discover patterns for these facts.

Preparation

To prepare for this lesson, you might briefly review the 8 and 9 facts. It would be equally appropriate to begin immediately with the investigation, if you prefer.

Investigation

Read with the children the material at the top of page 130. Help them understand that the question asks them to search for patterns in the facts that are illustrated and to use the ideas suggested by these patterns to complete the list of products. Encourage the children to study the given facts independently and try to discover some pattern on their own. After several minutes, allow the children an opportunity to share the patterns they discovered. Such a discussion should lead naturally into the discussion section in the text.

Are there patterns for the multiplication facts?

Investigating the Ideas

The set of products when 9 is a factor has some interesting patterns.

$2 \times 9 = 18$
$3 \times 9 = 27$
$4 \times 9 = 36$
$5 \times \quad 45$
$9 \times$

$6 \times 9 = 54$
$7 \times 9 = 63$
$8 \times 9 = 72$
$9 \times 9 = 81$

? Can you find the rest of the products without thinking about multiplying?

See Investigation and Discussion.

Discussing the Ideas

1. The products 5×9 and 6×9 both have the same digits, 4 and 5. Suppose you can't remember which is which. Can you think of something that would help you?
See Discussion.

2. Are there other pairs of products above that have the same digits? 18, 81; 27, 72; 36, 63

3. Here is a pattern for the nines. Test it by completing the list to 9×9.
See Discussion.

$1 \times 9 = 10 - 1$
$2 \times 9 = 20 - 2$
$3 \times 9 = 30 - 3$
$4 \times 9 = 40 - 4$
$5 \times 9 = 50 - 5$
$6 \times 9 = 60 - 6$
$7 \times 9 = 70 - 7$
$8 \times 9 = 80 - 8$
$9 \times 9 = ? - ?$
90 9

Discussion

The main purpose of this lesson is to provide children with an opportunity to experience enjoyment in discovering interesting patterns, which, incidentally, may serve as mnemonic devices for recalling the basic facts.

Some of the patterns the children might find for the "9 facts" are (1) the sum of the digits of each of the products is 9; (2) the products may be put in pairs according to reversed order of digits: 45, 54; 36, 63; 27, 72; 18, 81; (3) when listed vertically, the products show the pattern of increasing tens and decreasing ones; (4) in every product the digit in the tens' place is one less than the factor other than the 9 ($8 \times 9 = 72$).

As you discuss exercise 1, children might use one or two of the preceding patterns as an aid in remembering the products 5×9 and 6×9. You might also point out that since $6 \times 9 > 5 \times 9$ and $54 > 45$, 6×9 must equal 54 and not 45.

Some children may have discovered the pattern discussed in exercise 3. Such a pattern might help children see the relation of multiples of ten to products of 9; for example, if we know that $6 \times 10 = 60$, then 6×9 will be 6 less than 60 or 54. This may be viewed in terms of the multiplication-addition principle: $10 \times 6 = (9 \times 6) + (1 \times 6)$.

Using the Ideas

1. Continue this list to 9 × 5. What can you say about the product when 5 is a factor?
The ones' digit is either 0 or 5.

$2 \times 5 = 10$
$3 \times 5 = 15$
$4 \times 5 = 20$
$5 \times 5 = 25$
$6 \times 5 = 30$
$7 \times 5 = 35$
$8 \times 5 = 40$
$9 \times 5 = 45$

2. Some facts have a kind of rhyme. Can you find other facts that rhyme?
Sample answer: Seven times five is thirty-five.

Six times eight is forty-eight.

3. Here is a pattern for the eights. Check it by completing it to 9 × 8.

$1 \times 8 = 10 - 2$ 4×8=40-8
$2 \times 8 = 20 - 4$ 5×8=50-10
$3 \times 8 = 30 - 6$ 6×8=60-12
7×8=70-14
8×8=80-16
9×8=90-18
⋮

think

Give the pairs of numbers that could serve as both addends and factors in the gray spaces.

	Sums		Products	
A	5	2	3	6
B	7	3	4	12
C	10	4	6	24
D	11	3	8	24
E	6	3	3	9
F	4	2	2	4
G	2	1	1	1

	Sums		Products	
H	14	5	9	45
I	14	7	7	49
J	15	7	8	56
K	16	8	8	64
L	17	8	9	72
M	15	6	9	54
N	13	6	7	42

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Follow-up

Children might enjoy making posters which display the patterns they found in the investigation.

Resources for Active Learning

Developmental Math Cards, F[1]17, Addison-Wesley.

Mathex: Operations and Problem Solving No. 8, "Patterns in Multiplication," pupil pages 19, 20, Encyclopaedia Britannica Publications Ltd.

Using the Exercises

You might choose to use the exercises on page 131 as a basis for further discussion. If so, however, let the children enjoy the chance to discover the patterns on their own before they are discussed. You might suggest that some children work together to facilitate sharing of ideas. Exercise 3 may stimulate some children to investigate patterns with other factors, such as $1 \times 7 = 10 - 3$, $2 \times 7 = 20 - 6$, and so on.

Assignments (page 131)*

Minimum: 1–3. Average: 1–3.
Maximum: 1–3.

PAGES 132–133

Objectives

Given short word problems which require multiplication, the child will be able to solve the problem by applying his knowledge of multiplication concepts and the basic multiplication facts.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Any oral review, such as mental chain games, would serve as a suitable preparation for page 132 and page 133 as well. For example:

"Start with 8 . . . Multiply by 7 . . . Subtract 2 . . . Divide by 9. Your answer is ? " (6)

"Start with 63 . . . Divide by 9 . . . Add 7 . . . Subtract 5 . . . Multiply by 7. Your answer is ? " (63)

"Start with 6 . . . Multiply by 6 . . . Subtract 6 . . . Divide by 6. Your answer is ? " (5)

"Start with 4 . . . Multiply by 7 . . . Add 2 . . . Divide by 3. Your answer is ? " (10)

"Start with 5 . . . Multiply by 8 . . . Add 2 . . . Divide by 7 . . . Multiply by 3. Your answer is ? " (18)

Solving Short Story Problems



1 Tallest man on record: about 3 metres tall. Tallest giraffe: about two times as tall. How tall ? 6 metres

2 Pike fish: 1 metre long. Baby whale: 8 times as long. How long ? 8m



3 Rowboat: 3 metres long. Longest crocodile: almost 4 times as long. About how long ? 12m



4 Flea jump: 9 centimetres. Grasshopper jump: 8 times as far. How far ? 72cm

5 Ostrich egg weighs 2 kilograms. How much do 9 ostrich eggs weigh ? 18kg



6 Diamondback rattlesnake: 2 metres long. Longest python snake: over 5 times as long. About how long ? 10m



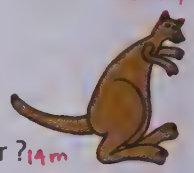
7 Measured speed for a snake: 9 kilometres per hour. Measured speed for an insect: 5 times as fast. How fast ? 45 km/h

8 Porpoise: 2 metres long. Whale: 9 times as long. How long ? 18m

9 1 octopus: 8 tentacles. 7 octopuses: How many tentacles ? 56 56



10 A fairly good frog jump: 2 metres. Long kangaroo jump: 7 times as far. How far ? 14m



11 Tall ostrich: 3 metres tall. Small jungle tree: 6 times as tall. How tall ? 18m

12 Slow bird: 9 kilometres per hour. Fastest duck: 8 times as fast. How fast ? 72 km/h

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More practice, page A-12, Set 20

Discussion

Have the children work through the word problems independently. You might suggest that children write equations for some problems.

Regardless of the manner in which you cover the problems, give the children an opportunity to do some additional research about animals. You might even have them verify some of the facts in these stories. If so, you will want to caution the children that, of course, not every kangaroo can jump 14 metres, and not every diamondback rattlesnake is two metres long. The children must recognize that these are only sample, and often maximum, figures. These are the sort of "soft facts" that may vary from one reference source to another.

We emphasize again the importance of allowing the children to digress from these exercises to topics other than arithmetic facts and correct answers. Such discussions stimulate children in their study of arithmetic and its applications to the physical world.

Assignments (page 132)

Minimum: Odd-numbered problems.
Average: 1–12. Maximum: 1–12.

Keeping in Touch with Addition Subtraction

1. Find the sums and differences.

A	B	C	D	E	F
32 + 57 = 89	87 − 24 = 63	54 + 29 = 83	53 − 26 = 27	134 − 67 = 67	87 + 69 = 156

2. Find the sums.

A	B	C	D	E	F
7 + 6 + 8 = 21	9 + 7 + 3 = 19	29 + 37 + 53 = 119	54 + 67 + 36 = 157	78 + 37 + 85 = 200	89 + 97 + 68 = 254

G	H	I
347 + 652 = 999	978 + 867 = 1845	763 + 898 = 1661

J	K	L
386 + 294 + 651 = 1331	418 + 690 + 789 = 1897	916 + 807 + 398 = 2121

think

Suppose a brick weighs 4 kilograms more than half a brick. How much does the brick weigh ? 8kg



3. Find the differences.

A	B	C
592 − 347 = 245	823 − 458 = 365	937 − 386 = 551

D	E	F
5832 − 2567 = 3265	506 − 219 = 287	8007 − 4234 = 3773

4. Find the sums and differences.

A $5 + 3 + 7 + 6 = n$ 21	E $403 - 395 = n$ 8
B $385 - 234 = n$ 151	F $613 + 56 + 9 = n$ 678
C $53 + 47 = n$ 100	G $485 + 386 + 67 = n$ 938
D $32 + 33 + 34 = n$ 99	H $807 - 467 = n$ 340



You are invited to explore

ACTIVITY CARD 5 Page 349

Using the Exercises

Page 133 may also be assigned as independent work. However, allow ample time for a discussion of the exercises when the children have completed the page. Give particular attention to parts D, E, and F of exercise 3. Some children may need a careful review of how to solve such problems.

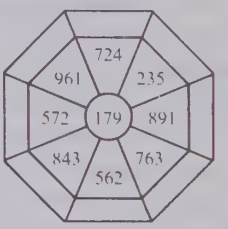
Give the children a chance to try the *Think* problem before you discuss the solution. Urge those who solve it to share their thinking with the rest of the class. Most children will benefit from a discussion of the various approaches to the solution.

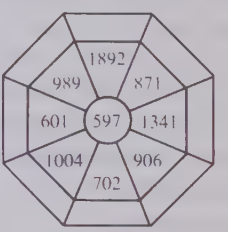
Follow-up

Encourage the children to make up short word problems of their own from any research about animals which they may do.

You might also prepare a worksheet of "practagons" to provide further practice in the basic operations.

Find the differences. Subtract the centre number from each of the other numbers.





Resources for Active Learning

Developmental Math Cards, F[4]12, F[1]19, Addison-Wesley. [A game]

Discovery, Section II, Units 5/21; 17/3, Encyclopaedia Britannica Educational Corp. [Reinforcement exercises]

PAGES 134–135

Objective

Given a division equation, the child will be able to find the quotient by thinking of missing factors.

Preparation

Materials

crayons; slips of paper, 5 by 10 centimetres (or supply the children with paper to cut to this size)

It would be appropriate to begin immediately with the investigation for this lesson. However, if you choose, use a short oral drill to review basic multiplication facts.

Investigation

Read the directions with the children. If possible, have the slips of paper cut out in advance. Have the children use crayons to mark the slips with fairly large symbols. Although the equations $3 \times 8 = 24$ and $8 \times 3 = 24$ use the same factors, accept them as different equations. Remind the children to record the equations they find. You might have children write equations on the chalkboard, grouping together those which use the same numerals.

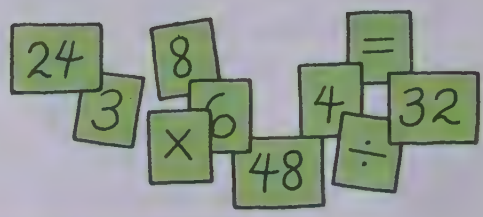
$3 \times 8 = 24$	$4 \times 6 = 24$
$8 \times 3 = 24$	$6 \times 4 = 24$
$24 \div 3 = 8$	$24 \div 6 = 4$
$24 \div 8 = 3$	$24 \div 4 = 6$
$4 \times 8 = 32$	$6 \times 8 = 48$
$8 \times 4 = 32$	$8 \times 6 = 48$
$32 \div 4 = 8$	$48 \div 6 = 8$
$32 \div 8 = 4$	$48 \div 6 = 8$

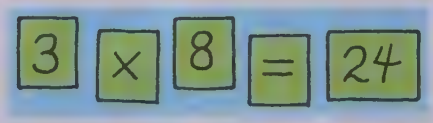


Let's explore products, factors, and quotients.

Investigating the Ideas

Cut out 10 slips of paper. Put one of these numerals or signs on each one.





? How many different equations can you "write" with your slips of paper?

See Investigation.

Record each equation on a sheet of paper.

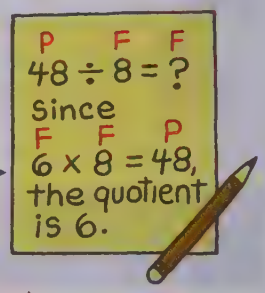
Discussing the Ideas

1. Which of the numbers in the Investigation could you use as factors? 3, 4, 6, 8

2. Which numbers were sometimes used as quotients? 3, 4, 6, 8

3. Which numbers were used as products? 24, 32, 48

4. Tim saw Fred's arithmetic paper. It looked like this. → Tim asked, "What do the red letters mean?" Can you answer Tim's question?



We can think of division in terms of factors and a product just as we think of multiplication.

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Discussion

As you guide the children in a discussion of questions 1 through 4, use the equations written on the chalkboard. Print *F* over each factor and *P* over each product. Stress for the children the idea that in multiplication they find the product of the factors.

$$\begin{matrix} \mathbf{F} & \mathbf{F} & \mathbf{P} \\ 9 & \times 8 & = n \end{matrix}$$

And in division they find one of the factors of the product.

$$\begin{matrix} \mathbf{P} & \mathbf{F} & \mathbf{F} \\ 72 & \div 8 & = n \end{matrix} \qquad \begin{matrix} \mathbf{P} & \mathbf{F} & \mathbf{F} \\ 72 & \div 9 & = n \end{matrix}$$

This labelling of a division equation in terms of factors and products is used in order to encourage children to think about multiplication when they are finding quotients. The product-factor language is most helpful in this respect.

Using the Ideas

1. Find the missing factors.

A To find this quotient, think 6 ? × 3 = 18 → 18 ÷ 3

B To find this quotient, think 7 ? × 5 = 35 → 35 ÷ 5

C To find this quotient, think 3 ? × 4 = 12 → 12 ÷ 4

2. Solve the equations.

A 18 ÷ 3 = *n* 6 B 35 ÷ 5 = *n* 7 C 12 ÷ 4 = *n* 3

In the following exercises, when you find the **missing factor** in equation A, you will have found the **quotient** in equation B. Write equation B with the correct quotient.

3. A *n* × 3 = 21 7 B 21 ÷ 3 = *n* 7
4. A *n* × 8 = 32 4 B 32 ÷ 8 = *n* 4
5. A *n* × 3 = 24 8 B 24 ÷ 3 = *n* 8
6. A *n* × 2 = 14 7 B 14 ÷ 2 = *n* 7
7. A *n* × 5 = 25 5 B 25 ÷ 5 = *n* 5
8. A *n* × 7 = 28 4 B 28 ÷ 7 = *n* 4
9. A *n* × 4 = 36 9 B 36 ÷ 4 = *n* 9
10. A *n* × 5 = 40 8 B 40 ÷ 5 = *n* 8
11. A *n* × 7 = 49 7 B 49 ÷ 7 = *n* 7
12. A *n* × 9 = 18 2 B 18 ÷ 9 = *n* 2
13. A *n* × 3 = 27 9 B 27 ÷ 3 = *n* 9
14. A *n* × 4 = 16 4 B 16 ÷ 4 = *n* 4
15. A *n* × 6 = 30 5 B 30 ÷ 6 = *n* 5
16. A *n* × 5 = 45 9 B 45 ÷ 5 = *n* 9
17. A *n* × 6 = 42 7 B 42 ÷ 6 = *n* 7
18. A *n* × 8 = 48 6 B 48 ÷ 8 = *n* 6
19. A *n* × 6 = 54 9 B 54 ÷ 6 = *n* 9
20. A *n* × 7 = 35 5 B 35 ÷ 7 = *n* 5

More practice, page A-12, Set 21



Using the Exercises

Use exercise 1 on page 135 as a basis for discussion, to remind children how to find quotients by thinking of missing factors. Assign the remaining exercises as independent work. When the children have finished and you are helping them correct their work, point out why one number is the answer for each pair of equations in exercises 3 through 20.

Assignments (page 135)
Minimum: Odd-numbered problems.
Average: 1–20. Maximum: 1–20.

Mathematics

The most important interpretation of division in *Investigating School Mathematics* is that which is based on its relation to multiplication (the basic working definition of division is in terms of multiplication). We give below a formal definition of division in terms of multiplication.

If a, b, and c are whole numbers and $b \neq 0$ and $a \times b = c$, then $a = c \div b$.

Observe that $c \div b$ is the factor a in the multiplication equation $a \times b = c$. That is, $(c \div b) \times b = c$. This simply means that $c \div b$ is the number that multiplies by b to give c. Therefore, when a child is looking for the quotient $c \div b$, he hunts for a number that will multiply by b to give c. For example, in the equation $24 \div 4$, children are encouraged to think, "What number times 4 equals 24?" When they discover that $6 \times 4 = 24$, they have found the quotient of 24 and 4.

Of course, since multiplication is commutative, this statement defines division in terms of either factor in a multiplication equation; hence, if multiplication is related to set concepts, we can think of division as finding either the number of sets or the number in each set.

Follow-up/"Challenge"

"Challenge" is an oral game designed to check children's understanding of the inverse relation between division and multiplication. Begin by telling the children that you are going to give some division equations orally and that they must listen carefully. After saying each equation, call on one member of the class to say "True" if the equation is true, or "Challenge" if he thinks it is incorrect. If a child challenges a particular equation, he must justify the challenge by giving a multiplication combination that is correct for the given product.

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, p. 81, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 25
Workbook, page 36
Skill Masters, page 25

PAGES 136–137

Objective

Given multiplication and division problems involving 0 and 1, the child will be able to find the products and quotients and, in particular, will demonstrate his understanding of the fact that 0 is never used as a divisor.

Preparation

If you wish to practice multiplication or division facts in an oral drill, limit such an activity to four or five minutes. It would be equally suitable, however, to omit such a preparation and begin immediately with the text.

Investigation

This investigation requires no materials other than the text. The children are to study the equations at the top of the page. The variety of solutions presented should help the children understand that $n \times 0 = 0$ has innumerable solutions—*any* number can be used as a replacement for n. The second equation, $n \times 0 = 5$, has *no* solution.

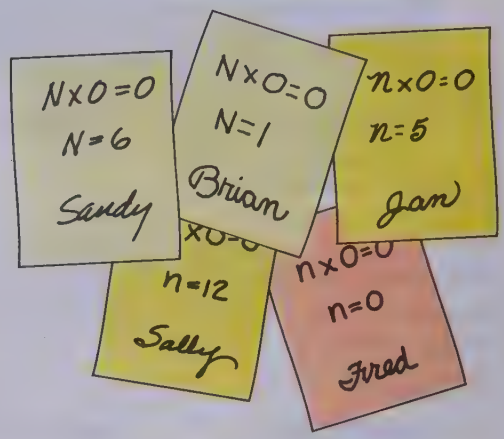
Give the children several minutes to study this section on their own. During this time you might allow free sharing of ideas or you might have them give their opinions to the whole class and work smoothly into the discussion section.

What are the 0 and 1 facts in division?

Investigating the Ideas

Check each paper to see who is correct. Do you see that there are many correct answers to the equation?

Solve: $n \times 0 = 0$



? How many correct answers can you find for this equation? None $n \times 0 = 5$

See Investigation.

Discussing the Ideas

1. Explain how the papers above show why we NEVER DIVIDE BY ZERO.

See Discussion.

2. Choose any number. Divide it by 1. What number do you get? Your original number

3. Choose any number (except 0). Divide it by itself. What number do you get? 1

4. Divide 0 by any number (except 0). What number do you get? 0

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Discussion

As you discuss exercise 1, it would be helpful to write the following equations on the chalkboard:

$n \times 0 = 0$ $\quad$ $n \times 0 = 5$

From the investigation children should realize that any whole number will be a solution to the equation $n \times 0 = 0$. Thus, if division by 0 were possible, the quotient of 0 divided by 0 would not be unique. On the other hand, since $n \times 0 = 5$ has *no* solution, 5 divided by 0 does not give any number as a quotient. Help the children realize that 5 is just an example of what happens no matter what number we use. To emphasize this, give other examples, such as:

$n \times 0 = 12$ $\quad$ $n \times 0 = 3$

These examples should help the children see why we NEVER DIVIDE BY ZERO. You might write this rule in large letters on the chalkboard so that those students who do not understand the reasoning will at least be impressed by the rule.

Present several examples for exercises 2 and 3, such as

$8 \div 1 = 8$ and $8 \div 8 = 1$,

to help children discover the special property of 1 in division.

Exercise 4 is related to the equation pair

$0 \times n = 0$ and $0 \div n = 0$.

Here we are not *dividing by* zero, we are dividing zero *by another number.*

Using the Ideas

DIVIDING BY 1, 2, 3, and 4

1. Find the quotients.

A	$8 \div 2 = n$ 4	H	$18 \div 3 = n$ 6	O	$27 \div 3 = n$ 9
B	$9 \div 1 = n$ 9	I	$36 \div 4 = n$ 9	P	$20 \div 4 = n$ 5
C	$24 \div 3 = n$ 8	J	$18 \div 2 = n$ 9	Q	$16 \div 2 = n$ 8
D	$16 \div 4 = n$ 4	K	$12 \div 3 = n$ 4	R	$15 \div 3 = n$ 5
E	$14 \div 2 = n$ 7	L	$21 \div 3 = n$ 7	S	$6 \div 2 = n$ 3
F	$9 \div 3 = n$ 3	M	$2 \div 2 = n$ 1	T	$8 \div 1 = n$ 8
G	$28 \div 4 = n$ 7	N	$32 \div 4 = n$ 8	U	$0 \div 2 = n$ 0

DIVIDING BY 5, 6, and 7

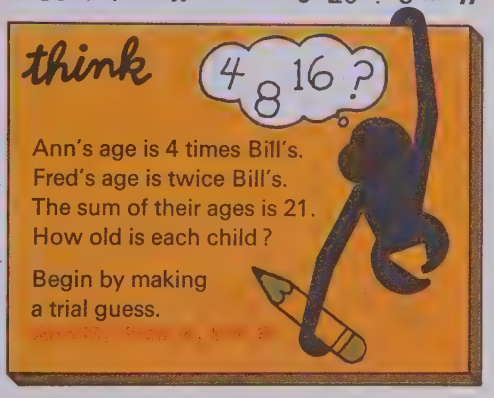
2. Find the quotients.

A	$25 \div 5 = n$ 5	H	$42 \div 7 = n$ 6	O	$56 \div 7 = n$ 8
B	$24 \div 6 = n$ 4	I	$30 \div 6 = n$ 5	P	$40 \div 5 = n$ 8
C	$14 \div 7 = n$ 2	J	$35 \div 5 = n$ 7	Q	$49 \div 7 = n$ 7
D	$36 \div 6 = n$ 6	K	$48 \div 6 = n$ 8	R	$5 \div 5 = n$ 1
E	$28 \div 7 = n$ 4	L	$21 \div 7 = n$ 3	S	$54 \div 6 = n$ 9
F	$0 \div 5 = n$ 0	M	$45 \div 5 = n$ 9	T	$63 \div 7 = n$ 9
G	$12 \div 6 = n$ 2	N	$35 \div 7 = n$ 5	U	$20 \div 5 = n$ 4

DIVIDING BY 8 and 9

3. Find the quotients.

A	$24 \div 8$ 3	H	$81 \div 9$ 9
B	$27 \div 9$ 3	I	$64 \div 8$ 8
C	$54 \div 9$ 6	J	$63 \div 9$ 7
D	$0 \div 8$ 0	K	$36 \div 9$ 4
E	$45 \div 9$ 5	L	$72 \div 8$ 9
F	$48 \div 8$ 6	M	$56 \div 8$ 7
G	$32 \div 8$ 4	N	$72 \div 9$ 8



More practice, page A-13, Set 22

Using the Exercises

On page 137, you may wish to have the children take turns answering the parts of exercise 1 orally. Either assign the rest of the exercises according to ability, or ask your class to write only quotients. If the children are very quick, you may wish to time their work and thus find out which facts, if any, they need to concentrate on.

In developing a strategy for solving the *Think* problem, we suggest that the children make a trial guess as a first step. They might use a table such as the one shown at the right, to help them in the organization of their guesses.

	Bill B	*Ann* $4 \times B$	*Fred* $2 \times B$	*Total* 21
Guess 1				
Guess 2				
Guess 3				

Assignments (page 137)

Minimum: 1–3. Average: 1–3.
Maximum: 1–3.

Mathematics

The fact that we do not divide by zero can be explained in a variety of ways. The explanation presented in the student text is simple and clear. A similar explanation can be found by thinking about the traditional method for checking division equations. For example, the check for $12 \div 4 = 3$ would be $3 \times 4 = 12$. Now if we consider $6 \div 0$, we see that any number times zero gives zero. In this case, when we try to divide a nonzero number by zero, no quotient can be found. In the case of dividing 0 by 0, any number will work. Since we wish for quotients, when they exist, to be unique we do not divide by zero.

Notice that in the formal definition of division, stated below, division by zero is excluded.

If a, b, and c are whole numbers and $b \neq 0$ and $a \times b = c$, then $a = c \div b$.

Follow-up

For a practice game, use division-game cards from a commercial arithmetic game, or make your own cards by ruling 30 two-centimetre squares on 10-by-12-centimetre pieces of tagboard. Give the children paper or cork discs to cover quotients and give the leader a check list so that he can verify the facts he has called. When a child has covered a row, column, or diagonal with the discs, he should call out "Combo."

C	O	M	B	O
9	4	8	2	6
7	3	5	0	1
2	7	FREE	6	4
5	0	9	8	3
1	2	6	4	7

A Sample Combo Card for Quotients

Duplicator Masters, page 26
Workbook, page 37
Skill Masters, page 26

PAGES 138–139

Objective

Given a set with a specified number of objects, the child will be able to divide the set into a given number of equivalent subsets and write the corresponding division equation.

Preparation

Materials

counters (24 per child); small boxes, cups, or containers in which to put the counters

Although it would be suitable to begin immediately with the investigation, you might prefer to have the children warm up with an oral drill of multiplication and division facts.

Investigation

In this investigation, children explore the basic concept of the division process—separating a set into a number of equivalent subsets—and then interpret the physical manipulation by means of a written division equation. You might choose to have the children work in groups of two or three. To be sure that they understand what they are to explore, stress the term "*equally*." The point brought out by these questions is that 24 counters *can* be divided equally into 4 sets but *cannot* be divided *equally* into 5 sets. Some children may need more specific questions in order to continue with the investigation, such as: "Can you divide the 24 counters equally into 7 sets? 8 sets?" and so on. Remind the children that they are to write a division equation to show each way they can divide 24 counters equally into sets. Help them understand that for questions such as "Can you divide them equally into 5 sets?" they should write no division equation because there will be a remainder after the counters have been divided into the stipulated number of equivalent sets.

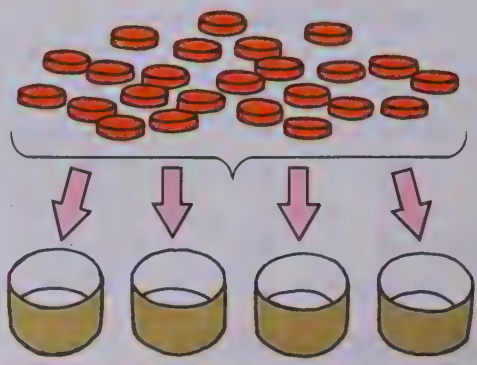


Let's explore division and sets again.

Investigating the Ideas

Try to divide 24 counters equally into 4 sets.

Can you divide them equally into 5 sets? No



24÷24=1 24÷6=4 24÷8=3 24÷12=2
24÷1=24 24÷4=6 24÷3=8 24÷2=12

? In how many ways can you divide 24 counters equally into sets?
See Investigation.

Record each result by writing a division equation.

Discussing the Ideas

1. You can use division to find how many sets. Solve the equation to find out.



18 strawberries

How many bowls of 6 strawberries can you serve? ➡ $18 \div 6 = n$ ³

2. You can use division to find how many in each set. Solve the equation to find out.



18 ice cubes

How many can you put in each glass if you divide them equally among 6 glasses? ➡ $18 \div 6 = n$ ³

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Discussion

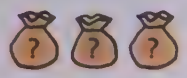
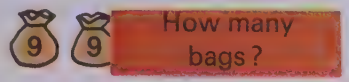
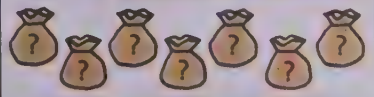
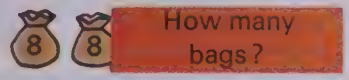
Before going on to the discussion exercises in the text, have children write equations on the chalkboard to show the ways they found to divide the 24 counters equally. Help the children realize how the equation describes division as the process of separating a total number of items equally into a given number of sets and also describes the number of items in each of these sets. For example, $24 \div 4 = 6$ corresponds to the first investigation question.

Discussion exercise 2 develops this set interpretation of division. Exercise 1 develops the interpretation of using division to find how many sets when the total number of items and the number of items in each set are given.

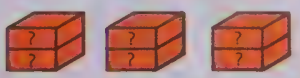
It would be helpful to provide some set demonstrations similar to those suggested by the illustrations on page 139. For example, show two or three bags beside a screen which obscures the total number of bags from the children's view. Tell the children how many items in all and how many items in each bag, for example, 20 marbles, 4 in each bag. Then have them tell you how many bags, and write $20 \div 4 = 5$ on the chalkboard. Also, show three bags in full view, explaining that there are 24 marbles equally separated into the

Using the Ideas

1. The bags in each exercise have the same number of marbles.

A		How many marbles in each bag? 9	27 marbles
B		9 marbles in each bag.	54 marbles
C		How many marbles in each bag? 8	56 marbles
D		8 marbles in each bag.	72 marbles

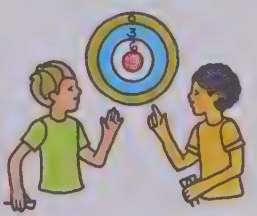
2. The boxes in each exercise have the same number of checkers.

A	24 checkers		How many in each box? 8
B	32 checkers		How many boxes? 4
C	48 checkers		How many in each box? 8
D	54 checkers		How many boxes? 6

3. A How many darts must Don throw into the red centre to get a score of 36? 4

B How many darts must Bob throw into the red centre to get a score of 63? 7

C How many darts must Glen throw into the white ring to get a score of 48? 8





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three bags, and ask the children how many marbles must be in each bag.

Using the Exercises

If page 139 has been preceded by set demonstrations, assign these exercises as independent work. When the children have finished, check the papers and allow time for a discussion in which they tell for each exercise whether they have found the number of sets or the number in each set.

Use exercise 3 as a basis for discussion. Help the children realize that they must refer to the picture of the dart board for the information needed to solve these problems.

Assignments (page 139)

Minimum: 1–3, oral. Average: 1–3. Maximum: 1–3.

Mathematics

As we have stated before, our basic working definition of division is in terms of multiplication. We have already looked at one of the set interpretations for division. In this lesson we propose to show that there are two distinct set interpretations for division.

Because multiplication is commutative, we can consider division as finding either missing factor in a multiplication equation. For example, if we choose to associate 4×5 with four sets of five each, then we should consider the equation $20 \div 5 = n$ as finding the number of sets and the equation $20 \div 4 = n$ as finding the number in each set.

Set interpretations provide meaningful physical activities for explaining certain concepts of division, and they trigger the division operation in certain word problems. In other words, if children read a word problem which requires finding the number of equivalent sets of a given size or the number of objects in each of so many equivalent sets, they should think about division. However, once the division idea has been triggered, encourage the children to find the quotient by thinking about the missing factor.

Follow-up

To help children reinforce their understanding of division, suggest that they use another divisible number of counters and find how many ways they can divide them equally into sets. You might write on the chalkboard problems such as the following.

Try to divide 21 counters equally into 7 sets; 3 sets.

Try to divide 16 counters equally into 4 sets; 8 sets.

Try to divide 12 counters equally into 3 sets; 6 sets.

Resources for Active Learning

Developmental Math Cards, G[13], G[116], H[11], Addison-Wesley.

PAGES 140–141

Objective

Given picture problems and short story problems which require multiplication or division (with the basic facts), the child will be able to solve the problems by finding the product or quotient.

Preparation

To prepare the children for these problems, develop oral examples similar to the If-Then problems on page 140. For example, use questions like the following.

If 7 cars hold 35 persons (in equal groups), then each car holds how many persons?

If 4 boxes hold 24 candy bars, then each box holds how many candy bars?

If 8 children drink 16 bottles of pop, then each child could drink how many bottles of pop?

Before you begin these examples, explain to the children that you are considering all the groups you talk about in a particular example as groups of equal amount or number. Give children several examples before you direct them to the text. You might suggest that some of the more able children volunteer some examples, but be sure that they use numbers from a basic fact.

Solving Short Picture Problems



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Discussion

Work through three or four of the If-Then problems to help children read them properly. For example, exercise 2 should be read, "If 6 children eat 18 lollipops, then each child eats how many lollipops?" and exercise 4, "If 8 cans hold 24 tennis balls, then 1 can holds how many tennis balls?" You might choose to use all of these If-Then picture problems as oral exercises and assign the word problems on page 141 as written work.

Before the children begin page 141, remind them that for each problem they must decide which operation, multiplication or division, is applicable. Exercise 10 is an exception in that it requires subtraction first, then division.

Encourage several children to try the *Think* problem. Some may use the trial-and-error method. Others may start at 12 and work backwards, using the inverse of each operation. Thus, when the step reads "Multiply by 4," they will divide by 4; when it reads "Subtract 4," they will add 4; and so on. Note that the problems are based on the multiplication facts and involve no remainders.

Assignments (page 140)

Minimum: 1–6. Average: 1–6.
Maximum: 1–8.

Solving Short Stories

1 32 buttons. 8 on each shirt. How many shirts? 4



2 6 boxes of pencils. 48 pencils in all. How many in each box? 8

3 5 tires per car. 8 cars. How many tires? 40

4 6 weeks. 7 days per week. How many days? 42

5 36 players divided equally into 6 teams. How many per team? 6



6 Apples: 9 cents each. You have 45 cents. How many can you buy? 5

7 Bicycling: 9 kilometres each hour. 45 km to go. How long will it take? 5h





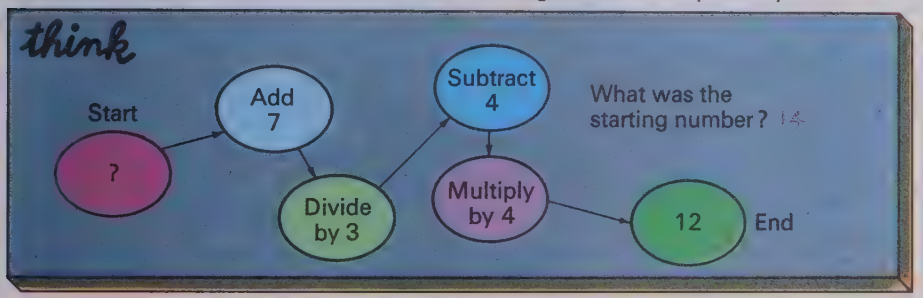
8 40 cookies. 8 hungry children. Divide the cookies equally. How many for each child? 5

9 9 jet airplanes. 4 jet engines per plane. How many jet engines? 36





10 Had 50 cents. Lost a dime and spent the rest on nickel candy bars. Bought how many candy bars? 8



More practice, page A-13, Set 23

Follow-up

Encourage children to make up their own picture or short story problems. However, to make sure their problems will be appropriate to the sequential development in the text, it would be helpful to write on the chalkboard pairs of numbers and instruct the children to develop their problems around these pairs, such as the following:

24,3	6,7	81,9
32,8	35,5	56,7
42,7	36,9	5,4

Duplicator Masters, page 27

Assignments (page 141)

Minimum: Odd-numbered problems.
Average: 1–10. Maximum: 1–10.

PAGES 142–143

Objectives

Given division exercises based on the multiplication facts, the child will be able to find the quotients.

Given word problems which require division, the child will be able to solve the problems by dividing to find the quotients.

Preparation

To prepare for this lesson, use a brisk oral drill to review multiplication and division facts. For example, adapt the "What's My Rule" game described on pages 64 and 65 to review these facts. You might use charts such as the following:

56	7
72	9
64	?
32	?

(Divide by 8.)

27	3
63	7
54	?
45	?

(Divide by 9.)

4	24
6	36
3	?
7	?

(Multiply by 6.)

Put in one or two answers and have volunteers guess your rule. You might do this orally by saying, "I'm thinking of the number 56. My answer is 7. What's my rule?" Instruct those who guess your rule to fold their arms. Then call on them to give the answer for the next number you mention (to which your rule should still be applicable).

Can you give the products and quotients?

The figure will help you review a different way to write division exercises. → $30 \div 5 = 6$ $\quad 5\overline{)30}$ (6)

1. Find the quotients.

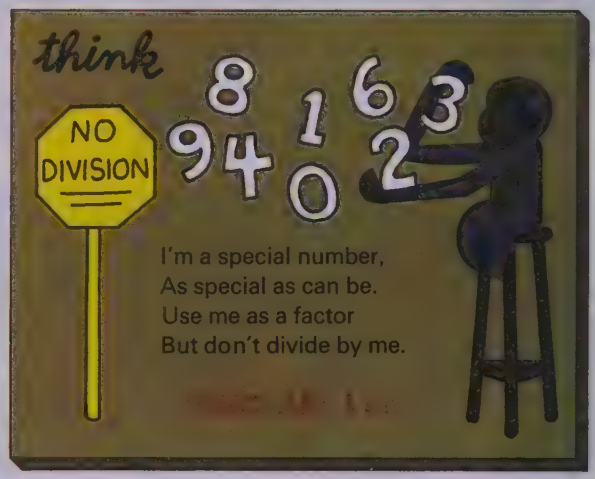
A $4\overline{)16}$ 4	F $7\overline{)56}$ 8	K $5\overline{)40}$ 8	P $5\overline{)35}$ 7	U $5\overline{)20}$ 4
B $8\overline{)32}$ 4	G $9\overline{)63}$ 7	L $1\overline{)9}$ 9	Q $4\overline{)36}$ 9	V $8\overline{)64}$ 8
C $8\overline{)8}$ 1	H $6\overline{)54}$ 9	M $6\overline{)24}$ 4	R $3\overline{)27}$ 9	W $1\overline{)8}$ 8
D $6\overline{)42}$ 7	I $8\overline{)72}$ 9	N $6\overline{)6}$ 1	S $7\overline{)49}$ 7	X $2\overline{)12}$ 6
E $7\overline{)0}$ 0	J $8\overline{)48}$ 6	O $7\overline{)42}$ 6	T $9\overline{)18}$ 2	Y $9\overline{)36}$ 4

2. Find the equation that has no solution. Find the equations that have many solutions. Solve the other equations. E has no solution. H and K have many solutions.

A $9 \times 0 = n$ 0	D $0 \times 0 = n$ 0	G $36 \div n = 4$ 9	J $0 \div 5 = n$ 0	
B $0 \div 8 = n$ 0	E $0 \times n = 18$ No sol.	H $n \times 0 = 0$ Many sol.	K $0 \times n = 0$ Many sol.	
C $n \div 8 = 4$ 32	F $9 \div 9 = n$ 1	I $0 \times 5 = n$ 0	L $28 \div 7 = n$ 4	

★ **3.** Find the quotients. Use whatever method you want.

A $52 \div 4 = n$ 13	F $102 \div 3 = n$ 34
B $45 \div 3 = n$ 15	G $112 \div 4 = n$ 28
C $46 \div 2 = n$ 23	H $56 \div 2 = n$ 28
D $102 \div 6 = n$ 17	I $144 \div 8 = n$ 18
E $112 \div 8 = n$ 14	



Discussion

Before assigning the exercises on page 142 as independent work, point out the more traditional way of writing division problems. Remind the class that the quotient is placed above the line. Note that exercise 3 is intended for the children who are competent in solving equations. It is *not* intended that the shortcut algorithm be taught here. The purpose of this exercise is to give the child an opportunity to invent his own way of finding the quotient. For example, he may use repeated addition or repeated subtraction. Note the relationship between 3D and 3F and between 3E and 3G.

Any children who wish to should be allowed to try the *Think* problem. The correct answer should be clear to all when it is given.

You might choose to use page 143 as a basis for discussion after the children have solved the problems. It may be helpful to have the children explain their answers to one another. You can do this effectively by having one child put the correct equation on the chalkboard and having another child explain how to arrive at this equation for the given exercise.

The children can benefit from discussing parts C and D of exercise 6, whether or not they are able to work them.

Solving Story Problems

AT THE GROCERY STORE

1. Jeff works at the supermarket after school. He put 32 cans of peaches on the shelf in 8 rows. How many cans were in each row? 4



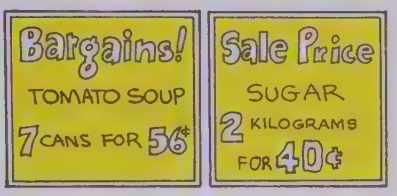
2. Juice comes to the store packed in boxes. There are 6 cans in each box. How many boxes should be ordered to get 54 cans? 9



3. Jeff put 72 apples into 8 plastic bags. He put the same number in each bag. How many apples did he put in each bag? 9



4. Jeff made these sale signs.
 - A How much does 1 can of tomato soup cost? 8¢
 - B How much does 1 kilogram of sugar cost? 20¢



5. Ted, Jeff's friend, came to the store. He had 45 cents. How many 5-cent candy bars could he buy? 9

6. A How much does one brand A frankfurter cost? 8¢
 B How much does one brand B frankfurter cost? 9¢
 ★ C Which would cost more: 3 packages of brand A or 4 packages of brand B? 4 pkg brand B
 ★ D How many packages of brand A must you buy to serve one hot dog each to 56 people? How much will it cost? $4.48 (7 pkg.)



Follow-up

Children might enjoy further work with the type of exercises on page 143. Perhaps you can encourage them to do some research at the grocery store so that they can make up their own problems. Some difficulty may arise because many prices will involve fractions or decimals, but the children can estimate the cost per item to the nearest whole number.

Resources for Active Learning

Mathex: Operations and Problem Solving No. 8, "Sale," pupil page 43, Encyclopaedia Britannica Publications Ltd.

Assignments (page 142)

Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Assignments (page 143)

Minimum: 1–6B, oral.
Average: 1–6B. Maximum: 1–6.

Objective

Given a function machine which involves two rules of operation, the child will be able to apply the rules step by step and find the output.

Preparation

To prepare for this lesson, you might use some mental chain games or the "What's My Rule" game. Some of the following mental chain games would be suitable.

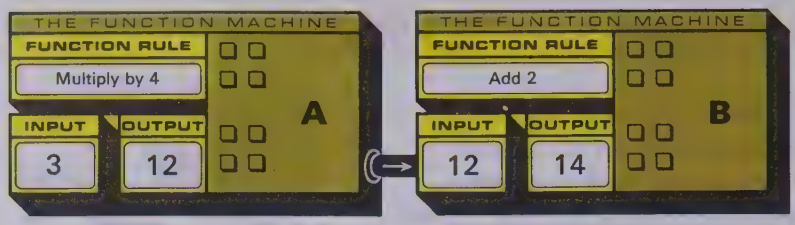
"Start with 45 . . . Divide by 5 . . . Multiply by 6 . . . Add 2 . . . Divide by 8. What's your answer?" (7)

"Start with 4 . . . Multiply by 7 . . . Subtract 1 . . . Divide by 3. What's your answer?" (9)

"Start with 18 . . . Divide by 6 . . . Multiply by 7 . . . Subtract 1 . . . Divide by 4 . . . Multiply by 8. What's your answer?" (40)

"Start with 72 . . . Divide by 8 . . . Multiply by 4 . . . Divide by 6 . . . Multiply by 4 . . . Divide by 8. What's your answer?" (3)

The Function Machine



Function machine A is connected to function machine B. The output number from machine A becomes the input number for machine B. We put in 3. Machine A operates. Machine B operates. We get 14.

Think about connected function machines and give the numbers and words you think should go in the gray spaces.

1. RULE: A Multiply by 2; B Add 1

	Input A	Output B
	3	7
A	6	13
B	9	19
C	10	21

2. RULE: A Multiply by 8; B Add 47

	Input A	Output B
A	4	79
B	6	95
C	7	103
D	9	119

3. RULE: A Add 37; B Subtract 37

	Input A	Output B
A	25	25
B	88	88
C	100	100
D	0	0

4. RULE: A Multiply by 7; B A — Divide by 7

	Input A	Output B
	4	4
	9	9
B	6	6
C	8	8

5. RULE: A Add 8; B A — Divide by 2

	Input A	Output B
	4	6
	6	7
B	2	5
C	10	9

6. RULE: A A — Multiply input A by itself; B Add 1

	Input A	Output B
	3	10
	5	26
B	4	17
C	8	65

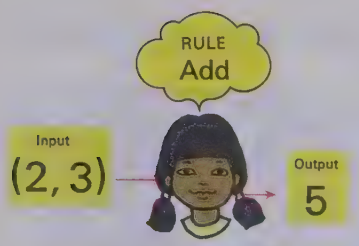
144

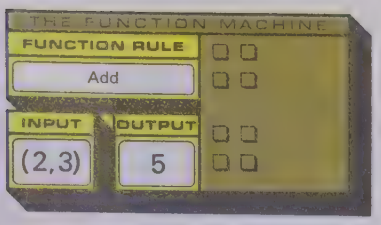
Discussion

Study with the children the two function machines at the top of the page. Notice with them that they are to think of two function machines and, thus, two function rules. Observe that in the first function machine there is an input A and an output A; the number 3 is put into the machine; the rule is "Multiply by 4"; so the output is 12. Explain to the children that output A of the first machine is input B of the second machine. That is, machine A takes the number 12 and puts it into machine B. Now machine B uses the rule "Add 2" and gives the final answer, 12 + 2 or 14.

Use other numbers as the input for machine A until children see how the two machines work together. You might have the children pretend that they are the function machines and have others pass them input cards. For example, the first input card is passed to function machine A, he gives the answer, and the second input card with that answer is passed to function machine B.

The material on page 145 presents still another variation of the function machine. Notice with the children that this time they are to put a pair of numbers into the machine and that this function machine performs an operation on the pair of numbers.





In the following exercises, you put a pair of numbers into a function machine and get a single number. Give what you think should go in the gray spaces.

7. Function Rule: Multiply

	Input	Output
A	(3, 7)	21
B	(4, 9)	36
C	(9, 9)	81
D	(7, 6)	42
E	(9, ▮) 5	45
F	(▮, 8) 9	72

8. Function Rule: Divide

	Input	Output
	(12, 4)	3
A	(18, 2)	9
B	(49, 7)	7
C	(0, 3)	0
D	(48, ▮) 6	8
E	(▮, 6) 54	9

9. Function Rule: Choose the greater

	Input	Output
	(39, 93)	93
	(107, 170)	170
A	(99, 999)	999
B	(384, 834)	834
C	(1005, 999)	1005
D	(617, 716)	716

10. Function Rule: Double the first Then add

	Input	Output
	(3, 4)	10
	(9, 3)	21
A	(7, 8)	22
B	(9, 18)	36
C	(8, 94)	110
D	(10, 10)	30

11. Function Rule: A ▮▮▮▮▮▮ Add

	Input	Output
	(6, 4)	10
	(9, 7)	16
	(14, 8)	22
B	(19, 8)	27
C	(79, 67)	146
D	(37, 63)	100

12. Function Rule: A ▮▮▮▮▮▮ Add then divide by 2.

	Input	Output
	(6, 4)	5
	(4, 10)	7
	(12, 8)	10
	(9, 9)	9
B	(7, 9)	8
C	(4, 8)	6

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To prepare the children for the exercises, you might give them other pairs of numbers to put into this function machine and then have them decide what comes out. You might say, "Suppose we put in the numbers 5 and 6." The children should observe that the output would be 11.

Have the children work the exercises on both pages. When they have finished, allow time for a discussion. Most would benefit from a discussion of the starred exercises even if not all can give the correct answers.

Assignments (page 144)
Minimum: 1–3. Average: 1–4.
Maximum: 1–6.

Assignments (page 145)
Minimum: 7–9. Average: 7–11.
Maximum: 7–12.

Follow-up

Children might enjoy making their own function machines. One similar to the two machines on page 144 would allow for interesting variations in rules. You might suggest that children experiment with the same operations, applying them first in one order and then in reverse order. For example, the pair

{Rule A: multiply by 5
{Rule B: subtract 3

yields an output different from that for the same operations in opposite order:

{Rule A: subtract 3
{Rule B: multiply by 5.

Resources for Active Learning

Mathex: Operations and Problem Solving No. 8, "What's My Rule?" p. 17; "Function Machine," p. 18, Encyclopaedia Britannica Publications Ltd.

Workbook, page 38

PAGES 146–147

Objective

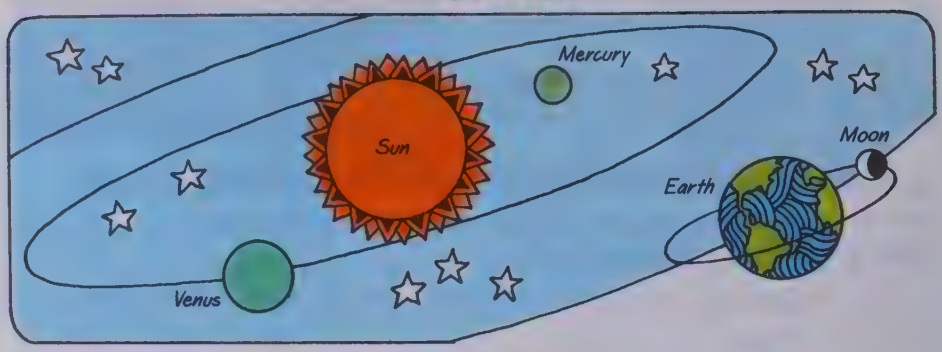
Given word problems which require application of one of the four basic operations, the child will be able to solve the problem by choosing and performing the proper operation.

Preparation

To heighten children's interest in the theme of these problems, spend a few minutes discussing facts about the moon and other bodies in our solar system. In particular, a discussion of gravity on Earth, the moon, and other planets would lead smoothly into the exercises on these pages. For example, you might point out that the reason objects weigh six times as much on the moon is that the Earth is six times as large as the moon.

Solving Short Stories

SPACE



1. Rocket to the moon. 63 kilometres in 9 seconds. How many kilometres per second ? 7



2. Closest distance to moon from Earth: 354 341 kilometres. Farthest distance: 404 336 kilometres. How much change in distance ? 49 995 km



3. Jupiter has 12 moons. Saturn: 10 moons. Uranus: 5 moons. Neptune: 2 moons. Mars: 2 moons. Earth: 1 moon. How many moons in all ? 32

4. Rocket going into Earth orbit. Speed: 8 kilometres per second. How far in 8 seconds ? 64 km



5. Diameter of moon: 3456 kilometres. Diameter of Earth: 12 672 kilometres. How much greater is Earth's diameter ? 9216 km

6. Weight. About 3 times as much on Earth as on Mars. Object weighs 27 kilograms on Earth. How much on Mars ? 9 kg

★ 7. Weight. About 3 times as much on Earth as on Mercury. 18 kilograms on Mercury. How many pounds on Earth ? 54 kg

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Discussion

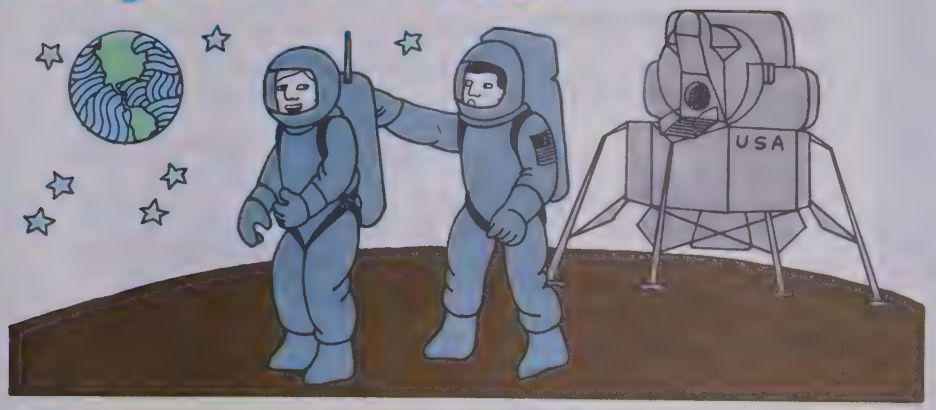
Explain to the children that the exercises on these pages involve addition, subtraction, multiplication, and division. If children seem to have difficulty in approaching word problems, give them simple guidelines to follow. For example, write the following italicized steps on the chalkboard. (The comments in parentheses are suggestions for a discussion of these steps.)

1. *What do I know?* (What information is given in the problem or in a chart or graph?)
2. *What must I find?* (How many altogether, how much more, how many times as?)
3. *What do I do?* (Which operation will answer the question?)
4. *Does my answer make sense?* (Reread the problem; does *my answer* really answer the question I was asked in the problem?)

On page 147, note that exercise 6 is challenging in that it requires work with fractions. However, many children will be able to do this since they will see that half of six is three; hence, the answer is 21. In discussing exercise 6, avoid getting into any complicated work concerning multiplication of fractions. This should be done strictly on an intuitive basis.

When the children have finished the exercises, allow time to check

Weights on the Moon



Objects weigh 6 times as much on Earth as they do on the moon.

Some children at Franklin School made this table to compare their Earth weights with their moon weights.

Name	Weight in kilograms	
	Moon	Earth
Jean	A 3	18
Kristin	B 4	24
Tom	4	C 24
John	3	D 18
Fred	E 5	30

1. Complete the table by giving the weight that should go in each space. See table.
2. Find the total moon weight of all the children. 19 kg
3. Find the total Earth weight of all the children. 114 kg
4. Bill's Earth weight is 24 kilograms. His dog has a moon weight of 2 kilograms. What is the total Earth weight of Bill and his dog ? 36 kg
5. Karen's bicycle has a moon weight of 3 kilograms. Karen's Earth weight is 24 kilograms. On Earth, how much more does Karen weigh then her bicycle ? 6 kg

★ 6. If Eric weighs $3\frac{1}{2}$ kilograms on the moon, what does he weigh on Earth ? 21 kg

papers and discuss anything pertinent to this lesson.

Assignments (page 146)
Minimum: 1–7, oral. Average: 1–7.
Maximum: 1–7.

Assignments (page 147)
Minimum: 1–5, oral. Average: 1–5.
Maximum: 1–6.

Follow-up

A lesson of this type is designed not only to provide experiences in working with arithmetic and word problems but also to stimulate the curiosity of children.

Encourage the children to do research on other interesting space facts in reference books. Ask them to make up some problems on their own, or give them a chart with some information about space on it, and ask each one to write four or five problems using the information.

SAMPLE DATA SHEETS

Stars in the Big Dipper	*Light Years Away*
Alpha	109
Beta	76
Gamma	79
Delta	74
Epsilon	72
Zeta	76
Eta	251

Planet	*Days for One Complete Orbit**
Mercury	88 Earth days
Venus	225 Earth days
Earth	365 Earth days
Mars	687 Earth days

*Trip around the sun

Workbook, page 39

PAGES 148–149

Objective

Given two sets of objects, the child will be able to find the product set by pairing objects in the sets and to write a multiplication equation related to the product set.

Preparation

Materials

crayons (4 colors per child); tracing paper

You may begin immediately with the investigation, or if you prefer, use a brisk oral drill to review multiplication and division facts.

Investigation

If possible, distribute paper which can be used for tracing and have the children trace the square, circle, and triangle as many times as they need to in order to color the figures according to the directions given. The exactness of the figure is not important to this lesson, however, so you might simply have children draw the figures freehand, without any loss to the concept development.

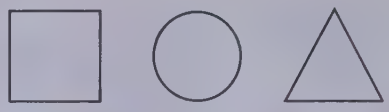
The graph referred to in the text need not be made with graph paper; children may simply make this large grid on tablet paper. You might point out that the colored figures should be placed where the lines intersect, not within the large squares.



How are pairing and multiplication related?

Investigating the Ideas

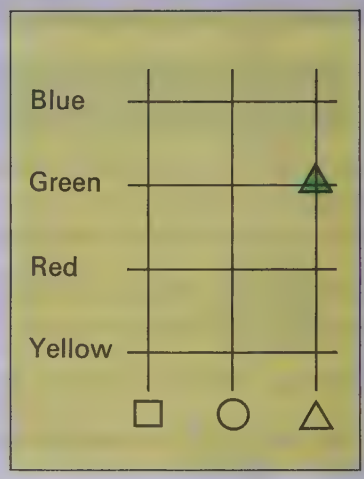
Use only these shapes →



and 4 different colors. →



Cut out and color (one color per figure) as many different figures as you can.



? Can you paste all your figures in their proper places on a graph like this one? See Investigation and Discussion.



Discussing the Ideas

1. A Did you **pair** each figure with each color? Yes
 B Did you get 4 rows of 3 on your graph? Yes
 C Solve: $4 \times 3 = n$ 12

2. How many different colored figures would you get with
 A 5 crayons? 15 B 6 crayons? 18 C 10 crayons? 30

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Discussion

Use a flannelboard and felt objects, an overhead projector, or colored chalk to demonstrate the colored figures on the grid. Make sure the children see that for each shape they should have four different colored figures. As you develop a discussion based on exercise 1, help the children interpret the multiplication equation $4 \times 3 = n$. Stress the relationship of the number of objects in each of the original sets to the factors in the equation, and relate the number of pairs they found to the product of the equation.

If necessary, provide demonstrations of other examples before working through the exercises on page 149.

Using the Ideas

1. Pam is making a sandwich. She plans to use white or brown bread (not both). She will use one of the following: beef, cheese, or peanut butter.



What kind of sandwich do you think Pam made?

A Which kind of bread would you choose?

B Which of the three would you put on your sandwich?

Children's choices will vary.

See Answers, T.E. page 149.

C Name all the different sandwiches that might be made.

D Solve the equation.

Number of kinds of bread	Number of things to put between the bread	Number of different sandwiches

$2 \times 3 = n$ 6

2. Bill can have milk, soda, or cocoa. Bill can have a doughnut, cookie, roll, or cake. What do you think Bill ate and drank?



Bill

A Name all the different choices Bill might have made.

See Answers, T.E. page 149.

B How many choices are there? 12

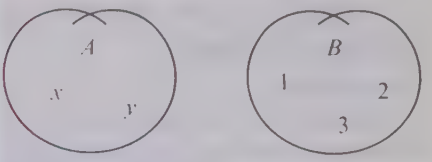
C Solve: $3 \times 4 = n$ 12

149

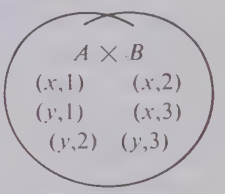


Mathematics

The idea of a product set is illustrated by considering two sets, A and B, as illustrated below.



Now consider the set of all pairs of objects such that the first object in the pair is from set A and the second object in the pair is from set B. We call this set of ordered pairs the *Cartesian product* of A and B (written: $A \times B$). The set $A \times B$ consists of six ordered pairs:



We now relate these set concepts to the equation $2 \times 3 = 6$.

Follow-up

Suggest that children make a list of all possible pairings between each of the following groups, and then write an equation for each group.

1.	brown skirt blue skirt	blue blouse white blouse orange blouse yellow blouse
2.	gold shirt blue shirt white shirt	green and gold tie brown and green tie yellow and brown tie
3.	white car blue car red car black car	red upholstery blue upholstery black upholstery white upholstery plaid upholstery

Children might enjoy making up their own sets to pair. Encourage them to print such sets on cards and then exchange them with one another.

Resources for Active Learning

Mathex: Operations and Problem Solving No. 8, "Counting Pairs," pp. 16–17, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Green Set, No. 35, Wiley.

Using the Exercises

You might use exercise 1 as a basis for further discussion. For example, list on the chalkboard or overhead projector all the different sandwiches the children suggest and then relate this list to the diagram in part D.

Assign exercise 2 as independent work. When the children have finished, allow ample time for discussion. For those who finish quickly, suggest other sets to be paired.

Answers, exercises 1D and 2A

1D	brown bread, beef	white bread, beef
	brown bread, cheese	white bread, cheese
	brown bread, peanut butter	white bread, peanut butter
2A	milk, doughnut	soda, roll
	milk, cookie	soda, cake
	milk, roll	cocoa, doughnut
	milk, cake	cocoa, cookie
	soda, doughnut	cocoa, roll
	soda, cookie	cocoa, cake

Assignments (page 149)*

Minimum: 1–2, oral. Average: 1–2. Maximum: 1–2.

PAGES 150–151

Objective

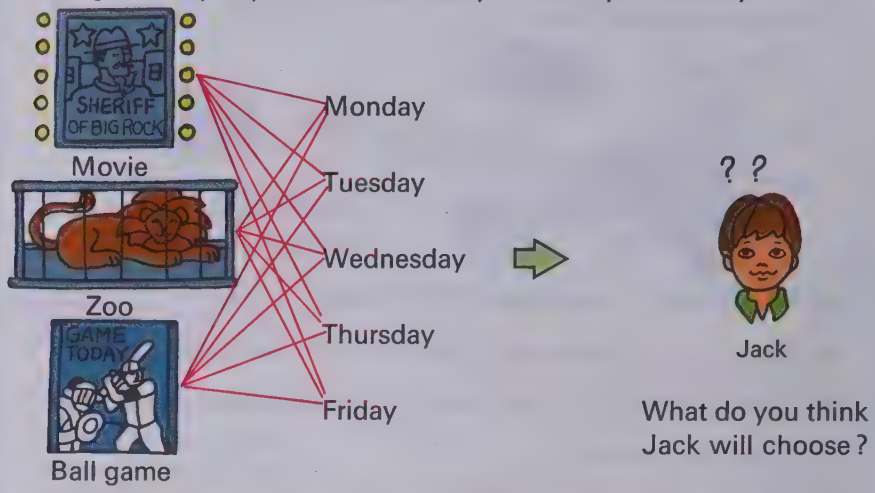
Given groups of sets with elements to pair, the child will be able to form the product set and relate it to a multiplication equation.

Preparation

To prepare for this lesson, conduct a brisk review of multiplication facts. You might show some rectangular arrays and have the children give a multiplication equation for each. You might also use a game such as "What's My Rule" to provide practice with the basic facts.

Pairing and multiplication

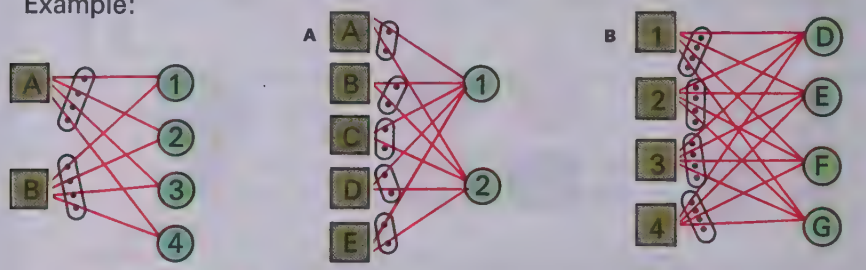
1. For his birthday, Jack gets to go to a movie, the zoo, or a ball game. He can go on any day of the week except Saturday or Sunday.



A Name all the choices (like the zoo on Tuesday) that Jack could make. How many choices are there? 15 (See above.)

B Solve: $3 \times 5 = n$ 15

2. Write a multiplication equation for each picture. The small dots help you count the red lines that pair the squares with the circles. Example:



$2 \times 4 = 8$ $5 \times 2 = 10$ $4 \times 4 = 16$

150

Discussion

Unless you think your class needs a discussion of the exercises on page 150, assign this page as independent work. The black dots in exercise 2 are included to help the children count the red matching lines, which in these exercises represent the product set. However, it is important to stress the number of squares and circles in each exercise and to relate these numbers to a multiplication equation.

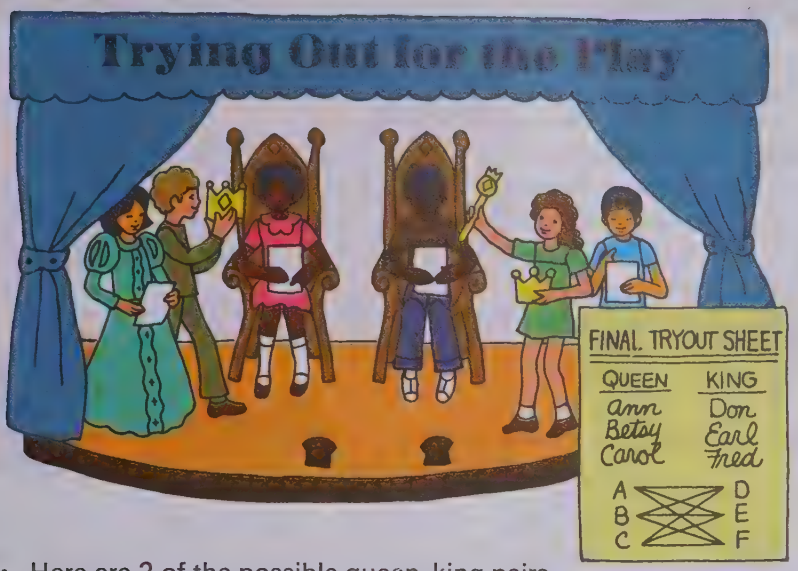
Before assigning page 151, study the chart at the top of the page with the children. Be sure that they understand from exercise 1A that A stands for Anne, B for Betsy, C for Carol, D for Don, E for Earl, and F for Fred. Observe the red matching lines. Have the children interpret several of the matching lines in terms of possible king-queen pairs.

Following this discussion, have the children do the exercises. Notice that exercise 6 is designed to be challenging. When the children have finished the exercises, give them an opportunity to discuss the exercises and ask questions.

Assignments (page 150)*

Minimum: 1, oral. Average: 1. Maximum: 1–2.

Solving Story Problems



1. A Here are 2 of the possible queen-king pairs.
 A, D (This is a short way of writing Ann and Don.)
 C, E (What does this represent?) Carol and Earl
 B How many different queen-king pairs are there? 9
 C Write a multiplication equation that tells how many pairs. 3x3=9

2. The children decided that Betsy should be a princess and not the queen. Now list the possible queen-king pairs.
 A,D; A,E; A,F; C,D; C,E; C,F

3. Earl had been reading about knights. He wanted very much to be a knight, so the children agreed. How many queen-king pairs were left? 4 (A,D; A,F; C,D; C,F)

4. Fred moved away before the play was presented. Who was the king? Don

5. Ann was not chosen to be the queen. Who was the queen? Carol

★6. If 8 girls want to be queen and 7 boys want to be king, how many queen-king pairs would be possible? 56

Follow-up

Encourage children to make up pairing problems of their own. You might suggest to the more capable children that they chart the pairs they can form and display them as a rectangular array. For example, consider how many choices of dipped ice cream cones can be made with the following selections.

vanilla ice cream — chocolate syrup
strawberry ice cream — marshmallow
chocolate ice cream — syrup

These may be charted as shown.

	chocolate	marshmallow
vanilla	x	x
strawberry	x	x
chocolate	x	x

The chart yields a 3×2 array, and it is clear that 6 choices are possible. Suggest to the children that they not only make up their own groups for pairing but also chart an array for each product set they form.

Workbook, page 40

Assignments (page 151)*
Minimum: 1–5, oral. Average: 1–5.
Maximum: 1–6.

PAGES 152–153

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

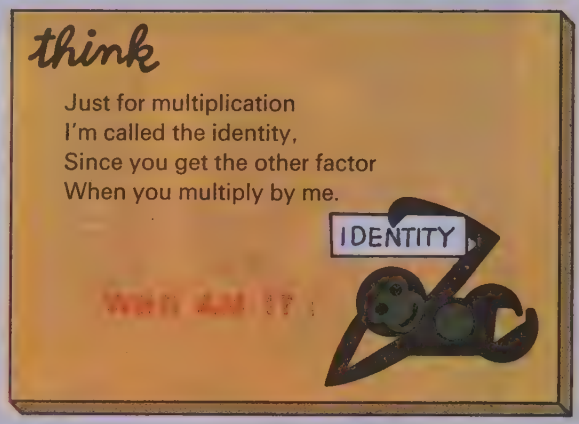
If you use the chapter review as an evaluation instrument, you might use the preparation time to review the basic concepts covered in this chapter. You might have the slower children review basic multiplication and division facts while the other children use this time to make up word problems for particular equations you display on the chalkboard. You might also let the children decide what they think they should review before working these pages.

If you use the chapter review as a review lesson, plan an oral warm-up before working through the exercises in the text.

Reviewing the Ideas

1. Find the products.

A 1×1 1
B 9×1 9
C 0×0 0
D 4×0 0
E 2×7 14
F 4×9 36
G 3×8 24
H 8×3 24
I 9×4 36
J 4×8 32
K 5×8 40
L 6×7 42
M 8×6 48
N 9×9 81
O 7×8 56
P 9×6 54
Q 9×8 72
R 7×9 63
S 9×10 90
T 10×10 100



2. Find the quotients.

A $24 \div 1$ 24
B $0 \div 7$ 0
C $9 \div 9$ 1
D $27 \div 3$ 9
E $36 \div 6$ 6
F $48 \div 8$ 6
G $48 \div 6$ 8
H $45 \div 9$ 5
I $42 \div 7$ 6
J $49 \div 7$ 7
K $54 \div 6$ 9
L $63 \div 7$ 9
M $63 \div 9$ 7
N $56 \div 8$ 7
O $64 \div 8$ 8
P $72 \div 9$ 8
Q $72 \div 8$ 9
R $81 \div 9$ 9
S $90 \div 9$ 10
T $100 \div 10$ 10

3. Solve the equations to find the products.

A $12 \times 7 = (6 \times 7) + (6 \times 7) = n$ 84
B $13 \times 5 = (7 \times 5) + (6 \times 5) = n$ 65
C $16 \times 7 = (8 \times 7) + (8 \times 7) = n$ 112
D $18 \times 8 = (9 \times 8) + (9 \times 8) = n$ 144

4. Find the quotients.

A $5\overline{)25}$ 5
B $6\overline{)54}$ 9
C $7\overline{)42}$ 6
D $9\overline{)54}$ 6
E $9\overline{)0}$ 0
F $7\overline{)56}$ 8

5. Give the missing numbers.

A Since $54 \times 398 = 21\,492$, we know that $398 \times 54 = n$. 21 492
B Since $54 \times 398 = 21\,492$, we know that $21\,492 \div 398 = n$. 54
C Since $4672 + 4672 + 4672 = 14\,016$, we know that $3 \times 4672 = n$. 14 016
D Since $144 \div 48 = 3$, we know that $3 \times 48 = n$. 144

Discussion

The first two sets of exercises review children's knowledge of the basic facts. Exercise 3 applies the multiplication-addition principle; remind the children to work the operation inside the parentheses first. Exercise 5 reviews the order principle and basic multiplication and division concepts. The word problems review all four of the basic operations.

Exercise 13 is starred since the children have not yet encountered divisions such as $56 \div 4$. However, in part A many will guess correctly that $56 \div 8$ is equal to $(56 \div 4) \div 2$. Some children may think that the expression $(56 \div 4) \div 2$ is equal to $56 \div 6$, since four and two are six. If the children should ask how they can go about dividing 56 by four, point out that they can do this problem by guessing and using multiplication. For example, one child might guess 10, and he would observe this is incorrect because $4 \times 10 = 40$. Another child might guess 12 and find that $4 \times 12 = 48$, which is incorrect. Through successive guesses, the children will probably arrive at the fact that 14 is the correct quotient for $56 \div 4$. Working exercise 13B should strengthen the understanding obtained from 13A.

6. How many weeks are there in 49 days? 7



7. 32 Boy Scouts were divided into 4 patrols. There were the same number of scouts in each patrol. How many scouts were in each patrol? 8

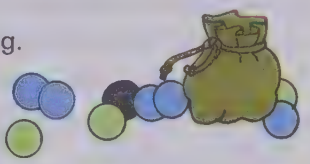
8. How much does it cost to buy seven 8-cent stamps and one 7-cent stamp? 63¢





9. How many pennies can you get for 7 nickels and 3 dimes? 65

10. Tom had 8 bags with 8 marbles in each bag. He had 29 extra marbles on the ground. How many marbles did he have in all? 93





11. Sally put 9 coins on each of 7 pages in her coin-collection book. She had 77 other coins in a box. How many coins did she have in all? 140

★12. Ted had 35 rocks. Jim had 63 rocks. Each boy decided to store his rocks in cans, with 7 rocks in each can. Who used more cans? How many more? 4
Jim



★13. A Does $56 \div 8 = (56 \div 4) \div 2$? B Does $42 \div 6 = (42 \div 3) \div 2$? Yes
Yes

Follow-up

If the children seem to have a good command of the basic multiplication facts but have difficulty finding quotients accurately, a worksheet re-emphasizing the inverse relation between multiplication and division may be helpful. You might duplicate one like this.

Study the pattern; then complete the table.

6 ×8 (48)	9 ×4 (36)	8 ×7	4 ×9	5 ×6	7 ×6	3 ×7	9 ×7
÷8 (6)	÷4 (9)	÷7	÷9	÷6	÷6	÷7	÷7

Resources for Active Learning

Discovery, Section II, Unit 17/4 Encyclopaedia Britannica Educational Corp. [Cross-number puzzle exercises]

Workbook, page 41

PAGES 154–155

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

To prepare for these pages, review any topics covered thus far in Book 4 with which the children have had difficulty. For example, you might make up short addition and subtraction word problems to give to the children orally. Or you might use the "What's My Rule" game to review addition and subtraction. If you use 2-digit numbers in some examples, you would be able to review the addition and subtraction algorithms, although the children would need to use paper and pencil.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Place value

1. Copy and find the sums.

A 538 + 24 + 365 + 4720 5647

B 4768 + 232 + 5496 10 496

C 6437 + 35 + 932 7404

D 375 + 893 + 5672 + 38 6978

2. Find the sums and differences.

A	B	C	D	E
4635	1386	8207	5003	5739
+2819	−539	−3649	−2216	+8654
7454	847	4558	2787	14 393

3. Write the numeral for each exercise.

A five hundred thirty-seven 537

B nine hundred fifty 950

C seven hundred six 706

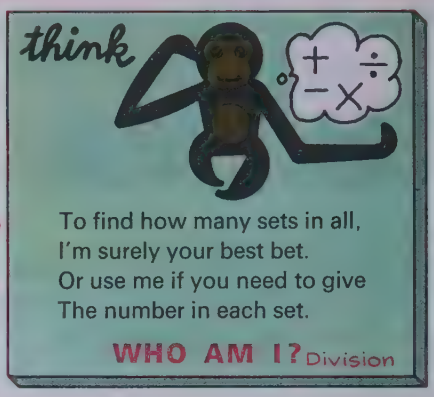
D four hundred nineteen 419

E eight thousand seven hundred 8700

F one hundred sixty-six thousand 166 000

G six thousand thirty-seven 6037

H ninety-eight thousand seventy 98 070



4. Give the next five numbers in each sequence.

A 1, 4, 9, 16, 25, . . . 36, 49, 64, 81, 100

B 1, 3, 5, 7, 9, . . . 11, 13, 15, 17, 19

C 1, 2, 4, 8, 16, . . . 32, 64, 128, 256, 512

5. A Add the first 2 numbers in exercise 4B.
Which number in exercise 4A do you get? 4

B Add the first 4 odd numbers in exercise 4B. You should get 4 × 4. 1+3+5+7=16

C Add the first 6 odd numbers. Do you get 6 × 6? Yes

D Add the first 7 odd numbers. Do you get 7 × 7? Yes

E What do you think is the sum of the first 8 odd numbers? 8×8, or 64

F Find the sum of the first 9 odd numbers. 9×9, or 81

G Find the sum of the first 10 odd numbers. 10×10, or 100

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Discussion

Exercises 1, 2, and 3 are straightforward review exercises. Children may need an explanation of the term *sequence* in exercise 4.

You may find that the children need to review the meaning of *odd numbers* in order to complete exercise 5. When discussing this exercise, guide the children to draw some conclusions, such as the fact that the sum of the first n odd numbers is the same as the product $n \times n$ (e.g., the sum of the first nine odd numbers is 81 and $9 \times 9 = 81$).

Most of the children will understand the *Think* problem when you give the solution, but allow them a chance to ponder the problem a bit before giving them the answer.

Allow ample time for a discussion of each problem in exercise 6. Part of the value of exercises of this type is the discussion which accompanies them.

You may find that the children enjoy problems such as those in exercise 6, since they do not have to do any computing. Even though there is no computation required, exercises of this type are extremely valuable because they focus attention on one of the most important purposes of story problems—having children think through the ideas and determine the correct operation.

6. No numbers are given in this exercise. Decide whether you would add, subtract, multiply, or divide if numbers were given. Answer **A, S, M,** or **D**.

A Joe had ▮ baseball cards.
Bill had ▮ baseball cards.
How many more baseball cards did Bill have than Joe? S





B We drove ▮ kilometres on Friday and ▮ kilometres on Saturday. How far did we drive altogether? A

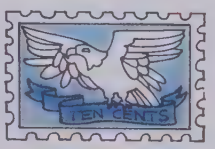
C In Jane's classroom there were ▮ rows with ▮ children in each row.
How many children? M





D Ted had ▮ hamsters. He gave ▮ away.
How many did he have left? S

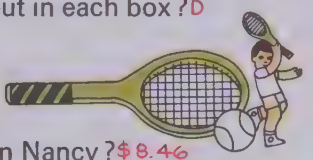
E Emily had ▮ stamps in her collection. She had ▮ stamps on each page. How many pages of stamps did she have? D

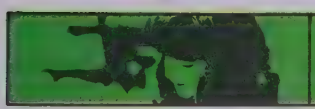




F Sandy made ▮ pieces of fudge. She put the same number of pieces in each of ▮ boxes. How many pieces did she put in each box? D

7. Nancy's tennis racket cost $9.99.
Ken spent $18.45 for his racket..
How much more did Ken spend than Nancy? $8.46





You are invited to explore

ACTIVITY CARD 6
Page 350

Naturally, once children decide what they are going to do for a given story problem, all that is left is straight computational drill.

Resources for Active Learning

[These resources provide review exercises for all operations.]

Developmental Math Cards, F[1]6, F[5]10, F[5]21, G[1]7, G[5]9, Addison-Wesley.

Discovery, Section II, Units 5/5; 6/4; 11/3; 18/4, Encyclopaedia Britannica Educational Corp.

Workbook, page 42

CHAPTER 6 | Special Products and Quotients

Pages 156–169

General Objectives

To develop skill in finding products that have factors of 10, 100, or 1000 or their multiples

To develop skill in finding quotients related to the products mentioned in the preceding objective

To generalize the order and grouping principles into one "rearranging" principle

To show how some of the principles are used to find products

To summarize all the principles studied thus far

To develop a sound foundation for Chapter 8

This chapter systematically develops products that are multiples of 10, 100, and 1000. The skills learned here are vital to Chapter 8, which deals with the multiplication algorithm. If the children are to work effectively with multiplication of large numbers, they must be skillful with products of this type.

Although we devote much attention to the development of particular skills, we expect you to recognize and emphasize the value of the basic principles, which are used to show how to arrive at certain points.

"Keeping in Touch" page 169 summarizes all the basic principles and may be considered an integral part of the chapter. You could even treat this as part of the chapter review. The basic principles of addition are not a part of the chapter review, but provide an opportunity to review the basic principles of multiplication.

Mathematics

The mathematical concepts encountered in this chapter are summaries of all the basic principles for whole numbers studied so far. Below is a summary of these principles.

Addition	Multiplication
For all whole numbers a, b, and c:	
1. $a + b = b + a$	$a \times b = b \times a$
2. $(a + b) + c = a + (b + c)$	$(a \times b) \times c = a \times (b \times c)$
3. $a + 0 = a$	$a \times 1 = a$
4. $a \times (b + c) = (a \times b) + (a \times c)$	

The first principle in each list is the commutative principle; the second is the associative principle; and the third is either the zero principle or the one principle. Number 4 is a statement of the distributive principle for multiplication over addition.

Teaching the Chapter

Materials

Overhead projector (if available) and transparencies

Objects that can be grouped easily by tens or hundreds (optional)

Vocabulary

multiple

Since one of the more important purposes of this chapter is to develop skill with particular types of multiplication exercises, the use of materials is limited. If you use materials at all, restrict their use to demonstrating multiples of 10 and 100 for slower children.

Multiple is the only new word in the chapter. Use the word often in context so that it will be readily understood.

Lesson Schedule

Plan to cover this material in about a week and a half, or seven to eight school days. However, you will want to vary this schedule according to the abilities and motivations of your pupils.

Evaluation of Progress

Evaluating the material in this chapter should be fairly simple, since mastery of routine factual material and skills is easily tested. None of the concepts in the chapter is new.

An important factor in evaluating the children's achievement is to determine whether they understand how certain basic skills grow out of the basic principles. For example, exercise 4 on page 158 depends on the grouping (associative) principle. Observe carefully whether or not the children understand the use of the principles in developing generalizations for finding particular products.

We cannot emphasize too strongly how important it is for the children to become skillful in finding special products. The purpose of this chapter is to give the children a background that will enable them to concentrate on the ideas behind the standard multiplication algorithm and the general procedures in Chapter 8, without worrying about special products.

We encourage you to provide many opportunities for oral practice on these simple products.

Resources for Active Learning

General Activities

[Since this chapter focusses on a special point of multiplication-division, the resources listed in Chapter 5 would be helpful here as well. Choose the favorite games and activities that will reinforce basic facts and computational skills.]

Games

Experiments in Mathematics, Stage 2, "Three-dimensional Noughts and Crosses," p. 52, Houghton Mifflin (Available from Thomas Nelson & Sons)

Franklin Series: *Patterns and Puzzles*, "Tit-tat-toe," pp. 7–14, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Math Activity Cards, "The Game of Bridges," B45; "Special Mill," C45, Macmillan

Notes on Mathematics in Primary Schools, "Squares," p. 233; "Go," p. 235; "Nim," p. 236, Cambridge University Press (Available from Macmillan of Canada)

Measure and Find Out, Book 1, Scott, Foresman. (Available from Gage Educational)

[Use the Metric System measures shown in the Tables of Measures (page A–38) to help with these mathematics-science activities.]

Commercial Games

[Both of these are games of strategy.]

Nine Men Morris (or Mill) (World Wide Games)

3-D Tic-Tac-Toe (Creative Publications; Math Media)

PAGES 156–157

Objective

Given two factors, one of which is 10, 100, or 1000, the child will be able to find their product.

Preparation

For this investigation, children need no materials other than the text. If several children still find some basic multiplication facts troublesome, use a few minutes before each lesson to practice these facts in particular.

Investigation

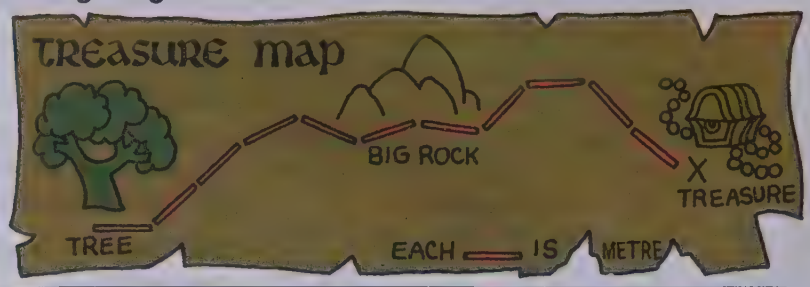
Give the children guidance in studying the map and the investigation questions beneath it. Help them observe the torn area of the map and how the missing portion relates to the investigation question. Suggest that they may use any method they can to answer the questions. For example, some may count by ones, then by tens, hundreds, and thousands. Others may count by ones and recall how to find 11×40 and 11×100 without counting. For those children who finish very quickly, write on the chalkboard an exercise similar to the following:

$2 \times 10 = ?$ $2 \times 100 = ?$ $2 \times 1000 = ?$
$3 \times 10 = ?$ $3 \times 100 = ?$ $3 \times 1000 = ?$
$4 \times 10 = ?$ $4 \times 100 = ?$ $4 \times 1000 = ?$
$\vdots$ $\vdots$ $\vdots$
$9 \times 10 = ?$ $9 \times 100 = ?$ $9 \times 1000 = ?$

6 Special Products and Quotients

● *Let's explore 10, 100, and 1000 as factors.*

Investigating the Ideas



Can you find the distance from the tree to the treasure if the missing number on the map is

A 1 ? 11m B 10 ? 110m C 100 ? 1100m D 1000 ? 11000m or 11km

See Investigation.

Discussing the Ideas

1. Solve the equations.
 - A $14 \times 1 = n$ 14
 - B $14 \times 10 = n$ 140
 - C $14 \times 100 = n$ 1400
 - D $14 \times 1000 = n$ 14,000

2. Continue the skip-counting for 20 more numbers.
 - A 10, 20, 30, 40, 50, 60, 70, 80, ... 250
 - B 100, 200, 300, 400, 500, 600, 700, 800, ... 2500
 - C 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, ... 25 000

3. Study this rule. Can you give a similar rule for finding the product of a number and 10?

 To write the **product** of a number and 100 just write this number in the **hundreds' place.**

 HUNDREDS: $6 \times 100 = 600$ | HUNDREDS: $87 \times 100 = 8700$

 See Discussion.

4. Can you think of a rule of your own for multiplying by 10, 100, or 1000? See Discussion.

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Discussion

As you discuss the questions in the investigation, lead into the equations in discussion exercise 1. Some children may realize that the skip counting in exercise 2 is the method many may have used to answer the investigation questions.

For exercise 3, you might expect the children to say, "To write the product of a number and 10, just write this number in the tens' place."

As children try to word a rule of their own in exercise 4, help them avoid the expression "adding zero to the number." You might suggest instead a phrase such as the following: "placing a zero (or two zeros or three zeros) at the end of the numeral."

Using the Ideas

1. How many cents?
 - A 1 dime 10
 - B 2 dimes 20
 - C 7 dimes 70
 - D 12 dimes 120
 - E 28 dimes 280
 - F 75 dimes 750
 - G 124 dimes 1240
 - H 526 dimes 5260

2. How many cents?
 - A 1 dollar 100
 - B 2 dollars 200
 - C 7 dollars 700
 - D 12 dollars 1200
 - E 28 dollars 2800
 - F 75 dollars 7500
 - G 124 dollars 12 400
 - H 526 dollars 52 600

3. Find the products.
 - A 5×10 50
 - B 17×10 170
 - C 12×10 120
 - D 18×10 180
 - E 5×100 500
 - F 17×100 1700
 - G 12×100 1200
 - H 24×100 2400
 - I 32×10 320
 - J 32×100 3200
 - K 48×10 480
 - L 48×100 4800
 - M 5×100 500
 - N 17×1000 17 000
 - O 12×1000 12 000
 - P 18×1000 18 000
 - Q 15×10 150
 - R 15×100 1500
 - S 15×1000 15 000
 - T 30×1000 30 000
 - U 40×100 4000
 - V 40×10 400
 - W 40×1000 40 000
 - X 70×100 7000

4. Each metre is 100 centimetres. How many centimetres in
 - A 5 metres? 500
 - B 14 metres? 1400
 - C 50 metres? 5000
 - D 100 metres? 10 000

★ 5. Solve the equations.
 - A $7 \times n = 7$ 1
 - B $7 \times n = 70$ 10
 - C $7 \times n = 700$ 100
 - D $7 \times n = 7000$ 1000
 - E $6 \times n = 600$ 100
 - F $18 \times 100 = n$ 1800
 - G $14 \times n = 14\,000$ 1000
 - H $n \times 100 = 900$ 9
 - I $n \times 1000 = 15\,000$ 15
 - J $23 \times n = 2300$ 100
 - K $79 \times n = 790$ 10
 - L $79 \times n = 7900$ 100
 - M $n \times 1000 = 51\,000$ 51



Using the Exercises

Exercises 1 and 2, page 157, use the children's familiarity with dollars and cents to stress the concept of products with 10 and 100 as factors. Help the children realize that 7 dimes is equivalent to 7×10 or 70 cents. Similarly, 12 dollars is the same as 12×100 or 1200 cents.

Work through exercise 4 with the children, helping them with the terms *metre* and *centimetre*.

You may wish to assign exercise 5 to the entire class, or you may prefer to treat it as enrichment. In either case, however, it would be helpful to work through at least a few of the equations with all the children.

Following completion of the exercises, allow time for checking papers and for any accompanying discussion.

Assignments (page 157)

Minimum: 1–3. Average: 1–4.
Maximum: 1–5.

Follow-up

An oral game such as "What's My Rule" or a team relay at the chalkboard frequently provides more effective practice in multiplying by 10, 100, and 1000 than an abundance of written drill.

To play "What's My Rule," call on children one at a time and ask for a number less than 1000. As each child responds, reply with a number according to your rule. For example, if your rule is "multiply by 100" and a child gives you the number 25, you should respond 2500. When the children guess your rule, they should fold their arms across their chests. As soon as you call on a child whose arms are folded, he should ask you for a number less than 1000 and respond to you by giving an answer that shows he has applied your rule. If he is correct, tell him so, but caution him not to tell anyone else your rule.

Continue the game by asking for numbers both from those who do not yet know the rule and from those with folded arms. Change the rule and announce "New game" when most of the children know your rule, or discontinue if only a few understand after several minutes of play.

A game like this, played for five minutes several times a week, can also provide a stimulating warm-up for the day's lesson.

Duplicator Masters, page 28
Workbook, page 43
Skill Masters, page 28

PAGES 158–159

Objective

Given two factors, one of which is a multiple of 10 and the other a single-digit number, the child will be able to find their product.

Preparation

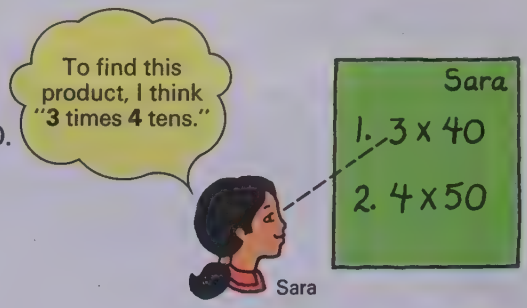
To prepare for this lesson, give the children practice finding products which have 10, 100, or 1000 as a factor.



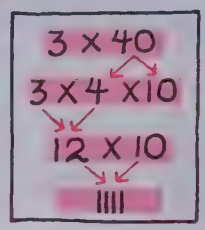
Let's explore products like 3 × 40.

Discussing the Ideas

1. A Explain how Sara is thinking about the product 3 × 40.
 B How would Sara think about the product 4 × 50?
 See Discussion.



2. The diagram shows how Sara is using the **grouping principle** to complete the problem. Find the final product. 120



3. In each exercise, explain why the two products are the same. Find the products.

A	B	C	D	E
3 × 50	4 × 60	5 × 300	4 × 600	6 × 3000
15 × 10	24 × 10	15 × 100	24 × 100	18 × 1000
150	240	1500	2400	18 000

4. Solve the equations.

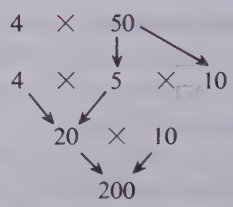
A $3 \times 20 = 3 \times 2 \times 10 = n$ 60
B $5 \times 30 = 5 \times 3 \times 10 = n$ 150
C $8 \times 40 = 8 \times 4 \times 10 = n$ 320
D $9 \times 50 = 9 \times 5 \times 10 = n$ 450
E $7 \times 60 = 7 \times 6 \times 10 = n$ 420
F $4 \times 70 = 4 \times 7 \times 10 = n$ 280
G $7 \times 80 = 7 \times 8 \times 10 = n$ 560
H $6 \times 90 = 6 \times 9 \times 10 = n$ 540
I $5 \times 70 = 5 \times 7 \times 10 = n$ 350
J $9 \times 70 = 9 \times 7 \times 10 = n$ 630
K $6 \times 40 = 6 \times 4 \times 10 = n$ 240
L $5 \times 60 = 5 \times 6 \times 10 = n$ 300
M $9 \times 80 = 9 \times 8 \times 10 = n$ 720
N $8 \times 90 = 8 \times 9 \times 10 = n$ 720
O $6 \times 80 = 6 \times 8 \times 10 = n$ 480
P $9 \times 60 = 9 \times 6 \times 10 = n$ 540
Q $3 \times 400 = 3 \times 4 \times 100 = n$ 1200
R $8 \times 600 = 8 \times 6 \times 100 = n$ 4800
S $5 \times 700 = 5 \times 7 \times 100 = n$ 3500
T $9 \times 8000 = 9 \times 8 \times 1000 = n$ 72 000

158

Discussion

As explained in the mathematics section for this lesson, exercises 1 and 2 rely on the grouping principle and reduce products such as 3 × 40 to an exercise in using basic multiplication facts and multiplication by 10. Direct the children to study the exercises Sara is working and her manner of thinking about them. You might expect the children's response in exercise 1B to be something like this: "To find this product (4 × 50), I think *4* times *5* tens."

After you discuss exercise 2 by working through the procedure step by step, work through the product 4 × 50 similarly:



If necessary, use other examples from exercise 3 with a development similar to that in exercise 2. You might find it worthwhile to have each equation in exercise 4 read aloud.

Using the Ideas

1. Find the products.

A	3 × 6 18	F	4 × 5 20	K	6 × 4 24	P	7 × 8 56	U	8 × 5 40
B	3 × 60 180	G	4 × 50 200	L	6 × 40 240	Q	7 × 80 560	V	8 × 50 400
C	5 × 3 15	H	5 × 7 35	M	3 × 8 24	R	9 × 3 27	W	9 × 7 63
D	5 × 30 150	I	5 × 70 350	N	3 × 80 240	S	9 × 30 270	X	9 × 70 630
E	5 × 300 1500	J	5 × 700 3500	O	3 × 800 2400	T	9 × 300 2700	Y	9 × 700 6300

2. Find the products.

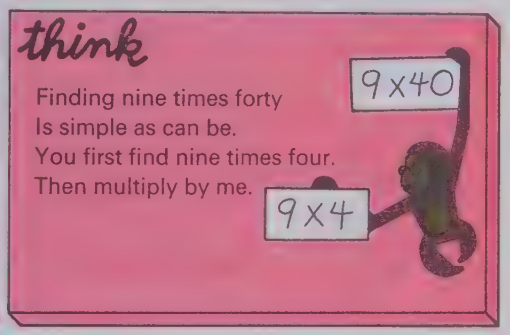
A	4 × 30 120	E	9 × 50 450	I	4 × 40 160	M	4 × 200 800	Q	7 × 600 4200
B	5 × 60 300	F	3 × 70 210	J	6 × 60 360	N	6 × 600 3600	R	5 × 500 2500
C	7 × 70 490	G	8 × 70 560	K	7 × 90 630	O	8 × 800 6400	S	3 × 900 2700
D	6 × 80 480	H	6 × 90 540	L	9 × 90 810	P	9 × 700 6300	T	8 × 500 4000

3. Find the products.

A	7 × 50 350	E	7 × 600 4200	I	9 × 700 6300	M	7000 × 4 28 000		
B	3 × 900 2700	F	9 × 800 7200	J	400 × 5 2000	N	4000 × 8 32 000		
C	6 × 300 1800	G	8 × 200 1600	K	700 × 7 4900	O	5000 × 6 30 000		
D	4 × 900 3600	H	2 × 900 1800	L	600 × 9 5400	P	9000 × 7 63 000		

★ **4.** Solve the equations.

A	6 × 90 = *n* 540	N	*n* × 900 = 6300 7	Q	7200 = 8 × *n* 900
B	*n* × 80 = 320 4	O	9 × *n* = 5400 600	R	*n* × 500 = 4500 9
C	*n* × 70 = 350 5	P	600 × *n* = 4200 7	S	9 × *n* = 8100 900
D	*n* × 9 = 630 70				
E	*n* × 8 = 560 70				
F	*n* × 5 = 400 80				
G	30 × *n* = 180 6				
H	*n* = 7 × 80 560				
I	6 × *n* = 420 70				
J	*n* × 40 = 360 9				
K	630 = *n* × 70 9				
L	540 = 6 × *n* 90				
M	*n* × 800 = 3200 4				



More practice, page A-14, Set 24



Using the Exercises

Have the children work independently on the exercises on page 159. Point out that some exercises are related and that they should use the pattern to help them find the products. You may find it helpful to select some parts of exercises 2, 3, or 4, and present these on the chalkboard with accompanying explanations of how the grouping principle is used to arrive at the products. (See the mathematics section of this lesson.) Encourage those children who are capable to arrive at generalizations for finding such products, but do not burden others with learning specific rules for finding these products.

Give all the children a chance to solve the *Think* problem. Since it is so closely related to the main point of the lesson, you will find that most children will benefit from discussion of the correct answer.

Assignments (page 159)
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Mathematics

In this lesson a simple illustration of the use of the associative (grouping) principle for multiplication shows how the children can arrive easily at products such as 3 × 40. The following are the steps involved in arriving at the correct product.

$$\begin{aligned} 3 \times 40 &= 3 \times (4 \times 10) \\ &= (3 \times 4) \times 10 \\ &= 12 \times 10 \\ &= 120 \end{aligned}$$

Note that in the first step 40 is written as 4 × 10 to get the expression 3 × (4 × 10). Then, using the grouping principle, this can be regrouped as (3 × 4) × 10. Since the children already know the product 3 × 4, they need only to find 12 × 10. The previous lesson has developed skills for multiplying by 10, and so the children are able to find the product 12 × 10, which is the same as the product 3 × 40.

Follow-up

If you think some children would benefit from further practice of this type, prepare a worksheet similar to the sample shown.

Study the patterns and then complete the equations.

5 × 7 = ___	4 × 6 = ___
5 × 70 = ___	4 × 60 = ___
5 × 700 = ___	4 × 600 = ___
5 × 7000 = ___	4 × 6000 = ___

5 × 80 = 5 × 8 × ___ = ___
4 × 90 = 4 × 9 × ___ = ___
6 × 700 = 6 × 7 × ___ = ___
7 × 400 = 7 × 4 × ___ = ___

Duplicator Masters, page 29
Workbook, page 44
Skill Masters, page 29

PAGES 160–161

Objective

Given the product of two factors, one of which is a multiple of 10, 100, or 1000, the child will be able to find the other factor.

Preparation

To prepare for this lesson, plan an oral review of the special products. For example, ask for products in a series such as 5×6, 5×60, 5×600, and 5×6000.

Investigation

After you read with the children the material at the top of the page, direct them to work on the investigation independently. Be sure they notice that one of the factors in Susan's equation ends in 0. As they are working, remind them to think of all the factors of 24 that they can remember. The equations which they may write are the following:

$1 \times 240 = 240$ $\quad 6 \times 40 = 240$
$2 \times 120 = 240$ $\quad 8 \times 30 = 240$
$3 \times 80 = 240$ $\quad 12 \times 20 = 240$
$4 \times 60 = 240$ $\quad 24 \times 10 = 240$

For those children who finish quickly, suggest a similar illustration on the chalkboard and ask them to write equations for it.

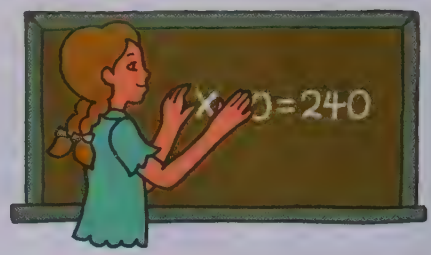
$\square \times \square 0 = 360$



Can you find missing factors?

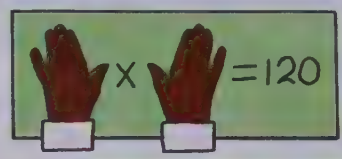
Investigating the Ideas

Note that one factor in Susan's equation ends in 0.

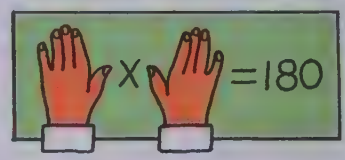


? Can you write all the equations Susan might be hiding?
See Investigation.

Discussing the Ideas



1. Suppose one of the factors ends in 0. Give possible pairs of factors that might be hidden.
1,120; 2,60; 3,40; 4,30; 6,20; 10,12



2. Can you find an easy shortcut for finding these pairs of factors?
See Discussion.

3. Practice with special products will help you find missing factors. Give the products.

A 8×10 80	E 50×8 400	I 47×10 470	M 10×96 960
B 13×10 130	F 60×10 600	J 10×65 650	N 0×10 0
C 60×9 540	G 9×80 720	K 10×93 930	O 831×10 8310
D 10×10 100	H 100×10 1000	L 132×10 1320	P 10×729 7290

4. Find the products.

A 20×9 180	E 7×90 630	I 9×400 3600	M 6×70 420
B 5×30 150	F 21×10 210	J 80×4 320	N 30×9 270
C 4×70 280	G 3×700 2100	K 34×10 340	O 4×500 2000
D 10×35 350	H 6×500 3000	L 8×700 5600	P 6×60 360

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Discussion

Have children write on the chalkboard their equations from the investigation. Then lead into discussion exercises 1 and 2. Continue to stress the relationship of the basic facts to the special products and to finding missing factors.

For exercise 2, you might expect the children to respond in a manner similar to the following: "Since 6×3 is 18, 60 and 3, or 6 and 30, are pairs of factors for 180; also, since 2×9 is 18, 20 and 9, or 2 and 90, are pairs of factors for 180."

Exercises 3 and 4 would best be treated as oral drill. If necessary, provide other examples for oral drill.

Using the Ideas

1. Solve the equations.

A $n \times 3 = 21$ 7	G $n \times 4 = 24$ 6	M $n \times 6 = 54$ 9
B $n \times 3 = 210$ 70	H $n \times 4 = 240$ 60	N $n \times 6 = 540$ 90
C $n \times 7 = 28$ 4	I $n \times 4 = 2400$ 600	O $n \times 6 = 5400$ 900
D $n \times 7 = 280$ 40	J $n \times 6 = 42$ 7	P $n \times 9 = 18$ 2
E $n \times 5 = 30$ 6	K $n \times 6 = 420$ 70	Q $n \times 9 = 180$ 20
F $n \times 5 = 300$ 60	L $n \times 6 = 4200$ 700	R $n \times 9 = 1800$ 200

2. Solve the equations.

A $n \times 3 = 240$ 80	G $n \times 8 = 400$ 50	M $n \times 3 = 2400$ 800
B $n \times 6 = 360$ 60	H $n \times 2 = 60$ 30	N $n \times 4 = 2800$ 700
C $n \times 2 = 160$ 80	I $n \times 4 = 120$ 30	O $n \times 2 = 1600$ 800
D $n \times 5 = 200$ 40	J $n \times 9 = 270$ 30	P $n \times 5 = 2000$ 400
E $n \times 7 = 210$ 30	K $n \times 5 = 450$ 90	Q $n \times 7 = 4900$ 700
F $n \times 4 = 320$ 80	L $n \times 7 = 420$ 60	R $n \times 6 = 3000$ 500

3. Solve the equations.

A $n \times 3 = 24$ 8	G $n \times 2 = 16$ 8
B $n \times 30 = 240$ 8	H $n \times 20 = 160$ 8
C $n \times 5 = 20$ 4	I $n \times 200 = 1600$ 8
D $n \times 50 = 200$ 4	J $n \times 4 = 28$ 7
E $n \times 4 = 32$ 8	K $n \times 40 = 280$ 7
F $n \times 40 = 320$ 8	L $n \times 40 = 2800$ 70

think



Maria baked less than 2 dozen cookies. When she tried to divide them equally among 2, 3, or 4 of her friends, there was always 1 cookie left over. How many cookies did she bake? 13

More practice, page A-14, Set 25

Using the Exercises

Assign the exercises on page 161 as independent work. Notice that the problems in exercises 1 and 3 are patterned to help the children go from the basic facts and special products to finding missing factors. When the children have finished, allow time for checking and discussing the exercises.

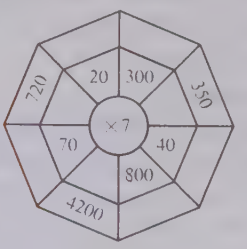
The entire class should benefit from a discussion of the *Think* problem, after those who wish to do so have had a chance to try to find the solution on their own. Help children see that if they list all the multiples of 2, 3, and 4 that are less than 24, they can quickly reduce the possibilities to 5, 7, 11, 13, 17, 19, and 23. Then they can find that the number of cookies she baked must have been 13 because it is the only number less than 24 that is just one greater than a common multiple of 2, 3, and 4.

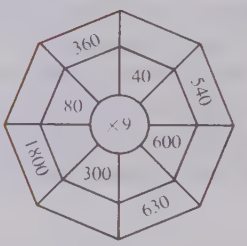
Assignments (page 161)

Minimum: 1–3. Average: 1–3. Maximum: 1–3.

Follow-up

To provide further practice with special products and finding missing factors, prepare on a duplicated worksheet "practagons" similar to the following:





Workbook, page 45

PAGES 162–163

Objective

Given division problems in which the dividend is a product of a multiple of 10, 100, or 1000 and the divisor is a single-digit number, the child will be able to find the quotient by thinking of missing factors.

Preparation

To prepare for this lesson, conduct a short oral drill that requires finding quotients related to the basic facts. For example, ask a child for the product 5 × 9. When he answers "forty-five," ask him for the quotients 45 ÷ 9 and then 45 ÷ 5. Continue with other examples to help the children see the relationship between the original multiplication problem and the two division problems.

Let's explore special quotients.

1. Solve the equations.

A $a \times 4 = 12$ (3) → $b \times 4 = 120$ (30) → $120 \div 4 = c$ 30
B $a \times 6 = 18$ (3) → $b \times 6 = 180$ (30) → $180 \div 6 = c$ 30
C $a \times 5 = 25$ (5) → $b \times 5 = 250$ (50) → $250 \div 5 = c$ 50
D $a \times 8 = 16$ (2) → $b \times 8 = 160$ (20) → $160 \div 8 = c$ 20
E $a \times 3 = 27$ (9) → $b \times 3 = 270$ (90) → $270 \div 3 = c$ 90
F $a \times 2 = 10$ (5) → $b \times 2 = 100$ (50) → $100 \div 2 = c$ 50

2. Solve the equations.

A $n \times 7 = 14$ 2	G $n \times 8 = 24$ 3	M $n \times 9 = 27$ 3
B $n \times 7 = 140$ 20	H $n \times 8 = 240$ 30	N $n \times 9 = 270$ 30
C $140 \div 7 = n$ 20	I $240 \div 8 = n$ 30	O $270 \div 9 = n$ 30
D $n \times 4 = 32$ 8	J $n \times 7 = 28$ 4	P $n \times 6 = 24$ 4
E $n \times 4 = 320$ 80	K $n \times 7 = 280$ 40	Q $n \times 6 = 240$ 40
F $320 \div 4 = n$ 80	L $280 \div 7 = n$ 40	R $240 \div 6 = n$ 40

3. Find the quotients.

A $21 \div 3 = n$ 7	G $40 \div 5 = n$ 8	M $27 \div 9 = n$ 3
B $210 \div 3 = n$ 70	H $400 \div 5 = n$ 80	N $270 \div 9 = n$ 30
C $14 \div 7 = n$ 2	I $48 \div 8 = n$ 6	O $49 \div 7 = n$ 7
D $140 \div 7 = n$ 20	J $480 \div 8 = n$ 60	P $490 \div 7 = n$ 70
E $24 \div 4 = n$ 6	K $36 \div 6 = n$ 6	Q $56 \div 8 = n$ 7
F $240 \div 4 = n$ 60	L $360 \div 6 = n$ 60	R $560 \div 8 = n$ 70

4. Find the quotients.

A $20 \div 5 = n$ 4	E $32 \div 8 = n$ 4	I $72 \div 9 = n$ 8
B $200 \div 50 = n$ 4	F $320 \div 80 = n$ 4	J $720 \div 90 = n$ 8
C $28 \div 4 = n$ 7	G $48 \div 6 = n$ 8	K $42 \div 6 = n$ 7
D $280 \div 40 = n$ 7	H $480 \div 60 = n$ 8	L $420 \div 60 = n$ 7

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Discussion

Although the exercises on these pages are not designated discussion exercises, you will find it worthwhile to handle exercise 1 at least as a class discussion activity.

As you work through exercise 1, encourage the children to explain in their own words how the three equations in each part are related. Help them see how the basic facts are related to the special products and how missing factors are related to division equations. Note also that each set of three consecutive parts of exercise 2 are related, as are each two consecutive parts of exercises 3 and 4.

After working through exercise 1, and perhaps a couple of parts of exercises 2 and 3, it would be helpful to write division equations on the chalkboard and discuss how thinking of missing factors helps in finding quotients. For example, you might develop a few equations in the following manner:

$360 \div 9 = n$

$9 \times n = 360$

Since $9 \times 4 = 36$, $9 \times 40 = 360$,
so $360 \div 9 = 40$.

5. Find the quotients. Think about the missing factors.

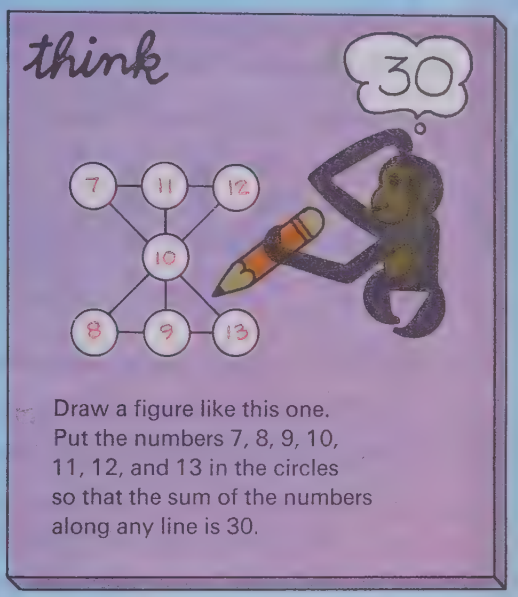
A $320 \div 8 = n$ 40	G $360 \div 9 = n$ 40	M $210 \div 3 = n$ 70	S $120 \div 20 = n$ 6	
B $270 \div 3 = n$ 90	H $180 \div 2 = n$ 90	N $560 \div 7 = n$ 80	T $150 \div 30 = n$ 5	
C $350 \div 5 = n$ 70	I $480 \div 8 = n$ 60	O $630 \div 9 = n$ 70	U $360 \div 40 = n$ 9	
D $240 \div 4 = n$ 60	J $420 \div 6 = n$ 70	P $720 \div 8 = n$ 90	V $480 \div 60 = n$ 8	
E $300 \div 6 = n$ 50	K $280 \div 4 = n$ 70	Q $490 \div 7 = n$ 70	W $560 \div 70 = n$ 8	
F $420 \div 7 = n$ 60	L $450 \div 5 = n$ 90	R $540 \div 6 = n$ 90	X $810 \div 90 = n$ 9	

6. Find the quotients. Think about the missing factors.

A 40 $3\overline{)120}$	E 80 $2\overline{)160}$	I 70 $9\overline{)630}$	M 80 $9\overline{)720}$	Q 6 $50\overline{)300}$
B 40 $5\overline{)200}$	F 50 $8\overline{)400}$	J 80 $8\overline{)640}$	N 90 $6\overline{)540}$	R 8 $60\overline{)480}$
C 30 $6\overline{)180}$	G 40 $9\overline{)360}$	K 70 $8\overline{)560}$	O 9 $30\overline{)270}$	S 7 $70\overline{)490}$
D 30 $7\overline{)210}$	H 80 $4\overline{)320}$	L 60 $7\overline{)420}$	P 5 $40\overline{)200}$	T 6 $90\overline{)540}$

7. Find the quotients.

A 400 $4\overline{)1600}$	K 800 $3\overline{)2400}$	U 700 $3\overline{)2100}$	
B 400 $5\overline{)2000}$	L 500 $6\overline{)3000}$	V 800 $5\overline{)4000}$	
C 200 $9\overline{)1800}$	M 700 $9\overline{)6300}$	W 800 $8\overline{)6400}$	
D 900 $3\overline{)2700}$	N 700 $4\overline{)2800}$		
E 800 $7\overline{)5600}$	O 900 $6\overline{)5400}$		
F 600 $8\overline{)4800}$	P 400 $7\overline{)2800}$		
G 600 $9\overline{)5400}$	Q 900 $8\overline{)7200}$		
H 600 $7\overline{)4200}$	R 900 $9\overline{)8100}$		
I 700 $3\overline{)2100}$	S 900 $7\overline{)6300}$		
J 800 $5\overline{)4000}$	T 800 $8\overline{)6400}$		



Draw a figure like this one. Put the numbers 7, 8, 9, 10, 11, 12, and 13 in the circles so that the sum of the numbers along any line is 30.

More practice, page A 93, Set H

Using the Exercises

Direct the children to do the other exercises on these pages independently. For children who have not yet developed the ability to work as speedily as some of their classmates, you may wish to assign only selected parts of each exercise. Make sure the children recognize the traditional form of writing division problems that is used in exercises 6 and 7.

As you correct exercise 6, you might have children recite the multiplication facts which check the divisions. For example, in exercise 6Q the children should have the answer 6, and you can have them read the matching equations, $6 \times 50 = 300$ or $50 \times 6 = 300$. Also, it may be helpful to have children read problems such as exercise 5A as follows: "What number times 8 gives 320?"

Since the type of *Think* problem on page 163 was discussed previously (see page 83), many children may be able to solve this one independently.

Assignments (pages 162–163)

Minimum: 3–6. Average: 1–7.
Maximum: 1–7.

Workbook, page 46

PAGES 164–165

Objective

Given multiplication equations in which both factors are multiples of 10, the child will be able to find the products.

Preparation

Conduct a short oral review of the products and quotients studied so far in the chapter. For example, present series of equations such as the following: $7 \times 8 = ?$, $7 \times 80 = ?$, $7 \times 800 = ?$ or $63 \div 7 = ?$, $630 \div 7 = ?$, $6300 \div 7 = ?$

Investigation

Your oral drill can lead into the problem illustrated in the investigation. Have the children spend a few minutes studying Carol's work. Next, have a child write the final product, 1200, on the chalkboard. Then, ask the children to work independently and show how to use Carol's method to find 40×60. While children are working, make a copy of Carol's work on an overhead transparency or on the chalkboard.

Is there an easy way to find products like 30×40?

Investigating the Ideas

Study the steps in Carol's notebook.

Carol

30×40

Clue 1: $30 = 3 \times 10$, $40 = 4 \times 10$ → $3 \times 10 \times 4 \times 10$

Clue 2: Rearrange → $3 \times 4 \times 10 \times 10$

Clue 3: Multiply → 12×100

? Can you show the same steps as Carol's for finding 40×60?

40×60
$4 \times 10 \times 6 \times 10$
$4 \times 6 \times 10 \times 10$
$24 \times 100 = 2400$

See Investigation.

Discussing the Ideas

1. Explain the steps and solve the equations.

 A $40 \times 60 = 4 \times 10 \times 6 \times 10 = n$ 2400
 B $50 \times 60 = 5 \times 10 \times 6 \times 10 = n$ 3000
 C $60 \times 70 = 6 \times 10 \times 7 \times 10 = n$ 4200
 D $70 \times 80 = 7 \times 10 \times 8 \times 10 = n$ 5600

2. Can you give a simple rule for finding 30×40?
 See Discussion.

3. Try your rule out on these products.

 A 20×60 1200 D 40×20 800 G 20×70 1400 J 90×20 1800
 B 30×30 900 E 60×10 600 H 50×50 2500 K 80×90 7200
 C 30×20 600 F 50×30 1500 I 60×30 1800 L 70×80 5600



Discussion

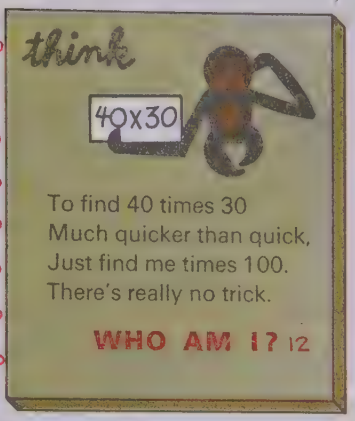
After allowing sufficient time for children to complete the investigation, ask a volunteer to show the steps for finding 40×60 beside the display of the steps Carol used for 30×40. Let other children present and discuss any alternative steps they might have shown in their own work. Be sure to help the children observe how the order and grouping principles permit the rearrangement of the factors.

As you discuss exercise 2, you might expect a response like, "Multiply the 3 times the 4, then multiply that product times 100." If necessary, work through other examples until children formulate some rule to use for these factors. Then let them apply their rule to the factors in exercise 3. The purpose of having children formulate a rule is to help them find products such as these mentally. Carol's steps are not to be imitated but should be understood as the basis for the mental procedure.

Using the Ideas

1. Find the products.

A $30 \times 60 = n$ 1800
B $80 \times 40 = n$ 3200
C $90 \times 30 = n$ 2700
D $70 \times 40 = n$ 2800
E $70 \times 90 = n$ 6300
F $40 \times 60 = n$ 2400
G $70 \times 30 = n$ 2100
H $40 \times 90 = n$ 3600
I $80 \times 70 = n$ 5600
J $60 \times 80 = n$ 4800
K $40 \times 40 = n$ 1600
L $80 \times 90 = n$ 7200
M $60 \times 90 = n$ 5400
N $90 \times 90 = n$ 8100
O $70 \times 70 = n$ 4900
P $60 \times 50 = n$ 3000



2. Find the quotient by solving the multiplication equation.

A $n \times 40 = 800$ 20 — $800 \div 40 = n$ 20
B $n \times 30 = 900$ 30 — $900 \div 30 = n$ 30
C $n \times 50 = 1500$ 30 — $1500 \div 50 = n$ 30
D $n \times 30 = 2700$ 90 — $2700 \div 30 = n$ 90
E $n \times 70 = 3500$ 50 — $3500 \div 70 = n$ 50
F $n \times 60 = 2400$ 40 — $2400 \div 60 = n$ 40
G $n \times 90 = 6300$ 70 — $6300 \div 90 = n$ 70
H $n \times 60 = 1800$ 30 — $1800 \div 60 = n$ 30
I $n \times 80 = 4800$ 60 — $4800 \div 80 = n$ 60

3. Find the quotients. Think about missing factors.

A $40\overline{)1200}$ 30
B $20\overline{)1400}$ 70
C $50\overline{)2500}$ 50
D $10\overline{)900}$ 90
E $30\overline{)2100}$ 70
F $60\overline{)1800}$ 30
G $70\overline{)4200}$ 60
H $30\overline{)2700}$ 90
I $10\overline{)800}$ 80
J $80\overline{)800}$ 10
K $20\overline{)1800}$ 90
L $40\overline{)3200}$ 80
M $50\overline{)4000}$ 80
N $60\overline{)3000}$ 50
O $40\overline{)3600}$ 90
P $80\overline{)5600}$ 70
Q $10\overline{)400}$ 40
R $90\overline{)6300}$ 70
S $10\overline{)500}$ 50
T $90\overline{)2700}$ 30

4. Find the products.

A $20 \times 700 = n$ 14000
B $40 \times 400 = n$ 16000
C $30 \times 800 = n$ 24000
D $60 \times 300 = n$ 18000
E $50 \times 900 = n$ 45000
F $70 \times 900 = n$ 63000
G $70 \times 800 = n$ 56000
H $80 \times 800 = n$ 64000
I $90 \times 800 = n$ 72000

More practice, page A-15, Set 27



Using the Exercises

You may treat parts of exercises 1 and 2 as oral exercises and then direct the children to number their paper and write only the answers for the remaining exercises. For exercises such as 2A, encourage the children to think, "What number times 40 equals 800?" If some children have difficulty with exercise 3 or 4, direct them to think of missing factors. You might want to help them work through a few exercises step by step. For example, in exercise 3L:

$40\overline{)3200}$

Think: $40 \times n = 3200$
$4 \times 8 = 32$, $40 \times 8 = 320$
so $40 \times 80 = 3200$;
or $40 \times n = 3200$
Since $4 \times 8 = 32$ and $10 \times 10 = 100$
then $4 \times 8 \times 10 \times 10 = 3200$
and $40 \times 80 = 3200$

Assignments (page 165)
Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Follow-up

To give the children a clearer understanding of the way rearranging factors can increase efficiency in finding products quickly and accurately, show problems like the ones below on the chalkboard or overhead projector or on a duplicated worksheet. Ask the children to rearrange and recombine the factors so that they only need to know the basic facts and how to multiply by 10, 100, or 1000.

Rearrange factors to make multiplication easier.
1. $10 \times 9 \times 60 \times 10$ $= 9 \times 6 \times 10 \times 10 \times 10$ $= 54 \times 1000 = 54\,000$
2. $50 \times 8 \times 10 \times 10 =$ _____
3. $6 \times 10 \times 10 \times 70 =$ _____
4. $4 \times 50 \times 10 \times 100 =$ _____

Duplicator Masters, page 30
Workbook, page 47
Skill Masters, page 30

PAGES 166–167

Objective

Given multiplication problems which contain computational errors or missing digits, the child will be able to find the error or the missing digits by applying his knowledge of multiplication and special products.

Preparation

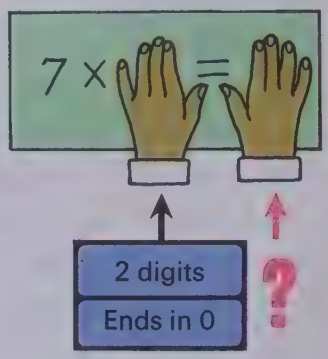
It would be appropriate to begin this lesson immediately with the investigation and delay any oral drill until the discussion section. However, if you prefer, use a short, lively review of multiplying with factors that are multiples of 10.

Investigation

Ask the children to study carefully the illustration and the information given for this investigation. Be sure they observe that one of the factors is given and that two important clues about the second factor are given. The children should work independently to answer the investigation question, but you may find it necessary to give some help to those who have difficulty making the kinds of inferences that are expected. If so, you might ask them such questions as these: "Does the fact that the second factor ends in zero tell you anything about the product?" "Think about how small a number and how large a number the second factor could possibly be. Does this tell you anything about the size of the product, or about how many digits it could have?"

Investigating the Ideas

Suppose you know two things about one of the factors.

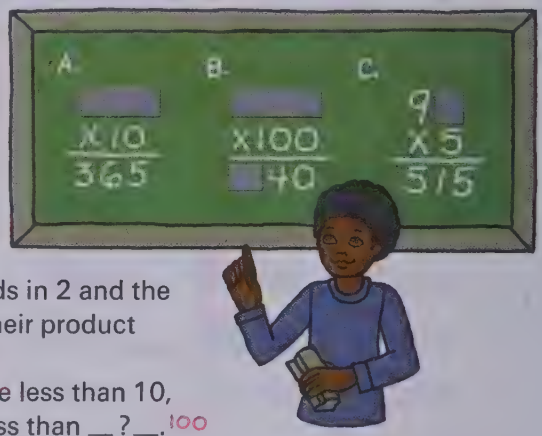


? How many different things can you tell about the product?

See Investigation and Discussion.

Discussing the Ideas

1. Explain the mistake in each problem.
See Discussion.



2. Give the missing words and explain your answers.
 - A If one number ends in 2 and the other ends in 5, their product ends in __?__. 0
 - B If two numbers are less than 10, their product is less than __?__. 100
 - C If two numbers are odd, their product is __?__. odd
 - ★ D If the product of two whole numbers is less than one of the numbers, then one of the numbers is __?__. 0



Discussion

After the children have had adequate time to try to formulate answers for the investigation question, let them freely discuss their conclusions. The discussion should bring out these ideas: (1) The product ends in 0; (2) it can be no less than 70 and no greater than 630 (because the unknown factor can be no less than 10 and no greater than 90); and (3) it must be either a 2-digit or a 3-digit number.

In discussing exercise 1, children should observe that problem A must be wrong because the product of any whole number and ten ends in zero. For problem B, they should explain that the product of a whole number and 100 ends with 2 zeros. For problem C, they should recognize that, since 99 is less than 100, 5 times the greatest possible 2-digit factor (99), must be less than 500 because $5 \times 100 = 500$.

Reinforce the ideas brought out in exercise 2 with several examples for each part. Not all the children will recognize at once for exercise 2A that the number described is zero, but all should be able to see why that is the answer once it is given.

Using the Ideas

1. Copy each problem. Give the missing digits.

A $\begin{array}{r} 50 \\ \times 1\square 0 \\ \hline 500 \end{array}$ B $\begin{array}{r} 7 \\ \square 0 \\ \times 10 \\ \hline 700 \end{array}$ C $\begin{array}{r} 4 \\ \square 0 \\ \times 9 \\ \hline 36\square 0 \end{array}$ D $\begin{array}{r} 0 \\ 9\square \\ \times \square 6 \\ \hline 540 \end{array}$ E $\begin{array}{r} 80 \\ \square\square \\ \times 7 \\ \hline 5\square 0 \\ 6 \end{array}$ F $\begin{array}{r} 70 \\ \square\square \\ \times 9 \\ \hline 6\square 0 \\ 3 \end{array}$

2. A store manager bought 9 coats. Some of the coats cost $40 and the others cost $80.

A The total cost of the coats could not have been $720.
Why? The greatest possible cost would be (8 x 80)+(1x40), or $680

B The total cost of the coats could not have been $360.
Why? The minimum cost would be (8x40)+(1x80), or $400.

C What is the most he could have paid for the coats? $680

D What is the least he could have paid for the coats? $400

E If 4 coats cost $80 each and 5 cost $40 each, how much did he pay for all the coats? $520

★ F If the total cost of the coats was $440, how many $40 coats did he buy? 7

3. Give the pairs of numbers that go in the gray spaces.

	Products	×	+	Sums
A	50	5	10	15
B	140	7	20	27
C	140	2	70	72
D	560	7	80	87
E	4200	6	100	706
F	3600	4	900	904

	Products	×	÷	Quotients
G	100	10	10	1
H	250	50	5	10
I	160	40	4	10
J	300	30	10	3
K	1000	100	10	10
L	2400	240	10	24



Using the Exercises

Assign exercise 1 as independent work, reminding children to use what they know about special products to help them.

The parts of exercise 2 may also be assigned as independent work. However, you might have some children work in groups of two or three and "think aloud" for each other.

You may need to provide assistance for some children for exercise 3. Certainly, those children who do not understand the process will recognize the answers once they are given. For example, it is easily understood that $5 \times 10 = 50$ and $5 + 10 = 15$ is the answer to exercise 3A. When the children have completed the exercises on this page, be sure to allow time for them to review the exercises and perhaps invent some problems of their own concerning the buying and selling of clothes.

Assignments (page 167)*

Minimum: 1–2E. Average: 1–2E, 3. Maximum: 1–3.

Follow-up

A worksheet such as the following will give children further practice in reconstruction problems.

Find the missing digits.

$\begin{array}{r} 7\square 0 \\ \times 70 \\ \hline 49\square\square \\ 00 \end{array}$ $\begin{array}{r} 3\square 0 \\ \times 8 \\ \hline \square\square 0 \\ 24 \end{array}$ $\begin{array}{r} 5\square 00 \\ \times 4 \\ \hline 20\square\square \\ 00 \end{array}$ $\begin{array}{r} 5\square\square 0 \\ \times 30 \\ \hline 150\square \\ 0 \end{array}$

$\begin{array}{r} 90 \\ \times \square 0 \\ \hline 36\square\square \\ 00 \end{array}$ $\begin{array}{r} 80\square\square 0 \\ \times 8 \\ \hline 640\square \\ 0 \end{array}$ $\begin{array}{r} 200\square\square \\ \times 6 \\ \hline \square\square 00 \\ 12 \end{array}$ $\begin{array}{r} 5\square 0 \\ \times \square 0 \\ \hline 450\square \\ 0 \end{array}$

Resources for Active Learning

Teaching Aids for Elementary Mathematics, "Alphametics," and "Cryptarithms," pp. 52–53, Holt, Rinehart and Winston.

Duplicator Masters, page 31

PAGES 168–169

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

An oral review of special products would be an appropriate preparation for page 168, whether you plan to use that page as an evaluation instrument or as a basis for class discussion.

If you plan to assign page 169 as independent work, you might review the basic principles by writing examples on the chalkboard. For instance, write the following equations:

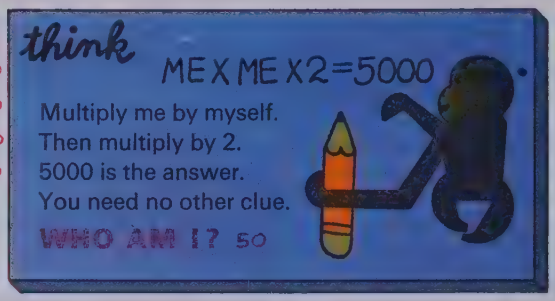
$7 \times 1 = 7$ (1 prin.)
$8 + 0 = 8$ (0 prin.)
$5 + 2 = 2 + 5$ (order prin., add.)
$3 \times 6 = 6 \times 3$ (order prin., mult.)
$(2 + 4) + 9 = 2 + (4 + 9)$ (grouping prin.)

Ask the children if they recall the names of the principles which each equation illustrates.

Reviewing the Ideas

1. Find the products.

70 A	10×7	G	6×100 600
180 B	18×10	H	100×18 1800
230 C	10×23	I	57×100 5700
500 D	50×10	J	70×100 7000
750 E	75×10	K	100×32 3200
830 F	10×83	L	86×100 8600



2. Give the missing numbers.
 - A Since $3 \times 2 = 6$, we know that $3 \times 20 = n$.60
 - B Since $7 \times 4 = 28$, we know that $7 \times 40 = n$.280
 - C Since $9 \times 6 = 54$, we know that $9 \times 600 = n$.5400
 - D Since $3 \times 10 \times 4 \times 10 = 12 \times 100$, we know that $30 \times 40 = n$.1200
 - E Since $8 \times 10 \times 9 \times 10 = 72 \times 100$, we know that $80 \times 90 = n$.7200

3. Find the products.

A	4×30 120	F	7×200 1400	K	70×20 1400	P	7×90 630
B	5×70 350	G	3×500 1500	L	30×40 1200	Q	800×2 1600
C	6×80 480	H	600×4 2400	M	50×60 3000	R	70×50 3500
D	40×4 160	I	8×300 2400	N	80×40 3200	S	6×700 4200
E	50×6 300	J	400×8 3200	O	30×90 2700	T	90×90 8100

4. Find the quotients.

A	$120 \div 4$ 30	D	$2500 \div 5$ 500	G	$2400 \div 30$ 80	J	$3500 \div 70$ 50
B	$240 \div 6$ 40	E	$2100 \div 7$ 300	H	$1600 \div 20$ 80	K	$4200 \div 60$ 70
C	$180 \div 2$ 90	F	$2400 \div 8$ 300	I	$2000 \div 40$ 50	L	$4500 \div 50$ 90

5. Find the quotients.

A	$4\overline{)280}$ 70	C	$9\overline{)270}$ 30	E	$8\overline{)1600}$ 200	G	$60\overline{)600}$ 10	I	$5\overline{)4000}$ 800
B	$5\overline{)100}$ 20	D	$6\overline{)540}$ 90	F	$40\overline{)280}$ 7	H	$20\overline{)140}$ 7	J	$70\overline{)630}$ 9

168

Discussion

You might choose to use page 168 as an evaluation tool. However, a discussion of the exercises would still be in order.

Many children will find the *Think* problem on page 168 particularly difficult. Even the answer to the riddle, once it is given, may mystify some children. Of course, the reason is not that they are unable to multiply 50×50 but that they have had no experience in multiplying 2×2500. You might remind those children who have difficulty that multiplying by 2 is the same as adding a number to itself and that they can simply think about $2500 + 2500$. Give the children considerable time to think about it before you ask for the correct answer.

Whether you work through page 169 with the children or have them work on their own, use the exercises to review the basic principles. After you have corrected their work, it would be helpful to name a principle and have the children give an example. In this way they not only must recognize the principles in equations but also must understand the principles well enough to give an example of their own.

Keeping in Touch with Basic principles

1. The equations below illustrate the order, grouping, and zero principles for addition. Write the corresponding equations for multiplication.

ORDER ⟶ $5 + 7 = 7 + 5$ $5 \times 7 = 7 \times 5$

GROUPING ⟶ $5 + (7 + 6) = (5 + 7) + 6$ $5 \times (7 \times 6) = (5 \times 7) \times 6$

ZERO ⟶ $5 + 0 = 5$ $5 \times 1 = 5$

2. Each exercise illustrates a basic principle. Name the principle used in each exercise.

A $30 \times (2 \times 10) = (30 \times 2) \times 10$ Grouping, for multiplication

B $54 + 37 = 37 + 54$ Order, for addition

C $93 + 0 = 93$ Zero, for addition

D $87 \times 0 = 0 \times 87$ Order, for multiplication

E $63 \times 1 = 63$ One, for multiplication

F $30 + (10 + 2) = (30 + 10) + 2$ Grouping, for addition

3. Using the multiplication-addition principle, give the number for each ▮.

A $23 \times 10 = (20 \times 10) + (▮ \times 10)$ 3

B $(30 \times 5) + (4 \times 5) = 34 \times ▮$ 5

C $67 \times 4 = (▮ \times 4) + (7 \times 4)$ 60

D $86 \times ▮ = (80 \times 7) + (6 \times 7)$ 7

★**4.** Name the principle used in each exercise.

A $(3 \times 2) \times (5 \times 4) = (5 \times 4) \times (3 \times 2)$ Order, for multiplication

B $(6 \times 7) + (4 \times 7) = (4 \times 7) + (6 \times 7)$ Order, for addition

C $(20 \times 8) + (3 \times 8) = 23 \times 8$ Multiplication-addition

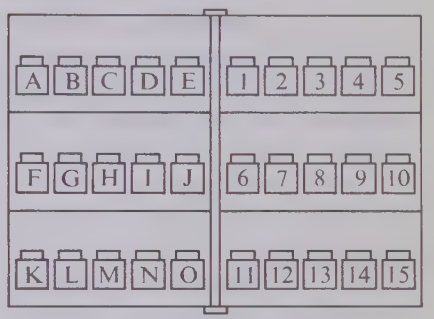
D $(25 + 7) + 3 = 3 + (25 + 7)$ Order, for addition

E $(8 + 5) + (4 + 7) = (4 + 7) + (8 + 5)$ Order, for addition

F $(5 + 9) + (6 + 8) = (9 + 5) + (8 + 6)$ Order, for addition

Follow-up/The Matching Game

To make the basic equipment for this matching game, glue thirty library-card pockets on two sheets of brightly colored poster board so that you have three rows of five pockets on each. Label the pockets on one sheet with the letters A through O, and the pockets on the other sheet with the numerals 1 through 15 (see the illustration). Cloth tape can be used to hinge the two boards together for easy folding and storage.



The Matching Game

This game can be adapted for many kinds of review. Initially, mark fifteen multiplication facts with one factor less than 10 and the other a multiple of 10, 100, or 1000 on small index cards. On another set of fifteen cards, write the corresponding products. Shuffle each set separately and place the cards face down at random in the pockets, a set on each side of the board.

To play the game, divide the children into two teams. The first child on one team chooses a letter from A through O and a numeral from 1 through 15. He should remove the two cards and show them to the entire class. If the product fits the exposed combination, give his team a point and remove the pair of cards. If the cards do *not* match, return them, face down, to their pockets. The first member of the other team then gets a chance to choose a pair of cards. The play continues until all pairs of cards have been removed. The winning team is the one with the greater number of points. Note that if children recall the position of particular products they will have a better chance of picking cards that match.

Workbook, page 48 (Use with page 168.)

CHAPTER 7 | Estimation

Pages 170–181

General Objectives

To provide a variety of experiences in estimation

To prepare for estimation in dividing

Mathematics

The opening lesson of the chapter provides a variety of experiences directed toward the general concept of estimation and some work in "rounding off" to multiples of 10 and 100 for easy estimations. Following this come lessons in estimating sums, differences, products, and quotients. The lessons are reinforced by word problems involving estimation.

The topic of estimation is seldom thought of as being entirely in the realm of pure mathematics. However, nearly everyone would agree that estimation plays a vital role in the practical application of mathematics. The pure mathematician may not want to claim estimation as a topic vital to his mathematical system, but in most instances estimation is a valuable tool. For example, the mathematician may want to use 1.414 as an approximation for $\sqrt{2}$ or 3.14159 for the value of π. These rational numbers are, of course, merely rational estimations for the irrational numbers $\sqrt{2}$ and π.

Because a precise definition of estimation would not be practical, we prefer to define it loosely and in terms easily understood by the children. Estimation is explained by giving several statements that the children themselves might use if asked to tell what they think estimation means (see the "page from Cindy's notebook" at the bottom of page 170). Our intent is to give the children a feeling for the general ideas involved in finding reasonably close approximations.

We might maintain that a certain element of logic is associated with this study. However, it is probably more accurate to assert that estimation is a function of common sense rather than of precise logical reasoning.

Teaching the Chapter

Materials

Demonstration number line
Jar full of marbles
Paper bags (optional)
Several small containers, such as a cup or small glass, and one large container, such as a mixing bowl or jug (optional)

Vocabulary

estimate

The materials mentioned are most appropriate for use with the word-problem lesson at the end of the chapter, but you may find it quite stimulating for the children to use these materials during the preparation and teaching of each lesson. For example, when first explaining estimation to the children, you can have them guess how many marbles are in a jar, or you can have a contest in which the children have a chance over a period of two or three days to guess the number of marbles in the jar. You might also give the children an opportunity to estimate how many times the contents of a small container must be put in a large container to fill it. Be sure to determine the best estimate by actual counting.

The demonstration with the paper bags requires a supply of blocks or other objects which can be used to fill the bags. For example, if you had a box of tennis balls, the children might guess how many tennis balls an empty bag would hold. Following the guesses, the children could count the exact number.

Although such "guessing activities" are not exactly at the heart of the topics taught in this chapter, they involve estimation and serve to stimulate children's interest. The children will often guess in numbers that are multiples of 10 or 100, which is one of the key points of the chapter: to accustom the children to "rounding off" to multiples of 10, 100, or 1000.

Lesson Schedule

Plan to spend about a week to a week and a half on this chapter. This chapter is not absolutely essential to future work in the text, but you may discover that the children find these lessons stimulating. In this case, two weeks spent here may produce great dividends.

Evaluation of Progress

Evaluation should not be a crucial consideration in this chapter, so do not put a high premium on achievement of skills. We do not expect the children to memorize any rules or to gain any great skill in estimation. We do hope that children acquire a general understanding of how to estimate; attain some skill in substituting multiples of 10, 100, and 1000 for other numbers; and arrive at some "common-sense" answers. Evaluating whether or not the children are able to do this is difficult. For example, on a test it is often difficult to tell whether a child has actually computed the answer or whether he has estimated it. We therefore recommend that you treat this chapter with a light touch. Try to see that the children enjoy and participate in the activities suggested for the chapter. You may use the chapter review as an evaluation instrument if you wish.

Resources for Active Learning

General Activities

Elementary Science Study: *Peas and Particles*, Webster, McGraw-Hill

Math Activity Cards, "The Bean Jar," B44; "Leaves," C38, Macmillan

Mathex: Measurement and Estimation No. 5, "Estimation," pp. 45–48 (pupil pages 55–64), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Green Set No. 44, Wiley

Commercial Games

Bali-Buttons—a game of logic (Creative Publications; Edmund Scientific; Math Media)

Wff 'N Proof—a game of modern logic (Cuisenaire Co.)

PAGES 170–171

Objective

Given numbers which are less than 1000, the child will be able to find the nearest multiple of 10 or 100 for each number.

Preparation

To stimulate interest for the investigation, ask the children to think of times when they have made a guess about something. Encourage several children to share these experiences of guessing. Present a covered glass container filled with beans or marbles and ask children to guess how many items it contains. Explain that the container will be kept in view for a number of days, so they will have time to make new guesses if they want.

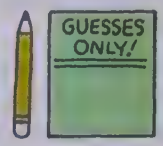
Investigation

Before you direct the children to the text, explain that in this investigation they will be asked to guess the answers to some problems. You might conduct a timed contest for this investigation to see who can come closest to the correct answer in 30 seconds. If you do conduct such a contest, make sure the children know what they are to do before they look at the text, so that they can begin immediately when they see the problems. Some children will realize that 987 is near 1000 so their answer must be less than but near 72 000. Similarly, 4987 is almost 5000, so their answer should be near 12 000. Observe whether any children make wild, unreasoned guesses; such children will need particular help in learning how to estimate.

7 Estimation

● *What does it mean to estimate?*

Investigating the Ideas



No pencil and paper
(except to record your guesses)

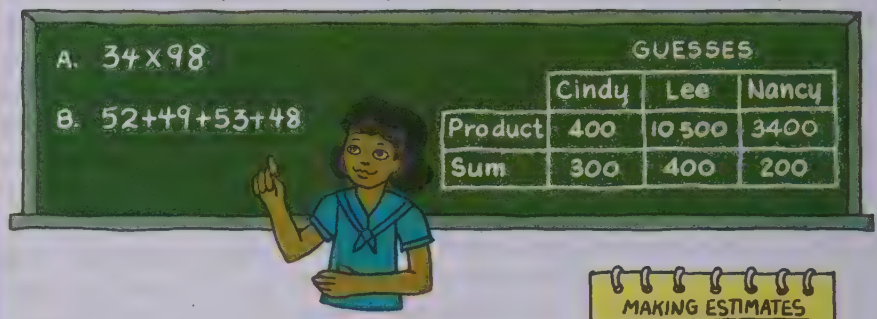
72 × 987
7056 + 4987

How close can you come to guessing this product and sum?
See Investigation.

Discussing the Ideas

1. Which guess do you think is best for the product? Why?

2. Which guess is best for the sum? Why?



3. We will use the word **estimate** instead of **guess**. Can you add anything about estimates to Cindy's notebook?
See Discussion.

MAKING ESTIMATES
1. An estimate is close to the correct answer.
2. Usually, I can make an estimate without pencil and paper.
3. Careful reasoning helps me make good estimates.
Cindy

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Discussion

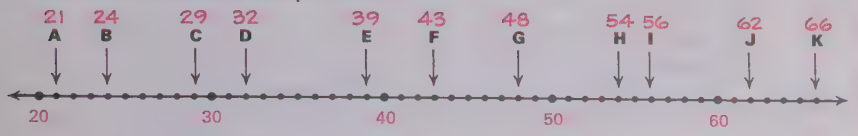
Have several children give their answers to the investigation questions and explain their reasoning.

Continue in the same manner with discussion exercise 1. Stress the convenience of thinking of multiples of 10 or 100. Discuss the word "estimate"; explain that it may be thought of as a "careful guess." Children might add to Cindy's notebook such comments as: estimates lead to many correct answers; errors in computation are easier to detect if an estimate has been made; and estimates made by rounding off numbers are found quickly compared to the time computation requires.

One of the main purposes of this lesson is to have children discover how to round off numbers to the closest multiple of 10 or 100. The guideline for rounding *up* or *down* is presented as a discovery exercise on page 171. (Numerals ending in 1, 2, 3, and 4 are rounded *down* to the closest multiple; numerals ending in 5, 6, 7, 8, or 9 are rounded *up* to the closest multiple.) Stress the importance of thinking about multiples of 10 and 100 during this discussion section.

Using the Ideas

1. Multiples of 10 and 100 are helpful in making estimates.
 To estimate 29 × 43, we can find the product 30 × 40.
 To estimate 48 + 73, we can find the sum 50 + 70.
 To estimate 312 × 67, we can find the product 300 × 70.
 To estimate 687 + 218, we can find the sum 700 + 200.
 The number-line pictures and the exercises will help you choose multiples of 10 and 100 to use in estimating.
 Give the numbers (A through K) that go with the points on this number-line picture.

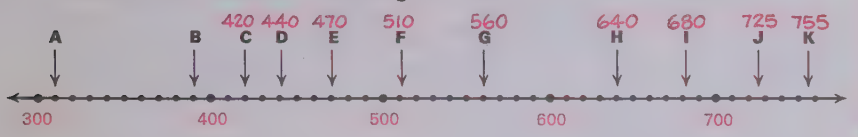


2. Give the multiple of 10 that is closest to each number in exercise 1. The number-line picture should help you.
 A20, B20, C30, D30, E40, F40, G50, H50, I60, J60, K70

3. Give the multiple of 10 that is closest to each number.

A	B	C	D	E	F	G	H
31	42	53	64	26	37	48	59
30	40	50	60	30	40	50	60

4. Here are the numbers for A and B: A 310, B 390.
 Give the numbers for C through K.



5. Give the multiple of 100 that is closest to each number in exercise 4.
 A300, B400, C400, D400, E500, F500, G600, H600, I700, J700, K800

6. Give the multiple of 100 that is closest to each number.

A	B	C	D	E	F	G
320	590	499	619	735	549	551
300	600	500	600	700	500	600

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Using the Exercises

Read the top of page 171 with the children. Suggest that they list the answers to exercise 1 in a column down their paper and put the answers to exercise 2 beside them. This might help them see the alignment of the numbers in relation to the digits in the ones' place. Have the children work the exercises independently. Then allow ample time for a discussion based on their work. Point out that when a number is directly between two multiples, such as 15 in relation to the nearest multiple of 10, or 250 in relation to the nearest multiple of 100, either the multiple below the number or above the number would give a rough estimate for a problem, but it is agreed that such a number ending in 5 is rounded *up* to the multiple directly above it. Thus, 15 is rounded off to 20, and 250 is rounded off to 300.

Assignments (page 171)
Minimum: 1–6, oral. Average: 1–6. Maximum: 1–6.

Follow-up

Children would benefit from extended practice in rounding off numbers and estimating products and sums. For this purpose you might prepare a worksheet similar to the following.

1. Give the multiple of 10 that is closest to each number. Use a number line only if you need to.

A 73	E 39	I 95
B 44	F 27	J 73
C 68	G 42	K 89
D 75	H 31	L 91

2. Give the multiple of 100 that is closest to each number.

A 304	E 629	I 980
B 536	F 212	J 150
C 865	G 510	K 145
D 483	H 673	L 286

3. Estimate the sums or products.

A 71 × 48	H 68 + 31 + 49
B 35 × 62	I 59 + 97 + 43
C 59 × 21	J 85 + 17 + 97
D 19 × 99	K 333 × 42
E 88 × 42	L 86 × 78
F 33 + 49 + 51	M 273 × 81
G 95 + 45 + 160	N 65 × 45

Workbook, page 49

PAGES 172–173

Objective

Given addition and subtraction problems with numbers less than 1000, the child will be able to estimate the sums and differences by thinking of multiples of 10 or 100.

Preparation

As preparation for this lesson, review rounding off numbers to the nearest multiple of 10 or 100. For example, give the number 28 and ask a child to respond with the multiple of 10 nearest to it. Or give the number 478 and ask a child to respond with the multiple of 100 nearest to it. Continue with similar examples, but remember to keep every preparation brief.

Let's explore estimates of sums and differences.

1. $\begin{array}{r} 79 \\ +49 \\ \hline \end{array}$

A Which of these do you think will give the best estimate of the sum in color? C

[A] $\begin{array}{r} 70 \\ +40 \\ \hline \end{array}$ [B] $\begin{array}{r} 80 \\ +40 \\ \hline \end{array}$ [C] $\begin{array}{r} 80 \\ +50 \\ \hline \end{array}$

B Check your estimate by finding the correct sum. 128

2. $\begin{array}{r} 151 \\ -89 \\ \hline \end{array}$

A Which of these do you think will give the best estimate of the difference in color? B

[A] $\begin{array}{r} 160 \\ -80 \\ \hline \end{array}$ [B] $\begin{array}{r} 150 \\ -90 \\ \hline \end{array}$ [C] $\begin{array}{r} 160 \\ -90 \\ \hline \end{array}$

B Check your estimate by finding the correct difference. 62

3. Give the multiples of 10 that are closest to the addends. Then give an estimate for the sum.

Example: $\begin{array}{r} 58 \rightarrow 60 \\ +96 \rightarrow 100 \\ \hline \text{Estimate} \rightarrow 160 \end{array}$

A $\begin{array}{r} 19 \rightarrow \blacksquare\ 20 \\ +48 \rightarrow \blacksquare\ 50 \\ \hline \blacksquare\ 70 \end{array}$ B $\begin{array}{r} 23 \rightarrow \blacksquare\ 20 \\ +78 \rightarrow \blacksquare\ 80 \\ \hline \blacksquare\ 100 \end{array}$ C $\begin{array}{r} 27 \rightarrow \blacksquare\ 30 \\ +69 \rightarrow \blacksquare\ 70 \\ \hline \blacksquare\ 100 \end{array}$

4. Give the number you think should go in each ■.

A To estimate 98 + 49, we can find the sum ■ + 50. 100
B To estimate 79 + 19, we can find the sum 80 + ■. 20
C To estimate 92 − 47, we can find the difference ■ − 50. 90
D To estimate 12 + 87, we can find the sum ■ + 90. 10
E To estimate 67 + 34, we can find the sum 70 + ■. 30
F To estimate 53 − 38, we can find the difference 50 − ■. 40

5. Give an estimate for each sum and difference.

A 59 + 19 80	E 69 + 69 140	I 44 + 86 130	M 12 + 23 + 29 60
B 92 − 48 40	F 93 − 27 60	J 97 − 46 50	N 58 + 63 + 47 170
C 43 + 69 110	G 86 + 59 150	K 97 + 98 200	O 69 + 46 + 44 160
D 67 − 27 40	H 87 − 28 60	L 77 − 38 40	P 86 + 84 + 98 270

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Discussion

As you discuss each exercise at the top of the page, give the children an opportunity to choose for themselves which set of rounded-off numerals will give the best sum or difference. Review the guideline for rounding off numerals and help the children see that 79 is nearer to 80, and 49 is nearer to 50. Similarly in exercise 2, 151 is nearer to 150, and 89 is nearer to 90. If necessary, use a number line to demonstrate the rounding off of these numbers. Select several exercises from parts of this lesson and display them on the chalkboard. Encourage children to discuss them and estimate the sums or differences.

Ask the children to explain to the class their methods for determining certain estimates. For example, a child who is estimating 53 − 38 might well explain that to arrive at this estimate he subtracted 40 from 50. Some of the children may be advanced enough in mental computation to arrive at the exact answer (15). However, for such an exercise, you should point out that they need only think of an estimate that is a multiple of ten and that in this case you are not really interested in the exact answer.

Since there will be children who are proficient enough at computation to arrive at correct answers faster

6. Give the multiples of 100 that are closest to the addends. Then give an estimate for the sum.

A
898 → ▮ 900
407 → ▮ 400
296 → ▮ 300
+513 → ▮ 500
▮ 2100

B
690 → ▮ 700
418 → ▮ 400
387 → ▮ 400
+209 → ▮ 200
▮ 1700

C
457 → ▮ 500
541 → ▮ 500
327 → ▮ 300
+273 → ▮ 300
▮ 1600

D
651 → ▮ 700
849 → ▮ 800
378 → ▮ 400
+764 → ▮ 800
▮ 2700

7. Find the correct sums in exercise 4. Which was your best estimate? C
A 2114, B 1704, C 1598, D 2642

8. Give the number you think should go in each ▮.

A To estimate 298 + 67, we can find the sum ▮ + 70. 300
B To estimate 702 − 397, we can find the difference ▮ − 400. 700
C To estimate 417 + 686, we can find the sum 400 + ▮. 700
D To estimate 641 + 856, we can find the sum ▮ + 900. 600
E To estimate 847 − 568, we can find the difference ▮ − 600. 800

9. Which sum will give the best estimate for 85 + 85? C

A 80 + 80 B 90 + 90 C 80 + 90

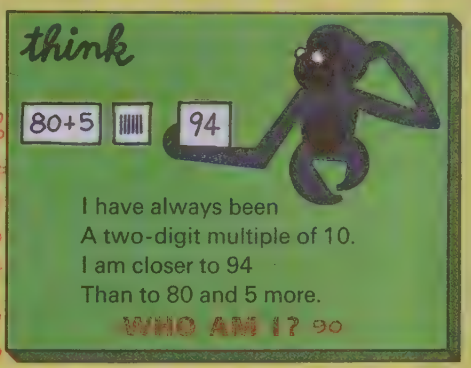
10. Give an estimate for each sum.

A 85 + 65 150 B 35 + 45 80 C 65 + 65 130 D 75 + 45 120

11. Give an estimate for each sum and difference. *

A 395 + 407 800
B 402 − 198 200
C 787 + 218 1000
D 479 − 324 200
E 963 + 439 1400
F 804 − 387 400
G 497 + 69 570
H 886 − 39 860
I 88 + 317 400
J 761 − 458 300
K 869 + 468 1300
L 729 − 346 400
M 301 + 649 950
N 649 − 301 350
O 489 − 92 400
P 86 + 215 300

* Estimates may vary.



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than some others can arrive at estimates, be sure to provide for individual differences.

Using the Exercises

After working through several exercises together, have the children complete the exercises independently. When they have finished, allow time for checking papers and a discussion during which the children explain the various ways of reasoning that they used to arrive at their estimates. This part of the lesson is vital; it is much more effective for the children to explain their reasoning to each other than it is to cite particular rules for all to follow.

Assignments (pages 172–173)
Minimum: 1–5, oral. Average: 1–5; 6–11, oral. Maximum: 1–11.

Workbook, page 50

PAGES 174–175

Objective

Given multiplication problems with factors that are less than 1000, the child will be able to estimate the products by thinking of multiples of 10 or 100.

Preparation

You might choose to begin immediately with the investigation. However, if you prefer, give the children further practice in rounding off numbers to the closest multiple of 10 or 100.

Investigation

Give the children ample opportunity to estimate the 8 products on Jack's paper. (Encourage them to prepare to explain during the discussion the 4 mistakes they find.) Have the children who finish quickly, and who are capable, try to find the exact answer to some of these problems. Some may recall the multiplication algorithm; others may use repeated addition for the first few problems.

● *Let's estimate products.*

Investigating the Ideas

Jack made 4 mistakes on his paper.

Jack

1. $3 \times 68 = 204$
2. $8 \times 23 = 154$
3. $4 \times 99 = 496$
4. $7 \times 52 = 364$
5. $6 \times 39 = 174$
6. $19 \times 19 = 401$
7. $21 \times 49 = 1029$
8. $98 \times 84 = 822$

Can you use estimation to find which problems Jack missed?
See Investigation and Discussion. (2,3,5,6,8)

Discussing the Ideas

Give the number you think should go in each ▮.

1. To estimate 3×51, we can find the product $3 \times$ ▮. 50
2. To estimate 8×42, we can find the product $8 \times$ ▮. 40
3. To estimate 6×39, we can find the product $6 \times$ ▮. 40
4. To estimate 9×78, we can find the product $9 \times$ ▮. 80
5. To estimate 32×29, we can find the product ▮ $\times 30$. 30
6. To estimate 98×18, we can find the product ▮ $\times 20$. 100
7. To estimate 45×53, we can find the product ▮ $\times 50$. 50
8. To estimate 9×207, we can find the product $9 \times$ ▮. 200
9. To estimate 7×496, we can find the product $7 \times$ ▮. 500
10. To estimate 32×387, we can find the product $30 \times$ ▮. 400

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Discussion

Ask the children to point out the mistakes they found in the investigation exercises and to give their reasoning in each case. In exercise 2, someone should point out that 8×20 is 160 so 8×23 could not be 154, which is less than 160. In exercise 3, children should estimate that, since $4 \times 100 = 400$, 4×99 must be less than 400 and, thus, 496 could not be correct. Since 39 is near 40, clearly, 6×39 must be greater than 6×30 or 180, so 174 in exercise 5 is incorrect. In exercise 6, 19×19 cannot be greater than 20×20, so 401 is incorrect. Finally in exercise 8, 98 is near 100, so the answer should be nearer to 8000 than it is to 800.

As you work through the discussion exercises, have the children give reasons for their answers. Note that exercises 8, 9, and 10 deal with 3-digit numerals. It would be helpful to have the children give their estimate of the product for each exercise.

Using the Ideas

1. Give the number you think should go in each ▮.
 - A To estimate 9 × 37, we can find the product 9 × ▮. 40
 - B To estimate 4 × 96, we can find the product 4 × ▮. 100
 - C To estimate 46 × 51, we can find the product ▮ × 50. 50
 - D To estimate 83 × 19, we can find the product 80 × ▮. 20

2. **Estimate** these products.

A 6 × 21 120	E 9 × 68 630	I 7 × 99 700	M 98 × 33 00
B 5 × 32 150	F 3 × 47 150	J 37 × 93 60	N 6 × 56 360
C 7 × 81 560	G 6 × 73 420	K 17 × 6 120	O 4 × 74 280
D 8 × 39 320	H 8 × 18 160	L 7 × 19 140	★ P 8 × 65 480 OR 560

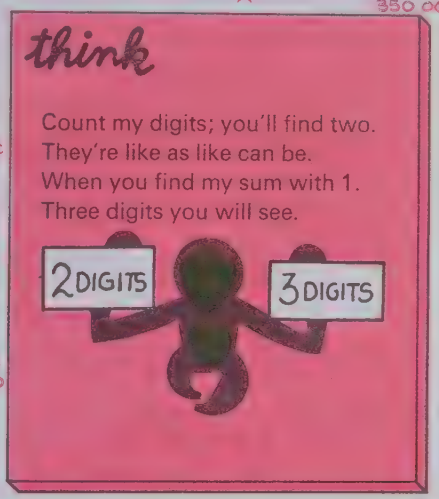
3. Give an **estimate** for each product.

A 31 × 29 900	F 64 × 56 3600	K 6 × 198 1200 ★	P 51 × 207 10 000
B 98 × 21 2100	G 38 × 88 3600	L 3 × 696 2100 ★	Q 39 × 289 12 000
C 69 × 19 1400	H 65 × 116 50	M 7 × 516 3500 ★	R 44 × 318 12 000
D 41 × 51 2000	I 45 × 452 000	N 8 × 999 8000 ★	S 469 × 34 15 000
E 82 × 89 7200	J 93 × 181 800	O 9 × 356 3600 ★	T 538 × 651 350 000

4. **Estimate** the answer.
 - A What did Joe have to pay for 6 books at 49¢ each? $3.00 or 300¢



 - B A pail holds about 8 litres. How many litres are in 98 pails? about 800
 - C 24 hours make one day. How many hours are in 62 days? about 1200



More practice, page A-16, Set 28

Using the Exercises

Have the children do the exercises on page 175 independently. When they have finished, allow time for checking papers and discussing the exercises.

Parts P through T of exercise 3 are starred because these estimates involve larger numbers. If time permits, accompany part B of exercise 4 with a demonstration in which the children estimate how many small glasses of water a given large container will hold. After the children have made their estimates, test these by having the children count to see how many glasses of water you can put in the large container.

Other demonstrations are also possible. For example, exhibit an empty paper bag and a set of like-sized objects and have the children guess how many of these objects will fit into the paper bag. After the children have made their estimates, fill the bag and then have the children count to see who made the best estimate.

Assignments (page 175)

Minimum: 1–2, oral. Average: 1–2O, 3A–O, 4. Maximum: 1–4.

Follow-up

Children might enjoy a different kind of estimation activity. Distribute or display a variety of pictures which show people, trees, cars, animals, etc., in great number. Have children estimate how many objects are in each picture. One way of doing this is to count some area, such as a 2-centimetre square, and use this as a basis for the estimation. (Of course, depending on perspective, the number of figures per 2-centimetre square could vary considerably from foreground to extreme background.)

Solution, Think, page 175

If some children have difficulty with the *Think* problem, you might suggest that they make a reconstruction problem to help them find the missing digits. For example:



Resources for Active Learning

Nuffield Project: *Computation and structure* 3, "Approximations," p. 47, Wiley.

Duplicator Masters, page 32

PAGES 176–177

Objective

Given division problems with a 1-digit divisor and a 2- or 3-digit dividend, the child will be able to estimate the quotient by thinking of missing factors and multiples of 10 or 100.

Preparation

To prepare for this lesson, you might review estimation of products with examples similar to those in exercise 2 on page 175. You might also review finding quotients by thinking of missing factors, drawing your examples from the basic multiplication facts.

Investigation

You might choose to have the children work as partners or in groups of three for this investigation. Thus, they could benefit from each other's reasoning as they work through each problem. Move around the room as the children work. You may need to remind some children that each division sentence has a related multiplication sentence. You may, need to remind others that, although division may be thought of as repeated subtraction, they are looking for *estimates* of the quotients and for this purpose repeated subtraction would not be as appropriate as thinking of missing factors. However, you might suggest to any children who finish quickly that they check their work by using repeated subtraction to find the quotient.

● *Let's estimate quotients.*

Investigating the Ideas

Jill made 4 mistakes on her paper.

Jill

1. 56 ÷ 2 = 38
2. 76 ÷ 4 = 19
3. 108 ÷ 6 = 18
4. 84 ÷ 3 = 31
5. 174 ÷ 6 = 19
6. 294 ÷ 3 = 98
7. 219 ÷ 3 = 53
8. 796 ÷ 4 = 199

? Can you use estimation to find which problems Jill missed?
See Investigation and Discussion. (1,4,5,7)

Discussing the Ideas

Thinking about missing factors will help you estimate quotients.

1. Use multiples of ten (10, 20, 30, 40, 50, 60, 70, 80, 90) to estimate these quotients.



? × 4 is about 156 — A 156 ÷ 4 40

? × 6 is about 132 — B 132 ÷ 6 20

? × 3 is about 144 — C 144 ÷ 3 50

? × 6 is about 114 — D 114 ÷ 6 20

? × 3 is about 111 — E 111 ÷ 3 40

? × 7 is about 637 — F 637 ÷ 7 90

2. Estimate these quotients. Use multiples of 10.

A 128 ÷ 4 30 C 60 ÷ 5 10 E 486 ÷ 6 80 G 129 ÷ 7 20 I 252 ÷ 5 50

B 57 ÷ 3 20 D 98 ÷ 2 50 F 236 ÷ 4 60 H 282 ÷ 3 90 J 222 ÷ 6 40

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Discussion

Have children point out the mistakes they found in the investigation. Since 2×30 is 60, $56 \div 2$ cannot be 38. Since 3×30 is 90, $84 \div 3$ cannot be 31. Since 6×20 is 120, $174 \div 6$ cannot be 19. Finally, since 3×60 is 180, $219 \div 3$ cannot be 53. It is important that the children give the reasons for each mistake they identify.

As you discuss the exercises on page 176, stress the convenience of thinking of missing factors when estimating quotients. Point out the "think clouds" above each problem in exercise 1 as guides to thinking of missing factors. Direct the children to work the problems in exercise 2 similarly.

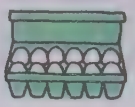
Using the Ideas

Three numbers are given for each problem. Choose the number that is closest to the correct answer.

1. 224 apples 
How many packages of 7? 30
A 100 B 30 C 80

2. 386 erasers 
How many boxes of 9? 40
A 400 B 4 C 40

3. 504 crayons 
How many boxes of 8? 60
A 60 B 100 C 7

4. 276 eggs 
How many cartons of 12? 20
A 20 B 10 C 100

5. 87 ÷ 3 How many threes in 87? A 3 B 30 C 20 30
6. 128 ÷ 4 How many fours in 128? A 10 B 300 C 30 30
7. 252 ÷ 6 How many sixes in 252? A 40 B 50 C 400 40
8. 686 ÷ 7 How many sevens in 686? A 10 B 1000 C 100 100
9. 5760 ÷ 8 How many eights in 5760? A 70 B 700 C 800 700
10. 217 ÷ 31 How many thirty-ones in 217? A 5 B 6 C 7 7
11. 152 ÷ 27 How many twenty-sevens in 152? A 10 B 5 C 8 5
12. $3\overline{)279}$ How many threes in 279? A 9 B 900 C 90 90
13. $5\overline{)525}$ How many fives in 525? A 90 B 100 C 200 100
14. $7\overline{)343}$ How many sevens in 343? A 5 B 40 C 50 50
15. $7\overline{)4760}$ How many sevens in 4760? A 600 B 70 C 700 700
16. $9\overline{)4680}$ How many nines in 4680? A 50 B 5000 C 500 500
17. $53\overline{)212}$ How many fifty-threes in 212? A 8 B 5 C 4 4
★ 18. $25\overline{)224}$ How many twenty-fives in 224? A 8 B 10 C 100 8

More practice, page A-16, Set 29



Follow-up

To give children practice in estimating quotients, suggest the following card game:

Use a deck from which all tens and face cards have been removed. One player is the dealer. This person will also be scorekeeper, though he himself will have no score. He deals one card to each player and 3 (or 4) cards to himself. He arranges his 3 (or 4) cards face down in any order he chooses. Then he turns his cards face up to form a 3- (or 4-) digit numeral and calls out the numeral he has formed.

Each player then uses his single card as a divisor and the dealer's numeral as the dividend. He *estimates* his quotient to the nearest multiple of 10 or 100. The dealer records each player's estimated quotient. All players should act as a check on each other to see that a player has not made an unreasonable estimate. Encourage children to develop their own criteria for "unreasonable" estimates and types of penalties to impose. The first player to reach a determined score (the sum of his correctly estimated quotients), for example, 1000, or the player with the highest score after a certain number of rounds, for instance, 5, wins.

Workbook, page 51

Using the Exercises

You might choose to use exercises 1 through 4 on page 177 as a basis for discussion and write an equation for each problem as you work through it. Then assign the remaining exercises as independent work. Remind the children to think of missing factors as they estimate the quotient for each exercise. When they have completed the exercises, discuss the reasoning underlying the various selections the children have made.

Assignments (page 177)
Minimum: 1–17, oral. Average: 1–17.
Maximum: 1–18.

PAGES 178–179

Objective

Given word problems which require multiplication or division, the child will be able to estimate the answers.

Preparation

To prepare for this lesson, you might give children oral practice in estimating quotients. For example, use questions such as the following: About how many fives are there in 264? About how many sixes are there in 192? About how many fours are there in 337? Encourage children to respond in multiples of 10 or 100 for all such questions. Practice such as this is an excellent preparation for developing skill in dividing.

Solving Short Stories

Estimation



Estimate the answer to each question.



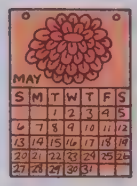
1 48 pills in each bottle
7 bottles. How many pills?
350 (7×50)

2 6 tomatoes in each bag.
32 bags. How many tomatoes? 180 (6×30)

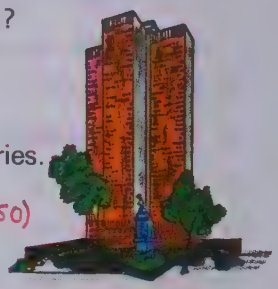
3 Travel 642 kilometres in 7 hours.
About how many kilometres each hour?
90 (630÷7)



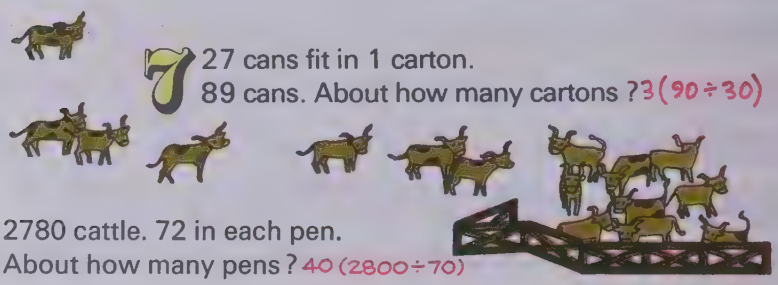
4 About 31 children in each class.
213 children. How many classes?
7 (210÷30)



5 Place Ville-Marie.
188 metres high. 49 stories.
How high is one story?
4m (200÷50)



6 About 365 days in a year. 7 days in a week.
About how many weeks in a year? 50 (350÷7)



7 27 cans fit in 1 carton.
89 cans. About how many cartons? 3 (90÷30)

★ 8 2780 cattle. 72 in each pen.
About how many pens? 40 (2800÷70)

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Discussion

The exercises on page 178 should, preferably, be assigned as independent work. When the children have completed the exercises, allow time for discussion and encourage the children to elaborate on their reasoning in arriving at various estimates. For such an exercise set, one of the most important elements of the lesson is to have the children explain their thoughts on a given exercise in order that they may learn from one another.

Assignments (page 178)

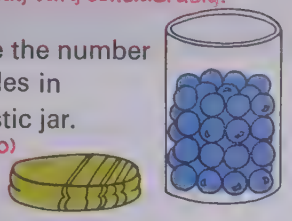
Minimum: 1–7, oral. Average: 1–7. Maximum: 1–8.

Answers may vary considerably.

Estimation for Fun

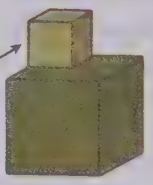
1 Estimate the number of marbles in this plastic jar.

160 (16×10)



2 If this block of lead weighs 22 kilograms, estimate the weight of this block.

160 (20×8)





3 Estimate the number of words in exercise 5 by counting the number of words in **one line** and counting the number of lines. 45 (9×5)

4 Count the number of breaths you take in 1 minute. Estimate the number of breaths you take

A in an hour.

★B in a 24-hour day.

Estimates will vary from child to child.





5 Sally took 4 drinks from the fountain on Monday. Tim took 6 drinks. Bill took 5 drinks. There are 33 children in Sally's class. Estimate how many drinks Sally's class takes from the fountain in A one day. B one week.

See Answers, T.E. page 179.

6 The tallest horse on record is 21 hands tall. Estimate this horse's height in centimetres.

See Answers, T.E. page 179.



On a 180-cm man this length is about one hand.

Using the Exercises

The exercises on page 179 are intended to provide fun and diversion rather than to serve any particular purpose or goal in the study of estimation.

For exercise 2, encourage the children to compare the small block with the large block. Help the children see that the volume of the large block is eight times the volume of the small block. You may wish to demonstrate with real blocks or with a picture.



Exercise 6 will involve some research by the children, since they must determine the approximate width of the hand of a man 180 centimetres tall. They should find that a man's hand is about 10 centimetres wide.

If you prefer to use these exercises as a class discussion activity, be sure to give the children an opportunity to find each answer on their own before discussing it.

Assignments (page 179)

Minimum: 1–6, oral. Average: 1–6. Maximum: 1–6.

Follow-up

Encourage children to make up problems for estimation. In order to avoid problems of great difficulty suggest that they make up problems of "concrete" estimation, such as how many pieces of paper in a tablet, or how many steps across the room, or how many kernels of corn in a can, etc.

Answers, exercises 5 and 6, page 179

5. A About 30×5 or 150

B 5-day week:
between 500 or 100×5 and 1000 or 200×5; about 750.
7-day week:
between 700 or 100×7 and 1400 or 200×7; about 1000.

6. 200 (20×10); 1 hand is about 10 centimetres.

PAGES 180–181

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

To prepare for this lesson, give children oral exercises in making estimates for sums, products, and quotients. Ask questions similar to the following:

The sum of 68 + 24 is close to what number? The sum of 35 + 97 is close to what number? The product 22 × 51 is close to what number? About how many eights are in 572? About how many threes are in 251?

Reviewing the Ideas

1. Give what you think should go in each gray space.
 - A To estimate 59 + 83, we can find the sum ▢ + 80. 60
 - B To estimate 396 + 507, we can find the sum 400 + ▢. 500
 - C To estimate 7 × 68, we can find the product 7 + ▢. 70
 - D To estimate 8 × 497, we can find the product 8 × ▢. 500
 - E To estimate 53 × 47, we can find the product ▢. 50×50
 - F To estimate 413 × 89, we can find the product ▢. 400×90
 - G To estimate 426 + 568, we can find the sum ▢. 400+600
 - H To estimate 65 × 9, we can find the product ▢. 65×10
 - I To estimate 403 − 196, we can find the difference ▢. 400−200

2. Estimate the sums, products, and differences.
 - A 69 + 23 90
 - B 82 − 49 30
 - C 8 × 37 320
 - D 33 × 58 1800
 - E 517 + 89 590
 - F 95 + 95 190
 - G 503 − 293 200
 - H 407 × 22 8000
 - I 659 + 435 1100
 - J 398 × 21 8000
 - K 49 × 313 15 000
 - L 301 × 297 90 000
 - M 205 + 298 500
 - N 39 × 604 24 000

think

I'm quite close to one hundred three.
Closer still to ninety-eight.
Yet for both numbers I am used
When you want to estimate.

Who am I? 100

3. Choose the best estimate for each exercise.

A	88 ÷ 4	How many fours in 88? 20	About 20? 30? 10?
B	434 ÷ 7	How many sevens in 434? 60	About 100? 70? 60?
C	558 ÷ 6	How many sixes in 558? 90	About 80? 90? 100?
D	1728 ÷ 32	How many thirty-twos in 1728? 50	About 100? 50? 6?

4. A cubit is a unit of length first used long ago. It is the distance from a man's elbow to the tip of his fingers. Goliath was reported to be over 6 cubits tall. Estimate Goliath's height in centimetres. 270 (45×6)



180

Discussion

Have the children do the exercises independently. When they finish, attempt to evaluate from the children's discussion of this page what they understood from this chapter and what caused difficulty. If the children have had trouble in estimating quotients, such as those in exercise 3, it might be wise to provide some additional work in this area. However, estimation of quotients will be covered again in much greater detail in the next chapter. In exercise 4 the children must do some research to find out the length of a man's forearm before they can determine the approximate length of a cubit. Of course, it is hoped that the children will arrive at the fact that a cubit is about 45 centimetres. Rather than disclose this fact, urge the children to try to find it independently.

Most of the children should be able to solve the *Think* problem, if they understand estimating at all, by using the nearest multiple of ten.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Place value

1. Find the missing numbers.

A $356 = 300 + 50 + n$ 6
B $872 = 800 + n + 2$ 70
C $639 = n + 30 + 9$ 600
D $5436 = 5430 + n$ 6
E $6947 = 6047 + n$ 900
F $4673 = 4603 + n$ 70

2. Find the sums.

A	B	C	D	E
39	56	856	638	5947
68	39	729	463	6879
+57	+87	+347	+806	+9435
164	182	1932	1907	22261

3. Find the differences.

A	B	C	D	E	F
234	195	156	117	78	39
−39	−39	−39	−39	−39	−39
195	156	117	78	39	0

4. Use exercise 3 to give the number of thirty-nines in 234. 6

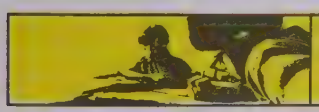
5. List the problems that have no whole-number answers. B, F
Then find the differences for the other exercises.

A	B	C	D	E	F
147	281	370	304	874	387
−108	−315	−106	−137	−396	−431
39		264	167	478	

G	H	I	J	K
500	1008	1870	9007	4976
−287	−369	−1798	−8438	−1098
213	639	72	569	3878

6. Find the products and quotients.

A 9×40 360
B 30×7 210
C 70×6 420
D 200×8 1600
E $360 \div 4$ 90
F $720 \div 8$ 90
G 20×50 1000
H 60×30 1800
I $800 \div 40$ 20
J 30×400 12 000
K $3600 \div 4$ 900
L 500×300 15 000



You are invited to explore

ACTIVITY CARD 7
Page 350

Follow-up

As a further exercise for more capable children, develop a worksheet similar to the following, to refine their skill in estimation.

Study the example that shows ways to find estimates.

$38 \times 47 = ?$

Rough estimate: $40 \times 50 = 2000$

Better estimate:

$(30 \times 50) + (8 \times 50) =$
$1500 + 400 = 1900$

Still Better:

$(30 \times 40) + (8 \times 40) + (7 \times 30)$

Estimate: $1200 + 320 + 210 = 1730$

Exact Answer:

$38 \times 47 = (30 \times 40) + (8 \times 40) +$
$(7 \times 30) + (7 \times 8)$
$= 1200 + 320 + 210 + 56$
$= 1786$

Now, you try it. $72 \times 68 = ?$

Workbook, page 52 (Use with page 180.)

Using the Exercises

If you wish to relate the work of this chapter to page 181, you might have children discuss how they could arrive at estimates for various exercises. For example, a child might estimate the sum for exercise 2C by finding the sum $900 + 700 + 300$; or he might estimate the difference in exercise 3D by subtracting 40 from 120.

For exercise 5, discuss very carefully those problems which have no whole-number answers. In the next chapter the children will be faced with the difficulty of attempting to subtract a larger number from a smaller one. That is, when using the division algorithm, the child's first estimate for the quotient may be too great and he will be faced with a subtraction such as in exercise 5B. In such a case, the child must be able to recognize that he cannot subtract and, therefore, must go back and try a smaller number for his estimated quotient.

CHAPTER 8 | Multiplying

Pages 182–205

General Objectives

To develop the multiplication algorithm for 1-, 2- and 3-digit multipliers

To provide word problems appropriate to this development

To maintain understanding of the multiplication concepts

Utilizing the groundwork which was laid in preceding chapters, this chapter moves directly to the development of the multiplication algorithm. Familiarity with the special products covered in Chapter 5 and an understanding of the distributive, or multiplication-addition, principle are essentially all the foundation necessary for a thorough development of the multiplication algorithm. Thus, all that remains is to demonstrate to the children the various processes involved in algorithm notation, which is simply a convenient method for writing multiplication exercises in such a way as to simplify computation.

The first pages of the chapter are devoted to reviewing the distributive principle, estimation of products, and inequalities. Then a review of 1-digit multipliers leads to multipliers that are multiples of ten, and finally, to multiplication by any 2-digit multipliers. This extension is little more than a formality, since the general process has been developed previously.

Mathematics

Because the concepts necessary for understanding the multiplication algorithm have already been developed, there are no new mathematical concepts presented in this chapter. The chief mathematical concept involved is the utilization of the distributive principle for "breaking apart" and adding partial products in the multiplication algorithm. The development of the algorithm, after a certain amount of groundwork has been provided, involves little more than helping the children learn to write the work in a special, convenient way.

Teaching the Chapter

Materials

Casein glue
Coins (optional)

Vocabulary

bar graph

In a chapter such as this, which is devoted primarily to the development of skills in working with an algorithm, there are no new mathematical concepts, and hence, little vocabulary is involved.

The *bar graph* provides a convenient way of organizing data to make it easier to interpret. *Bar graph* is really not a mathematical term, but the children need to become familiar with it to handle the lesson on page 189 and to make it an effective tool for handling other kinds of data.

Since our chief concern in this chapter is to teach children to organize their work in a certain manner on paper, little is required in the way of materials.

Lesson Schedule

Plan to spend about three weeks on this chapter. If the children are particularly slow at grasping the skills, you may decide to allow four weeks. However, if the children understand the ideas, these skills can be developed to a greater degree throughout the remainder of the text, which does provide for maintenance of the algorithm. Therefore, do not expect the children to have mastered this algorithm completely by the end of the chapter.

Evaluation of Progress

Evaluation of whether or not a child has mastered the algorithm is quite simple. However, the fact that the child can use the algorithm successfully does not guarantee that he understands it. In a cumulative evaluation for the entire multiplication process, you should include understanding of the algorithm as one criterion. An interview with each child during which you ask him to "think aloud" through exercises of various degrees of difficulty would facilitate your evaluation of his understanding of the algorithmic process.

Try to give more capable children as much experience as possible in creating and analyzing data from as many different kinds of graphs as they can reasonably find, but do not include this skill as part of your evaluation. Use pages 202 and 203, which provide a chapter review, and pages 204 and 205, a cumulative review, to assist you in your evaluation.

Resources for Active Learning

GENERAL ACTIVITIES

[Refer also to Chapters 5, 6, and 7 for appropriate resources for this chapter, since all these concepts deal with multiplication-division.]

Applied Mathematics Cards, "Numbers," Group 4/2, Schofield and Sims (Available from Mafex Associates, Willowdale)

Franklin Series: *From Fingers to Computers*, "Multiplication on the Counting Board," pp. 34–35; "Napiers Rods," pp. 38–45, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs," pp. 71–78, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Franklin Series: *Patterns and Puzzles*, "Napiers Bones," pp. 86–89, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Multiplication," pp. 118–119, Addison-Wesley

Math Activity Cards "Multiplying," C5, Macmillan

Mathex: Operations and Problem Solving No. 8, "Napiers Bones," pp. 19–22 (pupil page 22); "Lattice Method," pp. 22–23 (pupil pages 23–24); "Multiplication-Division Slide Rule," pp. 23–24, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and*

Structure 3, "The Distributive Property . . . ," pp. 38–47, Wiley

Nuffield Project: *Computation and Structure* 3 , "Tables," pp. 30–34; "Napiers Rods," p. 48; "The Calculating Machine," pp. 48–49, Wiley

MANIPULATIVE DEVICES

Multiplying Machine (Creative Publications; Hammett)

Napiers Rods (Hammett; Selective Educational Equipment)

SEE Calculator (Selective Educational Equipment)

Soma Cubes and Wooden Block Puzzles (Edmund Scientific; Selective Educational Equipment)

COMMERCIAL GAMES

Orbiting the Earth: Multiplication (Scott Foresman)

Quinto (Hammett; Selective Educational Equipment)

TUF (Creative Publications; Cuisenaire Co.; TUF)

PAGES 182–183

Objective

Given a multiplication problem with a 1-digit factor and a 2-digit factor, the child will be able to find the product by applying the multiplication-addition principle.

Preparation

It would be appropriate to begin with the investigation and let it serve as an introduction to this chapter. However, if you prefer, use a short drill of basic multiplication facts and multiplication with multiples of 10.

Investigation

If possible, have sets of coins available for use by those children who need them. You might have the children work in groups of two or three. Before they begin, make sure they realize that they are to find the cost of 3 hamburgers, of 3 hot dogs, and of 3 tacos. In each case they may simply add the cost of the item 3 times. However, suggest to them that, since they must find the cost of 3 of each of the items at the same price, they should try finding their answers by multiplying. Using multiplication will necessitate application of the multiplication-addition principle at least on an intuitive level.

8 Multiplying

Let's use the multiplication-addition principle.

Investigating the Ideas

You have a dollar to spend. The cost of each item is shown in coins.



Hamburger



Hot dog



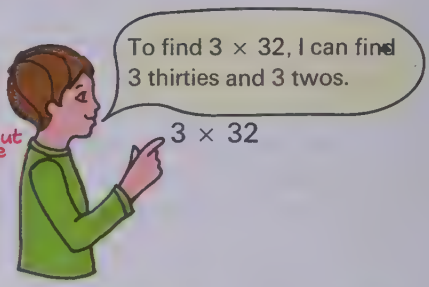
Taco



Can you buy 3 hamburgers? 3 hot dogs? 3 tacos?
See Investigation. Yes Yes No

Discussing the Ideas

1. A Explain how Steve is thinking about 3×32. He is thinking about breaking apart the 2-digit factor.
 B How would Steve think about 3×34?
 See Discussion.



2. Give the missing words.
 - A To find 4×26, we can find 4 twenties and 4 __?__. sixes
 - B To find 2×37, we can find 2 __?__ and 2 sevens. thirties
 - C To find 3×42, we can find 3 forties and 3 __?__. twos
 - D To find 5×24, we find 5 __?__ and 5 __?__. twenties, fours
 - E To find 3×25, we can find 3 __?__ and 3 __?__. twenties, fives
 - F To find 6×14, we can find 6 __?__ and 6 __?__. tens, fours

3. Can you find each product in exercise 2?
 A 104, B 74, C 126, D 120, E 75, F 84



182

Discussion

Have the children explain how they arrived at their answers for the investigation. Emphasize the application of the multiplication-addition principle. Help children see how Steve is thinking about 3×32. For exercise 1B, expect a response something like this: "To find 3×34, find 3 fours and 3 thirties. Then add 12 and 90 and get 102."

Exercise 2 provides excellent practice in "breaking apart" factors. As you discuss exercise 3, develop each problem in a way that will show how the multiplication-addition principle is applied. Do not use the vertical notation yet, but use an explanation similar to this: "In exercise 2D, to find 5×24, first think 5 twenties and 5 fours. Since 5 twenties is 100, and 5 fours is 20, altogether we have $100 + 20$, or 120."

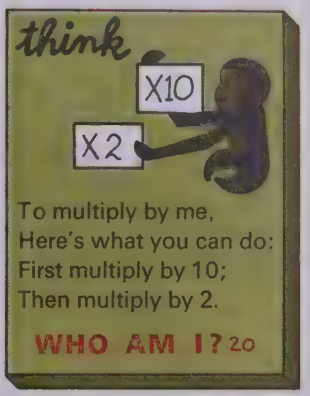
Using the Ideas

1. Solve the equations.

A $4\times25=(4\times20)+(4\times n)$ 5
B $6\times32=(6\times30)+(6\times n)$ 2
C $5\times18=(5\times10)+(5\times n)$ 8
D $3\times56=(3\times50)+(3\times n)$ 6
E $8\times21=(8\times20)+(8\times n)$ 1
F $7\times92=(7\times n)+(7\times2)$ 90
G $3\times76=(3\times n)+(3\times6)$ 70
H $9\times58=(9\times n)+(9\times8)$ 50
I $4\times35=(4\times n)+(4\times5)$ 30
J $8\times47=(8\times n)+(8\times7)$ 40

2. Solve the equations.

48 A $3 \times 16 = (3 \times 10) + (3 \times 6) = n$
115 B $5 \times 23 = (5 \times 20) + (5 \times 3) = n$
152 C $4 \times 38 = (4 \times 30) + (4 \times 8) = n$
378 D $9 \times 42 = (9 \times 40) + (9 \times 2) = n$
399 E $7 \times 57 = (7 \times 50) + (7 \times 7) = n$
512 F $8 \times 64 = (8 \times 60) + (8 \times 4) = n$
474 G $6 \times 79 = (6 \times 70) + (6 \times 9) = n$
430 H $5 \times 86 = (5 \times 80) + (5 \times 6) = n$
855 I $9 \times 95 = (9 \times 90) + (9 \times 5) = n$



3. Give the missing numbers.

A Since $4 \times 80 = 320$, we know that $4 \times 81 = n$. 324
B Since $7 \times 50 = 350$, we know that $7 \times 51 = n$. 357
C Since $6 \times 80 = 480$, we know that $6 \times 82 = n$. 492
D Since $4 \times 90 = 360$, we know that $4 \times 94 = n$. 376
E Since $8 \times 70 = 560$, we know that $8 \times 73 = n$. 584

★ **4.** Solve the equations.

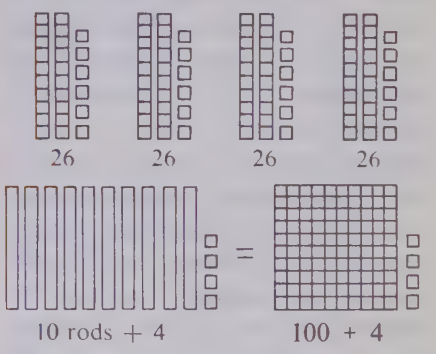
A $20 \times 43 = (20 \times 40) + (20 \times 3) = n$ 860
B $60 \times 74 = (60 \times 70) + (60 \times 4) = n$ 4440
C $30 \times 86 = (30 \times 80) + (30 \times 6) = n$ 2580
D $50 \times 29 = (50 \times 20) + (50 \times 9) = n$ 1450
E $90 \times 98 = (90 \times 90) + (90 \times 8) = n$ 8820
F $23 \times 34 = (23 \times 30) + (23 \times 4) = n$ 782
G $34 \times 45 = (34 \times 40) + (34 \times 5) = n$ 1530

More practice, page A-17, Set 30



Follow-up

Some children might benefit from working with concrete materials such as number blocks or centimetre rods and grids. For example, to multiply 4×26, a child would take 2 centimetre rods and 6 centimetre units, four times, and regroup his final amount.



Workbook, page 53

Using the Exercises

On page 183, have the children work the exercises independently. When they have finished, point out, or have the children explain, how the multiplication-addition principle is used in exercises 1 and 2. The use of the multiplication-addition principle is much less obvious in exercise 3 than in exercise 1. For example, in exercise 3A, we observe that since we know

$$4 \times 80 = 320,$$

we know that

$$4 \times 81 = 324$$

because

$$4 \times 81 = (4 \times 80) + (4 \times 1).$$

Hence, we just have one more four than when we multiplied 4×80.

Most of the children will be able to solve the *Think* problem.

Assignments (page 183)

Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

PAGES 184–185

Objective

Given a multiplication problem with one 2-digit factor, written in either horizontal or vertical notation, the child will be able to find the product.

Preparation

A short, brisk game of "What's My Rule" (see page 63) would be a suitable preparation for this lesson. However, you may choose to begin immediately with the investigation.

Investigation

The function table to be studied in this investigation demonstrates the use of the multiplication-addition principle and provides an introduction to the standard algorithm. As children are finishing the investigation, write other examples of similar function tables on the chalkboard, such as the following:

57
× 4

Multiply by 4

Input	Output
7	28
50	200
57	

74
× 6

Multiply by 6

Input	Output
4	24
70	420
74	

You might also have the children make a function table for other products which you present in vertical notation; for example:

38 × 5 82 × 3 46 × 8



Let's multiply with one 2-digit factor.

Investigating the Ideas

Here is a function table for finding this product. Find the product.

45
× 3

Function Rule

Multiply by 3

Input	Output
5	15
40	120
45	[illegible] 135

Multiply by 4

Input	Output
7	28
30	120
37	148

? Can you make a function table for this product? See Investigation.

37
× 4

Discussing the Ideas

1. Compare this method with the function table above. Find the final product.

45
× 3
15
120
135

2. Find the product 37 × 4 using the method in exercise 1.

37
×4
28
120
148

3. Explain each step in the shortcut.

26 × 3 → 18, 60 → 78

26 × 3 → 8 (1 ten noted)

26 × 3 → 78

Multiply 3×6 and write down the ones' digit (8) and note the number of tens; then multiply the tens' digit (2) and add the 1 ten noted in the first step.

4. Explain each example. See Discussion.

A 38 × 6 = 228 (4)

B 54 × 7 = 378 (2)

C 65 × 4 = 260 (2)

D 82 × 9 = 738 (1)

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Discussion

Use discussion exercise 1 to relate the function table in the investigation to the algorithm. Point out the partial products in the table and note that their sum equals the final product. Use several illustrations of the method in exercise 1 before continuing through exercise 2. Then work through the example in exercise 3 step by step with the children.

Children using this notation frequently make the mistake of adding the number carried before multiplying. For instance, in example 4A, the children might add the 4 and the 3 and then multiply 7 by 6 instead of multiplying 6 times 3 and then adding 4. The most effective way to combat such errors is to ensure that the children understand what they are doing. As you work through exercise 4A, for example, you might use commentary such as: "6 times 8 is 48; write down the 8, remember the 4. 6 times 3 is 18, plus 4 is 22. Write down the 22."

Using the Ideas

In exercises 1 to 4, copy each equation and give the missing number.

1. A $3\times12=(3\times10)+(3\times n)$ 2
 B $3\times12=30+n$ 6
 C $3\times12=n$ 36

2. A $4\times21=(4\times n)+(4\times1)$ 20
 B $4\times21=n+4$ 80
 C $4\times21=n$ 84

3. A $3\times25=(3\times n)+(3\times5)$ 20
 B $3\times25=n+15$ 60
 C $3\times25=n$ 75

4. A $6\times14=(6\times10)+(6\times n)$ 4
 B $6\times14=60+n$ 24
 C $6\times14=n$ 84

5. Copy each exercise and give the missing number.

A	B	C	D	E	F
46 ×3	64 ×7	18 ×4	43 ×5	84 ×2	65 ×3
18	[] 28	32	15	8	[] 15
120	420	40	[] 200	160	180
[] 138	448	[] 72	215	[] 168	195

6. Find the products.

A 48 ×2 = 96	B 32 ×6 = 192	C 16 ×4 = 64	D 14 ×6 = 84
E 35 ×9 = 315	F 62 ×4 = 248	G 53 ×8 = 424	H 71 ×3 = 213
I 55 ×8 = 440	J 46 ×7 = 322	K 39 ×2 = 78	L 18 ×5 = 90

7. Solve the equations.
 A $4\times6\times3=n$ 72
 B $5\times7\times4=n$ 140
 C $3\times6\times2=n$ 36
 D $9\times4\times5=n$ 180
 E $6\times7\times6=n$ 252
 F $8\times7\times4=n$ 224
 G $7\times2\times7=n$ 98
 H $9\times8\times7=n$ 504

think

Draw a figure like the one below, only larger. Now try to color this "map" with just 4 colors. All bordering regions should be different colors.

See Using the Exercises.



More practice, page A-17, Set 31



Follow-up

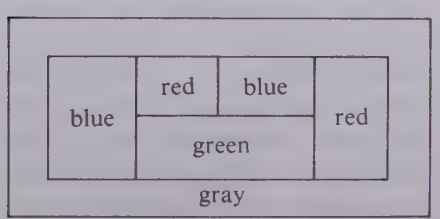
Encourage children to make their own function tables. You might put four or five on task cards for the children to exchange. The answers could be labelled on the back of each card or on a separate answer key.

Duplicator Masters, page 33
Workbook, page 54
Skill Masters, page 33

Using the Exercises

Note that the selection of problems on page 185 leads the child gradually into the independent use of the algorithm. Assign these exercises as independent work. Aid any child who needs help, and allow time for further discussion of the algorithm when checking the papers.

Many combinations are possible for the *Think* problem. One example:



Assignments (page 185)
Minimum: 1–4, 6. Average: 1–6.
Maximum: 1–7.

PAGES 186–187

Objective

Given a problem calling for multiplication of a 3- or 4-digit factor by a 1-digit factor, the child will be able to find the product by using the multiplication algorithm.

Preparation

To prepare for this lesson, provide the children with oral practice in finding products such as 5×40, 4×30, 8×50, 5×300, 3×200, 2×400, 5×700, 7×9000, 2×3000, 6×8000, etc. However, remember that the preparation should serve as a stimulus for the lesson and should not extend more than four or five minutes.



How can we multiply with a 3-digit factor?

Discussing the Ideas

1. Can you explain this method for finding this product?
See Discussion.

$$\begin{array}{r} 256 \\ \times\ 3 \\ \hline \end{array} \longrightarrow \begin{array}{r} 200 \\ 50 \\ 6 \\ \times\ 3 \\ \hline 18 \\ 150 \\ 600 \\ \hline \end{array}$$

What is the product? → 768

2. Explain each step and give the missing numbers.

Step 1	Step 2	Step 3	Step 4
$\begin{array}{r} 347 \\ \times\ 5 \\ \hline \end{array}$ 35	$\begin{array}{r} 347 \\ \times\ 5 \\ \hline 35 \end{array}$ 200	$\begin{array}{r} 347 \\ \times\ 5 \\ \hline 35 \\ 200 \end{array}$ 1500	$\begin{array}{r} 347 \\ \times\ 5 \\ \hline 35 \\ 200 \\ 1500 \end{array}$ 1735
5×7	5×40	5×300	$35+200+1500$

3. Explain the shortcut shown below. See Discussion.

$$\begin{array}{r} 436 \\ \times 4 \\ \hline 24 \\ 120 \\ 1600 \\ \hline 1744 \end{array} \Rightarrow \begin{array}{r} {\scriptstyle 12}436 \\ \times 4 \\ \hline 1744 \end{array}$$

186

Discussion

Give the children an opportunity to study the example in discussion exercise 1. Make sure they realize how the partial products are added to find the final product. In exercise 2, use the colored shadings to guide the discussion step by step through the algorithmic process. Point out the partial product for each step and how they are added in the last step. Finally, relate the shortcut in exercise 3 to the method developed in exercise 2. If children understood the use of the algorithm with a 2-digit factor, extending this use to a 3- or 4-digit factor should not cause difficulty. Present several examples similar to the one in exercise 3 and have children explain each step as they work the problems.

Using the Ideas

1. Copy each exercise and give the missing number.

A	B	C	D	E
254 × 3 = 762	621 × 7 = 4347	523 × 4 = 2092	562 × 5 = 2810	142 × 6 = 852
12	▓ 7	12	10	▓ 12
150	140	▓ 80	300	240
600	4200	2000	▓ 2500	600

2. Find the products.

A 213 × 3 = 639	B 416 × 2 = 832	C 532 × 4 = 2128	D 618 × 3 = 1854	E 729 × 5 = 3645	F 843 × 6 = 5058
G 348 × 4 = 1392	H 217 × 5 = 1085	I 618 × 2 = 1236	J 521 × 4 = 2084	K 603 × 9 = 5427	L 735 × 2 = 1470
M 642 × 3 = 1926	N 815 × 6 = 4890	O 921 × 3 = 2763	P 926 × 2 = 1852	Q 138 × 6 = 828	R 507 × 8 = 4056

★

S 7651 × 7 = 53557	T 3716 × 4 = 14864	U 5481 × 7 = 38367	V 6567 × 6 = 39402	W 8035 × 5 = 40175	X 7604 × 4 = 30416

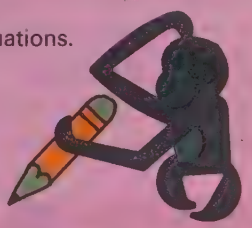
3. Find the products.

A $24 \times 6 \times 3$ 432 B $5 \times 47 \times 3$ 705 C $4 \times 56 \times 2$ 448 D $7 \times 25 \times 4$ 700

think

Study the pattern.
Then solve the equations.

$(1 \times 9) + 2 = 11$
$(12 \times 9) + 3 = 111$
$(123 \times 9) + 4 = 1111$
$(1234 \times 9) + 5 = 11{,}111$
$(12\,345 \times 9) + 6 = n$
$(123\,456 \times 9) + 7 = n$



Now check your answers.

More practice, page A-18, Set 32

Using the Exercises

On page 187, note that exercise 1 does not use the shortcut algorithm. It is essential that children understand the steps developed in exercise 1 before they attempt to develop skill in using the shortcut. Encourage them to use the shortcut in exercise 2 only if they can do so with understanding and confidence. However, it is hoped that most will use the shortcut method; this will facilitate the multiplication of two 2-digit factors, which is soon to be treated. Although the three factors in each part of exercise 3 may be multiplied in any order, the children might run into the difficulty of multiplying two 2-digit factors unless they work from left to right. When the children have finished, allow time for checking answers and for explaining several of these exercises.

Many children will solve the *Think* problem by observing the pattern. Encourage very capable children to extend the pattern if they can.

Assignments (page 187)

Minimum: 2A–L. Average: 1–2R. Maximum: 1–3.

Follow-up

A follow-up appropriate for this or any of the remaining lessons is the method of checking multiplication by casting out nines. Add the digits of the multiplicand, the multiplier, the product. Cast out nines. Find the product of the digit for the multiplicand and the digit for the multiplier. Combine the digits of this product, again casting out nines; if the multiplication problem is correct this digit should be the same as the digit resulting from the combination of digits in the product of the problem.

Examples:

(a)

$521 \rightarrow 5+2+1=8$; $\times\ 4 \rightarrow 4$; $2084 \rightarrow 14 \rightarrow 1+4=$ (5)

8 ×4 ; $32 \rightarrow 3+2=$ (5)

(b)

$603 \rightarrow 0$; $\times\ 9 \rightarrow 0$; $5427 \rightarrow 18 \rightarrow 1+8=9 \rightarrow$ (0) ↔ (0)

0 ×0

(c)

$6567 \rightarrow 6+5+6+7=24 \rightarrow 6$

$\times 6 \rightarrow 6$

$39402 \rightarrow 3+9+4+0+2 \rightarrow 18 = 1+8=9 \rightarrow$ (0)

6 ×6 ; $36 \rightarrow 3+6 = 9 \rightarrow$ (0)

Resources for Active Learning

Mathematics in Modules, WN13, Addison-Wesley.

Duplicator Masters, page 34
Workbook, page 55
Skill Masters, page 34

PAGES 188–189

Objectives

Given short problems which require multiplication with one 2-digit factor, the child will be able to solve the problem by finding products.

Given problems which relate to a bar graph, the child will be able to solve the problems by reading the bar graph and multiplying.

Preparation

To prepare for this lesson, you might give the children oral problems similar in structure to those on page 188. For example:

If 1 box holds 20 erasers,
then 5 boxes hold how many erasers?

If each classroom has 30 desks,
then 8 classrooms have how many desks?

If 1 film lasts 50 minutes,
then 6 films would last how many minutes?

Notice that the multiplication required for these problems involves multiples of 10, so children should be able to work them without pencil and paper. As usual, the preparation should be kept lively and short.

Short Picture Problems



Short Story Problems



1 Cost 6 cents each.
Bought 42.
Spent how much? $2.52 (252¢)



2 Cost 42 cents each.
Bought 6.
Spent how much? $2.52 (252¢)



3 2 dozen in a box.
Had 8 boxes full.
How many in all? 192 (16 doz.)

4 12 apples per kilogram.
7 kilograms.
How many apples? 84

5 5 school days a week.
36 weeks of school.
How many days? 180



6 12 girls, 15 boys.
Admission: 5 cents each.
How much for all? $1.35 (135¢)

188

Discussion

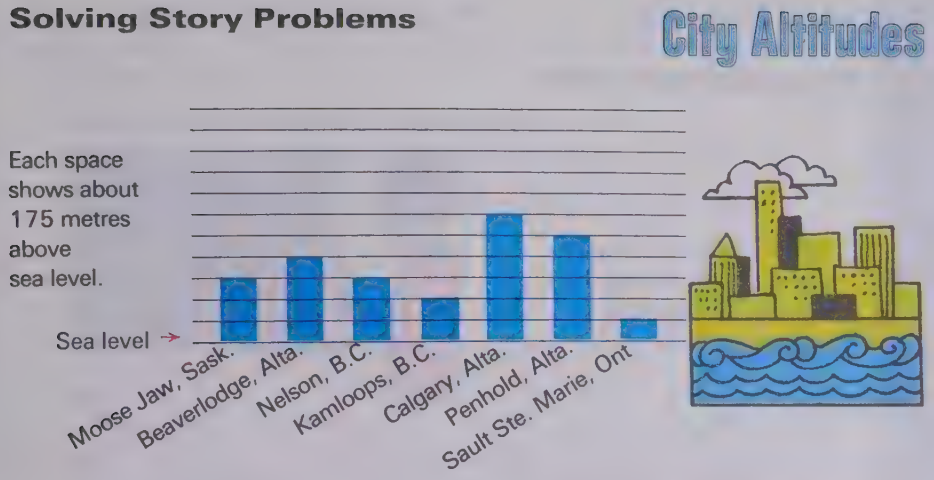
You might use the picture problems on page 188 as a basis for discussion and have children relate stories for each one before you have them explain how to work the problems. The story problems, however, might better be used as independent work.

When the children have completed the exercises, allow time for further discussion of these problems.

Assignments (page 188)

Minimum: 1–3, oral; short stories 1–6. Average: All. Maximum: All

Solving Story Problems



1. Study the bar graph. Then make a table like the one on the right. Give the missing altitudes.

City	Altitude
	(About)
Moose Jaw	525
Beaverlodge 700	
Nelson 525	
Sault Ste. Marie	175
Calgary 1050	
Penhold 875	
Kamloops 350	

2. About how much higher is Penhold then Nelson ? 350 m
3. The highest large city in the United States is Santa Fe, New Mexico. It is 2085 metres above sea level. How much higher is Santa Fe than Kamloops ? 1735 m
4. Armstrong, B.C., is about 350 metres above sea level. Jasper, Alta., is at an altitude about 3 times as high as Armstrong. What is the altitude of Jasper ? 1050 m
5. The surface of the Dead Sea in Palestine is 388 metres below sea level. How much higher is Calgary then the surface of the Dead Sea ? 1438 m
6. ★ The Dead Sea at its deepest point is 393 metres deep. What is the difference in altitude between the bottom of the Dead Sea and Kamloops, B.C. ? 1131 m

Using the Exercises

Treat page 189 primarily as an exercise in reading a bar graph. Study the graph with the children. Observe with them that Sault Ste. Marie is shown at an altitude of 175 metres and that this is equal to one of the spaces shown in the graph.

Have the children make a table on their paper as directed in exercise 1. If necessary, help children complete their tables. They must count spaces in order to determine the altitudes of the various cities. For example, to find the altitude of Beaverlodge, the children must count up 4 spaces and multiply 4×175. Encourage the children to check an almanac or encyclopedia to determine the altitude of their own, or a nearby city, and discuss geographic features which account for varying altitudes.

Following this discussion, ask the children to complete the page. Note that exercise 5 is a challenging problem because, in a sense, it involves a negative number. That is, the fact that the Dead Sea is 388 metres below sea level and that Kamloops is 350 metres above sea level indicates that the children must *add* these to get the difference.

Assignments (page 189)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–6.

Follow-up

Encourage the children to make up problems of their own based on the bar graph on page 189. These may be addition and subtraction problems as well as multiplication problems.

You might also suggest that they make a bar graph on their own or as a class project. A bar graph showing the height, in centimetres, of several children might be appropriate; or a graph showing the number of children with blue eyes, green eyes, brown eyes, gray eyes; or a graph developed from the birth dates of the children; and so on.

Duplicator Masters, page 35

PAGES 190–191

Objective

Given a multiplication problem in which one factor is a 2-digit number and the other is a multiple of 10, the child will be able to find the product.

Preparation

An oral practice in multiplying single-digit numerals by multiples of 10 would be an appropriate preparation for this lesson. However, you might choose to begin immediately with the investigation.

Investigation

Read the investigation question with the children. Then let them independently study the examples in the text to answer the question. It would be helpful to write similar problems on the chalkboard both for those who finish quickly and as an expansion of this investigation.

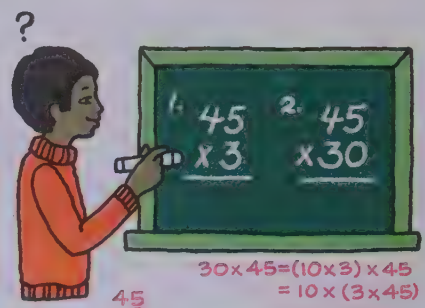
1.	37	2.	37	1.	82	2.	82
	× 4		× 40		× 7		× 70

Direct the children to ask themselves the same question for these exercises as that asked in the investigation. An investigation such as this should not require a great deal of time.

● *How is 30 × 45 like 3 × 45?*

Investigating the Ideas

Jim thought he could find the first product. He wasn't sure of the second.



30 × 45 = (10 × 3) × 45
= 10 × (3 × 45)
= 10 × 135
= 1350

45 × 3 = 135

See Investigation and Discussion.

? Can you find the first product and show on your paper how to use it to find the second product?

Discussing the Ideas

1. Find each of these products.
 - A 10 × 45 450
 - B 10 × 2 × 45 900
 - C 20 × 45 900
 - D 10 × 3 × 45 1350
 - E 30 × 45 1350
 - F 40 × 45 1800

2. A Explain step 1 in the diagram. 20 × 43
 B What principle is used for step 2? Grouping
 C Explain steps 3 and 4.
 D Solve: 43 × 20 = *n* 860
 E Explain this statement:

 [1] (10 × 2) × 43
 [2] 10 × (2 × 43)
 [3] 10 × 86
 [4] 860

 Since 43 × 2 = 86, we know that 43 × 20 = 860.
 Since 20 = 10 × 2, 43 × 20 must be 10 × (43 × 2) or 10 × 86.

3. Can you give an easy rule for multiplying by the 2-digit multiples of ten: 10, 20, 30 . . . ?
 Sample answer: Put a D in the ones' place and multiply by the number of tens. Or multiply by the number of tens and annex a 0.



Discussion

One of the main purposes of this lesson is to teach children how to write the product of a 2-digit factor and a 2-digit multiple of 10 on their paper.

Generally, the children have no trouble *finding* the product for such an exercise; rather, the difficulty is in becoming accustomed to the way they should *write* the product on their papers. For this reason, some children may have had difficulty with the investigation. Use these exercises to point out that the zero in the product is the result of multiplying by 10. Point out that, to derive the product 30 × 45 from the product 3 × 45, one needs merely to observe that 30 × 45 is 10 times as great as 3 × 45. This inference comes from the ideas presented in exercise 2. Thus, when the children work an exercise such as 43 × 20, they think of multiplying first by 10 and then by 2. When this exercise is written in vertical notation, they need simply to write the 0 in the ones' place and multiply by 2. Although children might find the product 43 × 20 by thinking 43 × 2 and multiplying that product by 10, they should learn and understand the method of writing this on paper so that they will be able to use the algorithm efficiently in the next lesson.

Using the Ideas

1. Find the products.
 - A Since $34 \times 2 = 68$, we know that $34 \times 20 = n$. 680
 - B Since $17 \times 3 = 51$, we know that $17 \times 30 = n$. 510
 - C Since $36 \times 7 = 252$, we know that $36 \times 70 = n$. 2520

2. Find the products.

A	36 × 4 = 144	36 × 40 = 1440	B	53 × 2 = 106	53 × 20 = 1060	C	78 × 3 = 234	78 × 30 = 2340
D	93 × 5 = 465	93 × 50 = 4650	E	86 × 4 = 344	86 × 40 = 3440	F	75 × 6 = 450	75 × 60 = 4500
G	47 × 8 = 376	47 × 80 = 3760	H	56 × 4 = 224	56 × 40 = 2240			

3. Study the example. Then find the products.

Step 1	Step 2	Step 3
47	^{3}47	^{3}47
× 50	× 50	× 50
0	50	2350

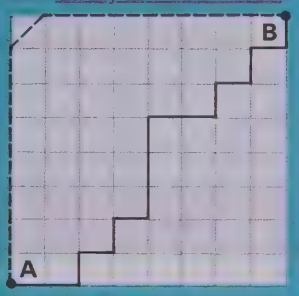
A	63 × 20 = 1260		B	75 × 30 = 2250		C	81 × 40 = 3240
D	67 × 30 = 2010		E	45 × 60 = 2700		F	54 × 80 = 4320
G	76 × 20 = 1520		H	49 × 60 = 2940		I	38 × 50 = 1900
J	67 × 90 = 6030		K	83 × 40 = 3320		L	92 × 50 = 4600

think

In the figure below, which path between **A** and **B** is shorter, the solid path or the dashed path? Dashed

Explain your answer.

See Using the Exercises.





More practice, page A-18, Set 33

Using the Exercises

Page 191 presents a progression of exercises reflecting the development of the procedures on page 190. You might consider it necessary to discuss the example in exercise 3 before you assign these exercises as independent work. Be sure that the children understand that the reason for writing the first zero as indicated in each example is that they multiply first by 10 and then by 5.

Following the completion of these exercises, allow time for checking papers and answering any questions the children may have.

Since the paths shown in the *Think* problem are so close to the same length, expect children to count each and say something like this: "The solid path is 16 units and the dashed path is 14 units and 1 shortcut, so the dashed path must be shorter."

Assignments (page 191)
Minimum: 1–3F. Average: 1–3.
Maximum: 1–3.

Duplicator Masters, page 36
Workbook, page 56
Skill Masters, page 36

PAGES 192–193

Objective

Given a multiplication problem with two 2-digit factors, the child will be able to find the product by using the multiplication algorithm.

Preparation

Since most children need frequent practice with the basic facts, you might want to use a warm-up activity such as a mental chain game like the following.

"Start with 35 . . . Divide by 7 . . . Multiply by 8 . . . Divide by 20 . . . Your answer is ? " (2)

"Start with 16 . . . Divide by 2 . . . Multiply by 30 . . . Divide by 6 . . . Your answer is ? " (40)

"Start with 64 . . . Divide by 8 . . . Multiply by 4 . . . Subtract 2 . . . Divide by 3 . . . Your answer is ? " (10)

"Start with 81 . . . Divide by 9 . . . Subtract 4 . . . Multiply by 7 . . . Add 1 . . . Divide by 4 . . . Multiply by 9 . . . Your answer is ? " (81)

Investigation

You might have the children work in groups of two or three for this investigation so that they can share ideas on how to find 35×43. Explain that the products in the arrow should help them figure out the product in the square. Write similar sets of problems on the chalkboard for those who finish quickly. For example:

$$\begin{array}{rrcr} 75 & 75 & \rightarrow & 75 \\ \times\ 3 & \times 50 & & \times 53 \\ \hline 82 & 82 & \rightarrow & 82 \\ \times\ 9 & \times 10 & & \times 19 \end{array}$$

● *Let's explore two 2-digit factors.*

Investigating the Ideas

Can you find each of these products?
See Investigation and Discussion.

You already know how to find these products.

$$\begin{array}{r} 43 \\ \times 5 \\ \hline 215 \end{array} \qquad \begin{array}{r} 43 \\ \times 30 \\ \hline 1290 \end{array}$$

They will help you find this product.

$$\begin{array}{r} 43 \\ \times 35 \\ \hline 1505 = (215+1290) \end{array}$$

Discussing the Ideas

1. Explain each step in the example below and give the missing numbers.

Multiply first by 3.	Then by 20	Add the partial products
Step 1	Step 2	Step 3
$\begin{array}{r} 46 \\ \times\ 2 \\ \hline 138 \end{array}$	$\begin{array}{r} 46 \\ \times\ 3 \\ \hline 138 \\ 920 \end{array}$	$\begin{array}{r} 46 \\ \times\ 23 \\ \hline 138 \\ 920 \\ \hline 1058 \end{array}$
3×46	20×46	$138 + 920$

2. Find this product and check your answer with your teacher.

$$\begin{array}{r} 42 \\ \times\ 36 \\ \hline 1512 \end{array}$$

192

Discussion

Have some children explain how they found the products in the investigation. Then discuss the example in exercise 1. It would be helpful to write on the chalkboard the products and sum in the red screen. Be sure the children understand that they are to think about multiplying in parts, first by 3 and then by 20. Point out the first step, which shows 3×46; then the second, which shows 20×46; and finally the third, which shows finding the sum of these two products.

After discussing this example, have the children explain another, such as 75×63. It may be beneficial to have them write the steps on the chalkboard as they explain the example. If necessary, select other problems from page 193 for similar explanations. You might add to exercise 2 other products for children to try. Then check and discuss the answers. Stress how the multiplication-addition principle is used in multiplying by parts and adding the partial products.

Using the Ideas

1. Find the products.

A 46 × 23 = 1058	B 54 × 12 = 648	C 71 × 24 = 1704	D 32 × 14 = 448	E 65 × 24 = 1560	F 56 × 57 = 3192	G 28 × 75 = 2100
H 61 × 78 = 4758	I 47 × 36 = 1692	J 55 × 26 = 1430	K 34 × 96 = 3264	L 48 × 48 = 2304	M 39 × 93 = 3627	N 57 × 61 = 3477
O 99 × 22 = 2178	P 68 × 45 = 3060	Q 15 × 48 = 720	R 49 × 60 = 2940	S 69 × 96 = 6624	T 25 × 68 = 1700	U 76 × 67 = 5092

2. Find the products.

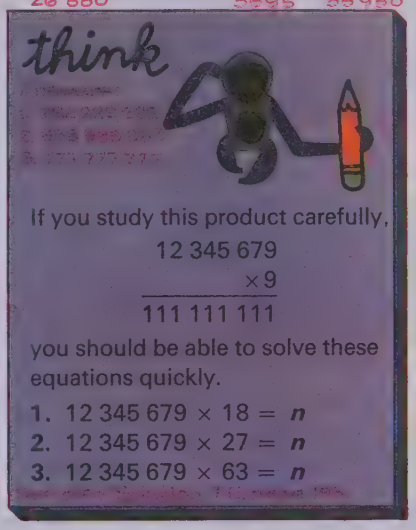
A 357 → 357 × 3 = 1071; 357 × 30 = 10 710

B 672 → 672 × 4 = 2688; 672 × 40 = 26 880

C 719 → 719 × 5 = 3595; 719 × 50 = 35 950

3. Find the products.

A 654 × 2 = 1308	654 × 30 = 19 620	→	654 × 32 = 20 928
B 467 × 5 = 2335	467 × 20 = 9340	→	467 × 25 = 11 675
C 721 × 3 = 2163	721 × 40 = 28 840	→	721 × 43 = 31 003
D 264 × 4 = 1056	264 × 40 = 10 560	→	264 × 44 = 11 616



4. Find the products.

A 391 × 12 = 4692; B 283 × 41 = 11 603; C 465 × 26 = 12 090; D 802 × 81 = 64 962; E 643 × 56 = 36 008; F 839 × 83 = 69 637

More practice, page A-19, Set 34

193



Using the Exercises

On page 193, select parts of exercise 1 for the children to do independently. Although exercises 2 and 3 carefully lead the children into the use of the algorithm with a 3-digit factor, you might use parts of them as a basis for discussion.

Solution, Think, page 193

Some children should be able to solve the *Think* problem by applying their understanding of multiplication. If 111 111 111 is the product for the given multiplicand of 9, then 18 times that multiplicand will be 2 times that product, since 18 is 2 times 9.

12 345 679 × 18 = (12 345 679 × 9) × 2. Similarly, 12 345 679 × 27 = (12 345 679 × 9) × 3.

Assignments (page 193)

Minimum: 1. Average: 1–4.
Maximum: 1–4.

Follow-up

Children might enjoy working on extensions of the *Think* problem, such as the following.

12345679 × 36 = 74074074 + 37037037 = 444444444	12345679 × 72 = 24691358 + 86419753 = 888888888

Let them discover on their own the relationship between the digit in the answer and the multiplier (which is a multiple of 9). Note also that "8" is omitted from the other factor.

Duplicator Masters, pages 37–38
Workbook, pages 57–58
Skill Masters, pages 37-38

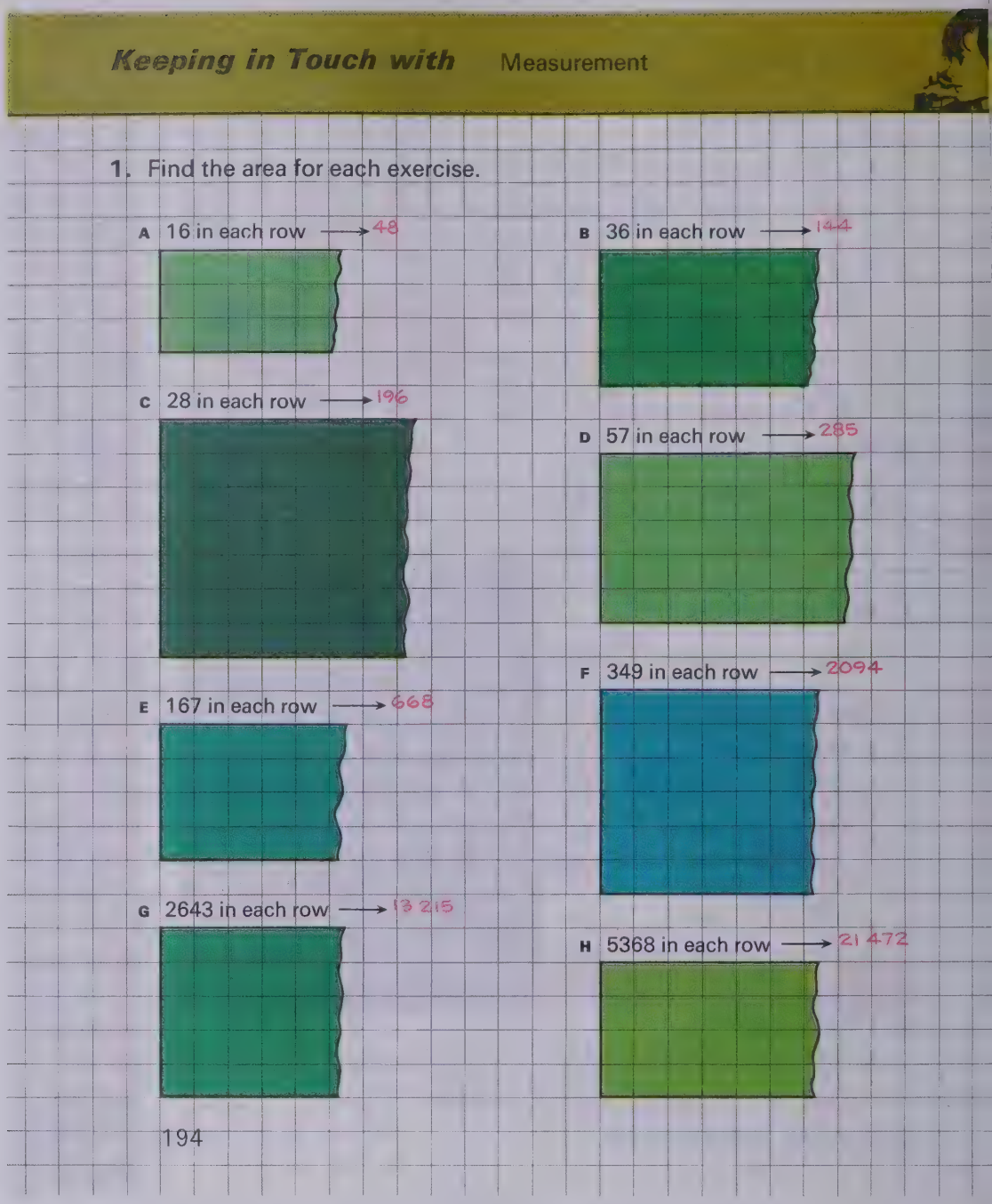
PAGES 194–195

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

To prepare for this lesson, show a large rectangular grid. Ask the children what is meant by unit of area and how they might find the area of the rectangle in terms of the unit on the grid. Compare the method of counting each unit with that of using multiplication. Also, show a large box marked in units of volume as in the pictorial representation on page 195.

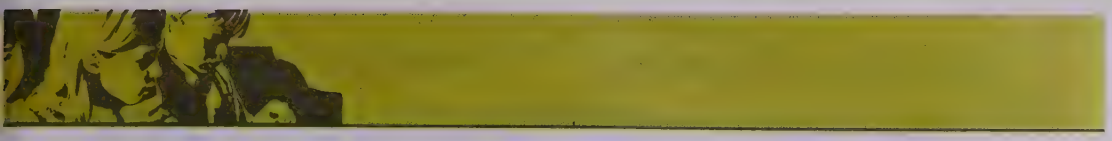


Discussion

According to the ability and needs of your class, conduct a discussion to help children realize how multiplication can be used to find the area for each exercise. Since only one dimension is given for each area, the children must discover how counting the number of rows and applying multiplication will enable them to find the area. You might ask some children to attempt these exercises on their own, challenging them to write a multiplication equation for each exercise. Others will need more clear-cut guidance before they realize how multiplication is applicable in these exercises.

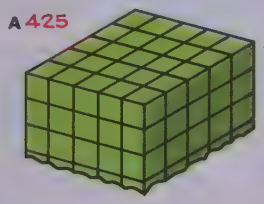
Exercise 2, which develops the application of multiplication to volume, should be treated similarly. Again, challenge some children with a question such as the following and direct them to try the exercises on their own: "Can you write a multiplication equation to show how to find the volume in each exercise?"

Help other children see the application of multiplication by leading a more carefully guided discussion. A few children may need actual manipulation with units of volume to build cubic objects. However, it is hoped that the pictorial representation is sufficient for most children at this level. In exercises 2E and 2F,

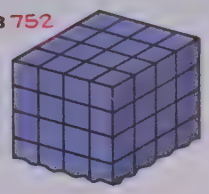


2. Give the volume for each exercise.

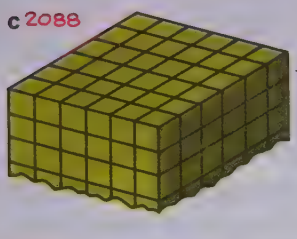
A 425



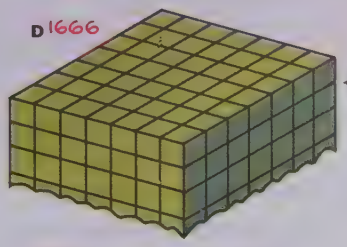
B 752



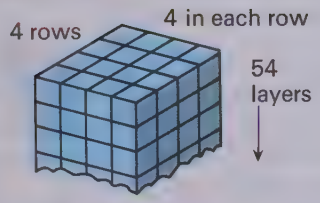
C 2088



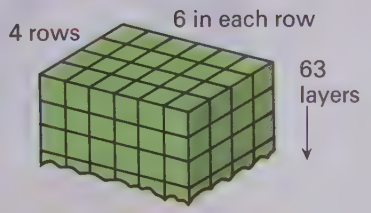
D 1666



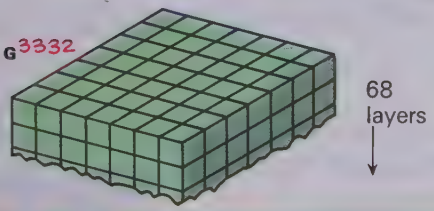
E 864



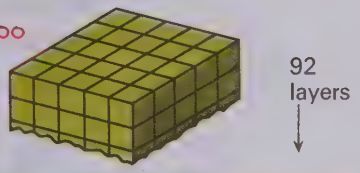
F 1512



G 3332



H 2300





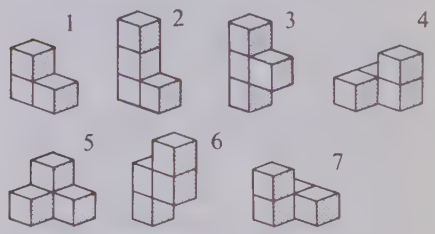
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ACTIVITY CARD 8 Page 351

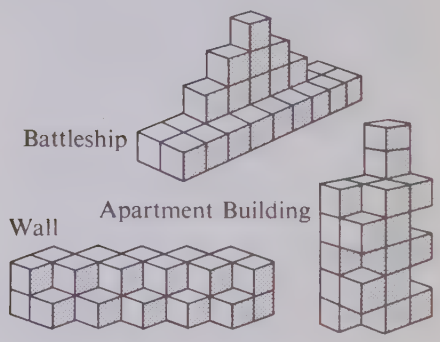
children must find the number of cubic units in each layer and then find the volume. In exercises 2G and 2H, children must count from the picture the number of units in a row and the number of rows in a layer before they can multiply to find the volume.

Follow-up

Many children may enjoy rearranging the pieces of a cube puzzle like the Soma cube devised by Danish writer Piet Hein. The seven pieces of the puzzle can be made by gluing 27 small cubes together, as shown in the illustrations. They can be put together in such a way as to make a 3-by-3-by-3 cube. The figures in the second group show some of the other forms the children can make. Encourage them to invent shapes using all the pieces. The children may enjoy giving their constructions names.



The Seven Soma Pieces



Sample Soma Constructions

Resources for Active Learning

Nuffield Project: *Computation and Structure* 3, "... Money, weights, and measures," pp. 56–60, Wiley.

Objective

Given picture and word problems which require any of the basic operations (except long division), the child will be able to solve the problem by choosing and performing the correct operation.

Preparation

As a preparation for this lesson and as a review of the basic facts, you might give the children some oral If-Then problems similar to those on page 196; for example:

If there are 54 pages in 9 folders, then how many pages are in 1 folder? (6)

If 1 wagon has 4 wheels, then 20 wagons have how many wheels? (80)

If 1 bus holds 40 children, then 7 buses will hold how many children? (280)

Short Picture Problems



Short Stories

1 Auditorium. 32 rows of seats. 25 seats in each row. How many seats? 800

2 Driving. 72 kilometres each hour. 25 hours. How far? 1800 km



3 74 children. 38 girls. How many boys? 36



4 17 teams. 12 on each team. How many in all? 204



5 34 sacks of cement. Weigh 40 kilograms each. What is the total weight? 1360 kg

6 468 girls. 397 boys. 39 teachers. How many people in all? 904

7 15 shelves. 38 books on each shelf. How many books? 570





8 Had 47 cents. Earned 15 cents. Spent 37 cents. Can buy how many 5-cent candy bars now? 5

196

Discussion

One of the main purposes of this problem set is to provide children with practice in multiplication with two 2-digit numerals, although other operations are reviewed. The three If-Then problems would best be developed orally. Have children give stories for each picture problem, and then work the problem on the chalkboard. The short story problems review all of the operations; these might be assigned as independent work. However, it will be important to discuss the children's solutions. Note that problem 8 deserves special discussion since it involves addition, subtraction, *and* division.

Assignments (page 196)

Minimum: 1–3, oral; even-numbered short stories. Average: 1–11. Maximum: 1–11.

Solving Story Problems



1. For some records, the turntable must be set to turn the record 45 times each minute (45 revolutions per minute). If it takes 3 minutes to play a "45 record," how many times does it turn around? 135

2. Some records play best when they go around about 33 times each minute. These are called long-playing (L.P.) records. If an L.P. takes 27 minutes to play, how many times does it go around? 891

3. Jan played 12 records one Saturday. Each one lasted 18 minutes. How long did Jan use her record player that day? 216 min

4. Some older records turn 78 times each minute. How many turns would a "78 record" make in 5 minutes? 390

★ 5. Betty played 6 records, and each one lasted 3 minutes. If each was a "45 record," how many times did the turntable go around while the records were playing? 810

More practice, page A-19, Set 35

Using the Exercises

Before assigning page 197, be sure the children understand the meaning of the word *revolution* as explained in problem 1. If you have a circular object in the classroom (a paper plate, for example), you can demonstrate a revolution; if you have a record player available, you could demonstrate the various speeds by setting the record player at $33\frac{1}{3}$, 45, or 78. Many of the children will be familiar with record players and their various speeds.

When the children complete the exercises, allow time for checking papers and discussing solutions.

Assignments (page 197)

Minimum: 1–4, oral. Average: 1–4. Maximum: 1–5.

Follow-up

Various events, imaginary or otherwise, lend themselves to situations in which multiplication and division problems arise. For example, planning a Valentine Day party might pose the following problems for more capable children.

1. Miss Bartlett wants to buy some large manila envelopes for the 25 children in her class to use in making valentine bags. The clerk says a gross (12 dozen) costs $3.95, or she can buy them for 3 cents each. Assume that Miss Bartlett can use any extra envelopes; which way should she buy them?
2. Red construction paper costs 4¢ a sheet or three sheets for 10¢ What is the least Miss Bartlett could pay for 25 sheets of red paper?
3. The children have volunteered to bring cupcakes for their party. If 4 children bring a dozen cupcakes each, will there be enough for each of the 25 children and the teacher to have 2 cupcakes apiece? If not, how many more cupcakes will be needed?

Other children might create less involved problems of their own.

Workbook, page 59

PAGES 198–199

Objective

Given multiplication exercises with factors of 2 or 3 digits, the child will be able to find the product.

Preparation

To prepare for this lesson, give the children some mental chain games such as these:

"Start with 85 . . . Subtract 4 . . . Divide by 9 . . . Multiply by 3 . . . Add 1 . . . Divide by 4 . . . Your answer is ? " (7)

"Start with 77 . . . Subtract 5 . . . Divide by 8 . . . Multiply by 4 . . . Divide by 6 . . . Your answer is ? " (6)

"Start with 15 . . . Divide by 3 . . . Add 3 . . . Multiply by 3 . . . Divide by 4 . . . Divide by 2 . . . Your answer is ? " (3)

"Start with 6 . . . Multiply by 8 . . . Subtract 3 . . . Divide by 9 . . . Add 2 . . . Multiply by 4 . . . Add 2 . . . Divide by 5 . . . Your answer is ? " (6)

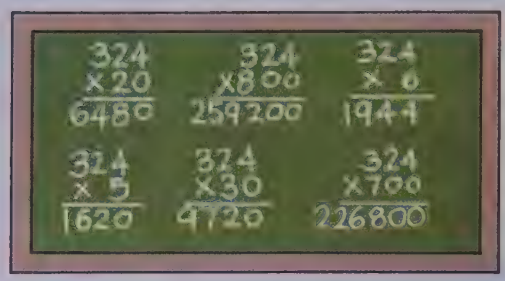
Investigation

Read through the investigation question with the children, pointing out that they can use the work shown on the illustrated chalkboard in any way they wish, to obtain the product 324 × 736 without multiplying. You may also need to caution them that they do not have to use all the products that are shown; they are simply to choose those they can use to help them find the desired product. If any children have difficulty getting started, you might suggest that they think of how they have used the multiplication-addition principle in finding products in previous lessons; but let the children work on their own insofar as possible.

Let's explore larger products.

Investigating the Ideas

This chalkboard shows some products when 324 is one of the factors.



? Can you find this product without doing any multiplying?

$$\begin{array}{r} 324 \\ \times\ 736 \\ \hline 238464 \end{array}$$

See Investigation and Discussion.

Discussing the Ideas

1. Explain how you used the products shown on the chalkboard to find 324 × 736. See Discussion.

2. Find as many of these products as you can. Then check your answers with your teacher.

A $245 \times 3 = 735$; $245 \times 30 = 7350$; $245 \times 300 = 73500$; $245 \times 333 = 81585$

B $162 \times 4 = 648$; $162 \times 40 = 6480$; $162 \times 400 = 64800$; $162 \times 444 = 71928$

3. Explain the steps below. Give the missing numbers.

Step 1	Step 2	Step 3	Step 4
514×263	514×263	514×263	514×263
1542	30840	102800	135182



198

Discussion

When children have completed the investigation, let them discuss their answers; have volunteers show their solutions at the chalkboard. Most children will have found the product by adding the partial products 324 × 6, 324 × 30, and 324 × 700. Of course, you should also recognize as correct an answer found by adding other suitable combinations such as (324 × 6), (324 × 5), (324 × 5), (324 × 20), and (324 × 700); you might wish to point out, though, that such a combination, while correct, is more time consuming.

One of the main purposes of exercise 3 is not only to help the children understand what they are doing when they find products but also to help them learn a convenient way to write them. As you discuss exercise 3, ask such questions as: "In step 2, is 514 being multiplied by 6 or by 60?" "In step 3, is 514 being multiplied by 2 or by 20, or by 200?" It would also be helpful to write and discuss this equation in relation to step 4: 263 × 514 = (200 × 514) + (60 × 514) + (3 × 514).

Using the Ideas

Find the products.

1. 50 × 36 = 1800	2. 48 × 90 = 4320	3. 327 × 5 = 1635	4. 619 × 7 = 4333	5. 823 × 9 = 7407	6. 695 × 8 = 5560
7. 824 × 9 = 7416	8. 907 × 50 = 45 350	9. 762 × 48 = 36 576	10. 929 × 88 = 81 752	11. 654 × 28 = 18 312	12. 802 × 43 = 34 486
13. 765 × 563 = 430 695	14. 807 × 444 = 358 308	15. 700 × 64 = 44 800	16. 930 × 607 = 564 510	17. 920 × 307 = 282 440	18. 526 × 710 = 373 460
19. 621 × 243 = 150 903	20. 517 × 444 = 229 548	21. 823 × 615 = 506 145	22. 264 × 222 = 58 608	23. 575 × 314 = 180 550	
24. 605 × 333 = 201 465	25. 316 × 634 = 200 344	26. 824 × 111 = 91 464	27. 967 × 121 = 117 007	28. 542 × 596 = 323 032	
29. 568 × 666 = 378 288	30. 397 × 260 = 103 220	31. 908 × 307 = 278 756	32. 967 × 384 = 371 328	33. 659 × 555 = 365 745	

34. 803 × 327 = 262 581

35. 9 × 7 × 4 × 6 = 1512

36. 6 × 7 × 8 × 5 = 1680

37. 4 × 9 × 8 × 6 = 1728

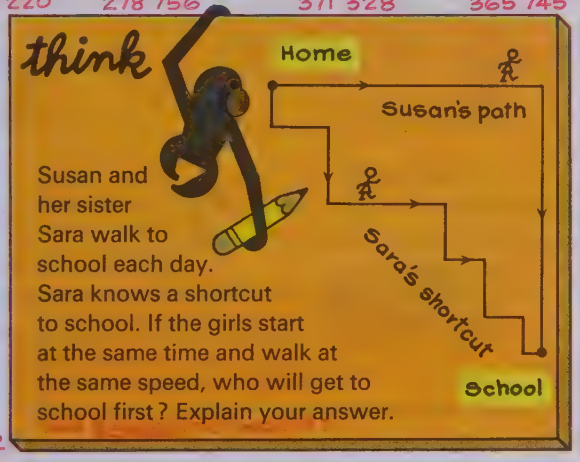
38. 8 × 8 × 8 × 8 = 4096

39. 8 × 7 × 326 = 18256

40. 921 × 3 × 47 = 129 861

41. 4 × 389 × 0 = 0

42. 2 × 12 × 13 = 312



More practice, page A-20, Set 36



Using the Exercises

Since there is a considerable amount of work in this section, you may not wish to assign all the exercises. Certainly, some children would require a great amount of time to work all the exercises, and the others would become bored by too much repetition of those things that they can do very well. Note that exercises 34 through 42 may be completed in several ways. Some children may naturally work from left to right, but point out to them that they have a choice of approaches.

Solution, Think, page 199

Children may explain the *Think* problem with a statement like this: "Even though Sara goes west and south a few blocks at a time, Susan and Sara should get to school at the same time since each goes the same distance west and south."

Assignments (page 199)*

Minimum: 1–12. Average: 1–33. Maximum: 1–42.

Follow-up

To encourage children to check their work for accuracy, you might prepare a worksheet similar to the following. The appropriate corrections are shown in parentheses for your convenience, and should not be included on the worksheet.

Can you find the errors in each problem? Give the correct product for each exercise.

1. 654 × 289 = 5856(*5886*) + 52320 + 130800 = 188976(*189006*)

2. 36 × 51 = 36 + 1500(*1800*) = 1536(*1836*)

3. 725 × 316 = 4350 + 7200(*7250*) + 287500(*217500*) = 299050(*229100*)

4. 29 × 64 = 116 + 174(*1740*) = 290(*1856*)

Duplicator Masters, page 39
Workbook, page 60
Skill Masters, page 39

Objective

Given word problems which require multiplication with 2- and 3-digit factors, the child will be able to solve the problems by finding the products.

Preparation

If you use the first three exercises on page 200 as oral exercises, you might prefer to begin the lesson without preparatory activity. However, if you choose, give the children oral practice with multiplying multiples of 10 and 100. You might ask children for products such as the following: 30×12, 400×5, 300×9, 4×800, 40×12, 40×25. For some of these, allow children to use paper and pencil if they wish.

Short Picture Problems

1. IF Each SCHOOL 37 THEN 8 SCHOOL ? 296

2. IF 1 PEACHES 24 THEN 6 PEACHES ? 144

3. IF 4 CANDY 29 THEN 24 CANDY ?

Short Stories

1 48 sacks.
75 kilograms each.
How many kilograms ? 3600

2 12 eggs in a carton.
94 cartons.
How many eggs ? 1128



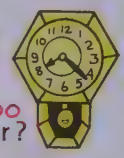


3 72 players.
8 on a team.
How many teams ? 9

4 72 teams.
8 on a team.
How many players ? 576

5 72 on each team.
8 teams.
How many players ? 576

★**7** 60 minutes in an hour.
24 hours in a day.
365 days in a year.
How many minutes in a year ? 525 600





6 640 sites in 1 campground.
Park: 54 campgrounds.
How many campsites ? 34 560

★**8** 26 rows of seats.
18 seats in a row.
295 people.
How many extra seats ? 173

More practice, page A-20, Set 37

Discussion

The three If-Then exercises are intended as oral exercises. Have children give a story for each and show the multiplication on the chalkboard. The short story problems may be assigned as independent work. However, allow ample time for discussion when the children have finished. Note that exercises 3, 4, and 5 are different problems but involve the same two numbers. Use these to remind children of the correct understanding of multiplication and division.

The starred exercises are intended as special challenges, but the entire class would benefit from an explanation of the solutions.

Assignments (page 200)

Minimum: 1–2, oral; short stories 1–6. Average: 1–2, 1–6. Maximum: All.

Solving Story Problems

Niagara Falls



Niagara Falls is a group of waterfalls located partly in Canada and partly in the United States. The area is world famous for its beauty. Electricity is produced by great generating stations located there.

The Falls is 50 metres high, and the river is nearly 1600 metres wide at the Falls. The Falls is about 25 000 years old. It moves back approximately 120 centimetres every year because of falling rocks.

1. How many centimetres is it from the base of the Falls to the top ? 5000
2. In January, 1931, nearly 70 000 metric tons of rock fell to the bottom of the American side of the Falls. Another 185 000 metric tons fell in July, 1954. What was the total number of tons that fell ? 255 000
3. The Falls were first described by Father Hennepin, a Franciscan missionary who saw them in 1678. How far have they moved back since then ? Answer will depend on year solved
4. How many metres has the Falls moved back since its beginning about 25 000 years ago ? About 30 000 metres or 30 kilometres

Using the Exercises

Page 201 may serve as a basis for discussion, or you may assign the problems as independent work after the explanatory paragraphs have been read aloud.

Assignments (page 201)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–5.

Follow-up

Encourage children to use the data on page 201 to make up problems of their own. Others may do research on dimensions of another landmark, local or national, and create problems from the data they collect. If practicable, you might have the children actually measure parts of a local statue or small landmark and let an extended class project evolve.

Resources for Active Learning

Math Activity Cards, D24, 28, 36, Macmillan. [Three multiplication problems]

Duplicator Masters, page 40
Workbook, page 61

PAGES 202–203

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Unless you prefer an oral review as a preparation, use the preparation time to create enthusiasm for the topic treated in the problems on page 203. For example, discuss the time difference between the sight of lightning and the sound of thunder, or ask the children if they have seen someone at a distance hit a golf ball before hearing the snap of the club striking the ball, or if they have been at a ball park and seen the batter strike the ball before hearing the sound of impact. You might lead this brief discussion into the opening paragraphs of page 203.

Reviewing the Ideas

Find the products.

1. 54 × 4 = 216	2. 26 × 3 = 78	3. 75 × 5 = 375	4. 82 × 2 = 164	5. 94 × 7 = 658
6. 67 × 8 = 536	7. 361 × 5 = 1805	8. 278 × 6 = 1668	9. 763 × 4 = 3052	10. 403 × 9 = 3627
11. 921 × 3 = 2763	12. 3462 × 5 = 17 310	13. 5278 × 3 = 15 834	14. 6439 × 7 = 45 073	15. 8432 × 6 = 50 592
16. 32 × 44 = 1408	17. 67 × 28 = 1876	18. 46 × 59 = 2714	19. 38 × 51 = 1938	20. 60 × 75 = 4500
21. 98 × 67 = 6566	22. 347 × 20 = 6940	23. 618 × 36 = 22 248	24. 514 × 48 = 24 672	25. 396 × 52 = 20 592
26. 821 × 95 = 77 995	27. 367 × 72 = 26 424	28. 280 × 43 = 12 040	29. 546 × 61 = 33 306	30. 904 × 55 = 49 720
31. 831 × 723 = 600 813	32. 766 × 803 = 615 098			
33. 685 × 720 = 493 200	34. 864 × 271 = 234 144			

35. 7 × 9 × 8 = 504

36. 9 × 6 × 7 × 7 = 2646

37. 27 × 2 × 314 = 16 956

38. 218 × 2 × 13 = 5668

39. 34 × 26 × 5 = 4420

40. 26 × 538 × 1 = 13 988

think

Study the pattern. Then copy the equations. Give the missing numbers.

1 × 8 = 10 − 2
2 × 8 = 20 − 4
3 × 8 = 30 − 6
4 × 8 = 40 − 8
5 × 8 = ▮▮ − ▮▮
6 × 8 = ▮▮ − ▮▮
7 × 8 = ▮▮ − ▮▮
13 × 8 = ▮▮ − ▮▮



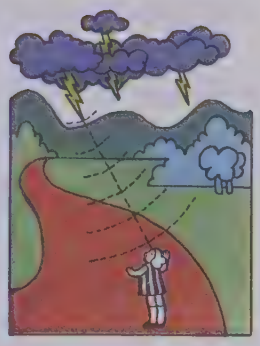
Discussion

The exercises on page 202 may serve as an evaluation instrument. However, it is not necessary that children do all of the exercises—not all at one time, certainly. Be selective in assigning these exercises; for example, if the children do two or three of the columns of exercises (such as exercises 1, 6, 11, 16, 21, 26, 31, 33), they will demonstrate their ability to work with the algorithm for problems representing various degrees of difficulty. Plan to help any children who show a need for remedial work. If necessary, have a few children explain one or two of the more difficult problems to the rest of the class.

The *Think* problem provides another interesting number pattern. This one occurs because 8 is 2 less than 10. Thus, in finding multiples of 8, multiples of 2 are subtracted from multiples of 10.

SPEED OF SOUND

Have you ever seen lightning before you heard its thunder? We often **see** something happen before we **hear** it. This is because light travels much faster than sound.



Speed of light . 300 000 kilometres per **second**
Speed of sound 1187 kilometres per **hour**

The speeds of supersonic aircraft are often given according to the speed of sound. A speed of Mach 1 is the speed of sound, Mach 2 is two times the speed of sound, Mach 3 is three times the speed of sound, and so on. Mach is the name of the scientist who made some important discoveries about sound.

1. How fast is A Mach 2? 2374 km/h B Mach 3? 3561 km/h C Mach 4? 4748 km/h
2. The X-15 rocket plane can fly at speeds greater than Mach 5. How fast is Mach 5? 5935 km/h
3. Some satellites travel at a speed that is about 23 times the speed of sound. How fast do they travel? 27 301 km/h
4. The earth travels in its orbit about the sun at a speed about 90 times the speed of sound. How fast does it travel? 106 830 km/h
5. The planet Mercury has an orbital speed that is about 146 times the speed of sound. How fast does Mercury travel in its orbit around the sun? 173 302 km/h

★ 6. Light travels 1 071 360 000 kilometres per hour. How many kilometres per hour faster is the speed of light than the speed of sound? 1 071 358 813

Follow-up

Encourage children to extend any interest they exhibit in a study of sound or of supersonic travel. They may also use the information given in the text to create problems of their own, such as other problems using the concepts of Mach 2, Mach 5, and so on.

Workbook, page 62

Using the Exercises

For page 203, continue an introductory discussion as suggested in the preparation section. Point out that in the illustration the straight dotted line shows the boy's line of sight and the curved dotted lines show sound waves. You may suggest that children work these problems together, in groups of two or three. Or you may use these problems to evaluate the children's ability to solve problems independently. In exercise 6, you may need to point out to the children that the speed of light given here is in kilometres per hour, whereas that given at the top of the page is in kilometres per second.

PAGES 204-205

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

To prepare for this lesson, use any oral drill which reviews the basic operations; for example, calling out various basic facts quickly, switching from one operation to another, stimulates children's thinking. Reading the combinations from a prepared list will help you maintain a rapid pace.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Inequalities

1. Answer **T** (true) or **F** (false).

A 374 652 > 374 562 T
B 483 007 < 483 010 T
C 921 354 = 912 354 F
D 676 767 > 767 676 F
E 208 302 < 208 203 F
F 543 210 > 543 120 T

2. Match these.

A 6 less than 243 (Answer: d)
B 243 divided by 6 c
C 243 more than 6 a
D the product of 6 and 243 e
E the difference of 243 and 6 b

(a) 6 + 243
(b) 6 − 243
(c) 243 ÷ 6
(d) 243 − 6
(e) 6 × 243

3. Find the sums and differences.

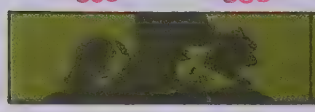
A	B	C	D	E
364	614	852	916	650
287	301	361	743	843
+521	+295	+725	+120	+921
1172	1210	1938	1779	2414

F	G	H	I	J
7642	8603	7625	9003	8080
−385	−105	−3468	−27	−808
7257	8498	4157	8976	7272

4. Find the products.

A Since 6 × 9 = 54, we know that 6 × 90 = *n*. 540
B Since 8 × 7 = 56, we know that 80 × 70 = *n*. 5600
C Since 9 × 8 = 72, we know that 900 × 8 = *n*. 7200

D	E	F	G	H	I
60	80	90	60	80	90
×5	×7	×9	×50	×70	×90
300	560	810	3000	5600	8100



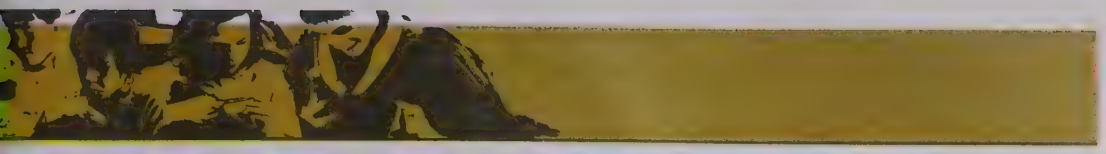
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ACTIVITY CARD 9
Page 351

Discussion

Review any topics with which the children had previous difficulty. You may want to review the addition and subtraction algorithms before you assign the exercises.

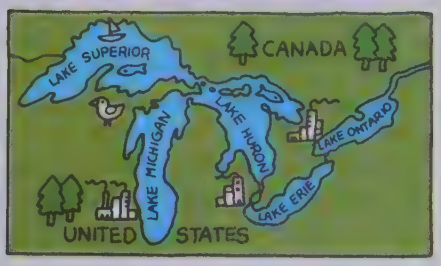
When the children have finished the exercises on page 204, allow time for checking papers and any discussion which may follow. Place particular emphasis on the discussion concerning exercise 2. The translation of the words in the first part into mathematical symbols in the second part is an extremely important skill in solving word problems. Be sure children see the relationship between the equation pairs in exercise 4.



The Great Lakes

Together the Great Lakes form the largest body of fresh water in the world.

Lake Michigan is the only lake that is entirely within the United States. The other four are shared by the United States and Canada.



Lake	Length (km)	Greatest depth (m)	Area (km^2)
Superior	563	400	82 400
Michigan	494	277	58 000
Huron	331	225	59 600
Erie	388	63	25 700
Ontario	312	241	19 700

1. What is the area of Lake Erie and Lake Ontario together? 45 400 km^2
2. What is the area of Lake Superior, Lake Michigan, and Lake Huron together? 200 000 km^2
3. What is the total area of all five lakes? 245 400 km^2
4. How much greater is the area of Lake Huron than that of Lake Erie? 33 900 km^2
5. How much deeper is Lake Superior than Lake Huron? 175 m
6. The Nile River (world's longest) is about 21 times as long as Lake Ontario. About how long is the Nile? 6552 km
7. The average depth of the Pacific Ocean is about 15 times as deep as the greatest depth of Lake Michigan. What is the average depth of the Pacific? 4155 m
8. The distance around the earth is about 129 times the length of Lake Ontario. About how far is it around the earth? 40 248 km

Using the Exercises

Use the material at the top of page 205 as a basis for discussion. Then have the children work the problems independently. The terminology used in problem 7 may cause some difficulty. Note that it concerns the average depth of the Pacific Ocean as compared to the greatest depth of Lake Michigan. (All these depths are approximations.) You might best explain the difference between *average* and *greatest* by pointing out that if the Pacific Ocean were the same depth all over, which it is not, then the average depth would be about the same as the answer to this problem. Demonstrate this for the children by placing sand in a box and mounding it into hills and gullies. Ask the children to imagine that this is the ocean floor. Point out that they can see that some parts of the floor are lower than others and therefore the ocean's *depth* would be greater in those places. Explain that for "average depth" they are to imagine that the highs and lows have been evened out. Demonstrate this idea by smoothing the sand to a level surface. Point out that some parts on the ocean floor are lower than others, so some places are deeper and some places are not as deep as the average.

Follow-up

Suggest that the children make up number games, riddles, or puzzles which can be solved by manipulating patterns or applying the algorithms they have learned. Here are a few to get them started. (Answers are in brackets.)

1. How much is: $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 0$? [0]
2. Can you change one sign in this combination of figures to make them produce a value of 100?
 $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$
 Hint: Be sure to enclose in parentheses the numerals on both sides of the sign you change.
 [$1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$]
3. A little girl asked her neighbor how many cats she had. The neighbor replied, "Well, they just jumped up on the fence and I saw a cat in front of two cats, a cat behind two cats, and a cat between two cats."
 How many cats did the neighbor have? [3]

General Objectives

To introduce the cone and cylinder

To introduce the circle, radius, and diameter

To provide experience in drawing circles with a compass

To introduce the concept of a line tangent to a circle

To introduce congruent figures

One of the main concepts of this chapter, the circle, is introduced through physical experiences with cones and cylinders. Children construct a cone and cylinder and thereby actually manipulate a circle before studying a circle drawn on paper. Different methods of drawing a circle are explored, and the terms *radius* and *diameter* are explained. The study of the circle is extended in a lesson which treats tangents to a circle and inscribed circles. Finally, the concept of congruent figures is introduced and studied in relation to symmetric figures.

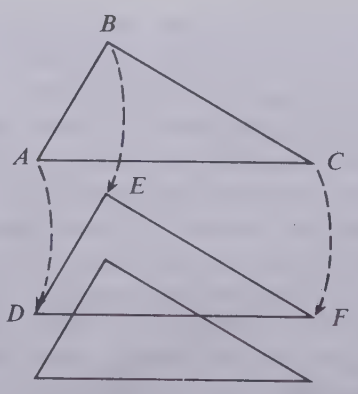
Mathematics

In the study of geometry, our point of view is that there are two kinds of geometry: physical geometry and mathematical geometry. Physical geometry is the geometry of the drawing board or of the physical world. Mathematical geometry is an abstract mathematical system that unfolds from a set of undefined objects and assumptions about these objects. Physical geometry provides an excellent model and a motivation for the development of mathematical geometry. While, in theory, we could completely separate mathematical geometry from any dependence upon the physical world, such a development would hardly be practical in an early study of geometry.

The geometry lessons in this series primarily present the geometry of the physical world. Thus, the black outline shown on the paper can be called a circle or square. When the child gains maturity, he will learn that the black mark is a picture or model of an idea existing in our minds. Such abstractions need not be presented in this early study of geometry.

In keeping with the concept of getting ideas of geometry from the physical world, the introductory lesson of the chapter deals with models of familiar space figures such as cylinders, cones, and spheres. By considering appropriate cross sections of these space figures, one gets plane figures which are circles. Tangents to circles, circles inscribed in triangles, and circles circumscribed about triangles are explored by means of paper-folding activities.

Some introductory ideas of congruent figures and symmetry also appear in this chapter. Congruent figures are often described as figures which have the same size and shape. To check whether two polygons are congruent, a student may use the method of *superposition*, that is, placing a model of one polygon on top of another to see if all their points coincide. But since geometry is concerned with abstract ideas, one cannot really "pick up" two polygons to see if they are congruent, hence a mathematical definition of congruence is necessary. In the case of triangles this definition is fairly simple and is given below.



Triangle ABC is congruent to triangle DEF if $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$, and if $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$.
We write $\triangle ABC \cong \triangle DEF$.

Teaching the Chapter

Materials

Backing material, such as a square of cardboard, to protect desk tops from compass-tip holes
Centimetre rulers
Compass
Cylinders (coffee cans, bean cans, etc.)
Heavy paper
Models of cones and cylinders
Paper cups or glasses, or other models of circles
Paper for folding
Paper for tracing and construction
Paste
A rectangular sheet of paper for each child
Scissors

Vocabulary

centre point	inscribed
circle	inscribed circle
compass	length of radius
cone	quadrilateral
congruent	radius
cylinder	simple closed curve
diameter	tangent

Since much of this chapter deals with construction of geometric shapes and figures, there is not much need for demonstrations by the teacher. However, displaying commercial or well-made geometric models would help to crystallize the children's concepts of the shapes they themselves construct and would encourage them to use care in their constructions. Give poorly co-ordinated children help when necessary.

Lesson Schedule

Allow one to one-and-a-half weeks for this chapter. The concepts are not difficult enough to warrant more time. The activities are to be enjoyed, and if the children want to continue them, you might use some of the suggested materials as supplementary activities for those who show particular interest.

Evaluation of Progress

In evaluating children's progress throughout this chapter, keep in mind that understanding the mathe-

matical concepts is more important than making neatly constructed geometric figures. You will be able to judge fairly well which children grasp the concepts correctly by their reactions to your questions during the activities and in the discussions.

Resources for Active Learning

GENERAL ACTIVITIES

Activities in Geometry for Primary Pupils, Activities 7, 9, 10, 18, 27, 36, 37, 48, Addison-Wesley.

Developmental Math Cards, G²11, Addison-Wesley

Geometry in the Classroom: New Concepts and Methods, pp. 37, 61-62, 123-126, Holt, Rinehart and Winston.

Math Activity Cards, "Whirls," C18, Macmillan

Nuffield Project: *Problems*—Green Set, No. 51, Wiley

MANIPULATIVE DEVICES

Caliper (Edmund Scientific; Selective Educational Equipment)

Geo Blocks (Webster, McGraw-Hill; Selective Educational Equipment)

Geoboards (Addison-Wesley)

Geometric Figures and Solids (LaPine Scientific; school supplier)

Mirror Cards—a study in symmetry (Selective Educational Equipment; Webster, McGraw-Hill)

Pattern Blocks (Webster, McGraw-Hill)

Spirograph (Cuisenaire Co.)

COMMERCIAL GAMES

Beeline (Selective Educational Equipment)

Psyche-Paths (Cuisenaire Co.)

PAGES 206–207

Objective

Given cylindrical or conical objects, the child will be able to classify them as cylindrical or conical.

Preparation

Materials

paper for tracing and construction; rectangular sheets of paper (1 per child)

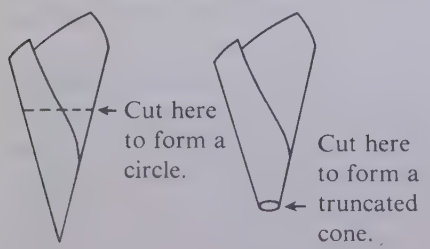
or, duplicated patterns of cylinder and cone (*Duplicator Masters*, pages 71–72)

To prepare for this lesson, you might display shapes with which the children are familiar, such as rectangles, squares, parallelograms, and circles, and ask the children to name them. After you have briefly reviewed these shapes, ask the children if they can think of objects which have shapes different from any of these. Explain that in this lesson they will investigate a few such different shapes.

Investigation

Aside from the experience of making the cylindrical and conical shapes, the important element of this investigation is the children's manipulation of these shapes. After working with these shapes in the form of three-dimensional models, the child will more readily interpret the pictorial representation of these objects.

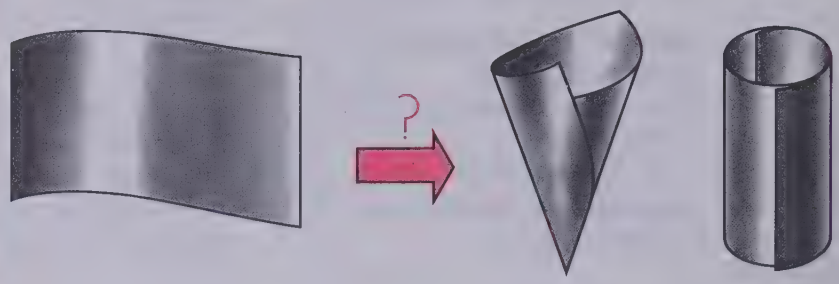
While working with the paper, some children may make a "truncated cone." That is, the cone may not come to a point but look as if the pointed tip has been cut off. You might also have children observe what happens if they cut off a portion of the cone at the wide end to "make it even." By cutting through the cone in this manner, they will form a circle. It is important that you use opportunities such as those that should arise from this investigation to emphasize various geometric concepts.



9 Geometry

Let's explore cylinders and cones.

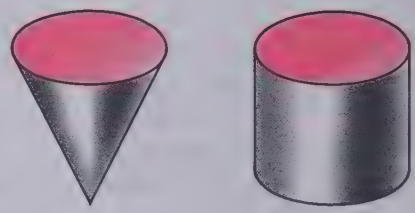
Investigating the Ideas



? Can you roll a sheet of paper into shapes like these?
See Investigation.

Discussing the Ideas

We think of the cone and cylinder as having flat "lids" (shown in color).



Cone Cylinder

1. What is the shape of each of the lids? Circular
2. How many lids does it take to close the cone? 1
3. How many lids does it take to close the cylinder? 2

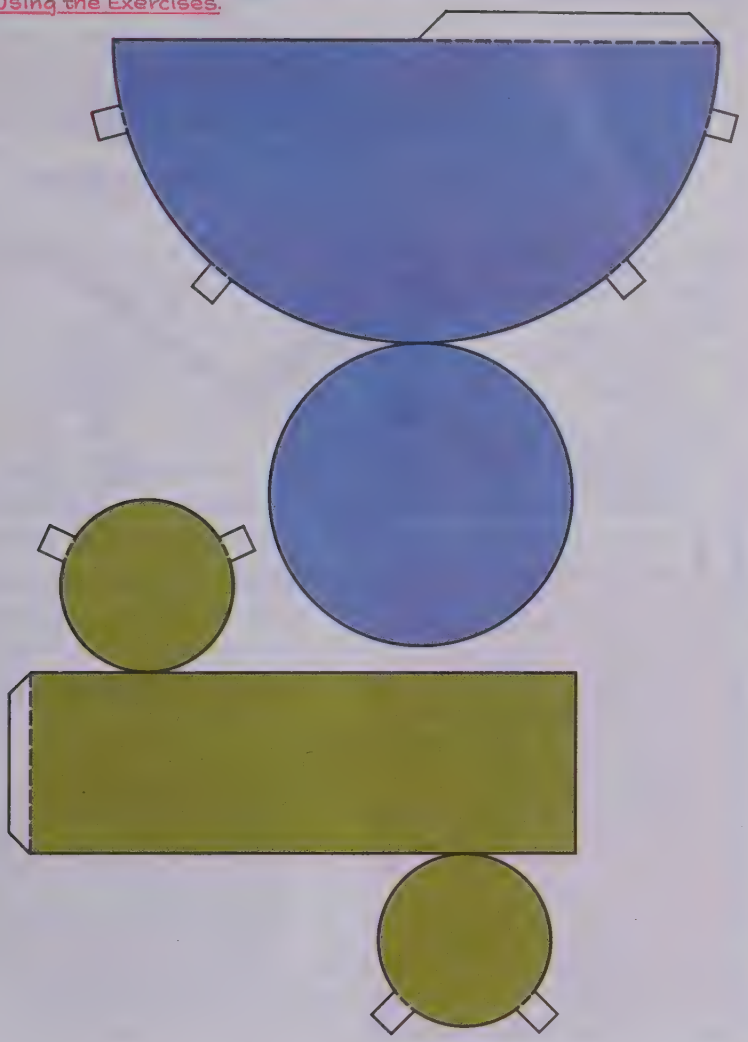
206

Discussion

One of the main topics treated in this chapter is the circle. In this lesson children investigate three-dimensional shapes which, if cut properly, produce this familiar plane figure, the circle. This concept is treated in discussion exercise 1. In exercises 2 and 3, discuss the differences between a cone and a cylinder. Use part of the discussion to have children name things that are cone-shaped or cylindrical. If you brought out the idea of a truncated cone in the investigation, children might use this term to identify a wastebasket. Examples of many different kinds of cylinders can be found in ordinary tin cans, sardine and herring cans, luncheon meat cans, and canned ham tins. However, you might choose right circular cylinders as your basic examples. Paper drinking cones and ice cream cones are examples of cones. You can squeeze a paper cone to show cone models with oval and irregular bases. A number of oddly shaped cans such as those in which some corned beef and plum puddings are packaged are examples of conic sections.

Using the Ideas

To make your own cylinder and cone, trace and cut out these patterns.
See Using the Exercises.



Using the Exercises

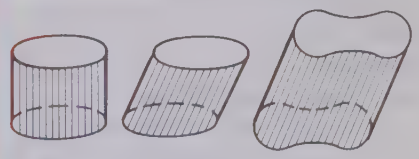
The patterns on page 207 give children an opportunity to make a cone and a cylinder according to fixed dimensions. Stress the importance of tracing and cutting very carefully on the line, for a small error in tracing or cutting will be magnified in construction. In the pattern of the cone, the radius of the semicircle must be exactly twice the radius of the circle. In the pattern of the cylinder, the length of the rectangle must be π ($3\frac{1}{7}$) times the diameter of the circle.

Assignments (page 207)

Minimum: All. Average: All.
Maximum: All.

Mathematics

Most cylinders which children see are *circular cylinders*, that is, the base of the cylinder is a circle. However, the base may be any simple closed closed curve.

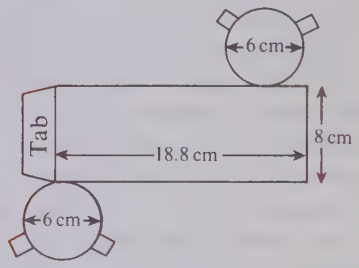


If the lateral surface is perpendicular to the base the cylinder is a *right cylinder*. The common tin can is a model of a *right circular cylinder*. Cones are classified in much the same way as cylinders.



Follow-up

You may wish to furnish less coordinated children with duplicated patterns for making cylindrical and conic constructions. The dimensions given in the following illustration are suitable for a pattern drawn on A4 (210×297 mm) paper.



If you have sturdy models of cones and cylinders, suggest that the children study and compare various properties of each. For example, they might roll a cone along a large piece of paper on the floor, tracing the path of the tip with one color pencil and the path of the base with another. After rolling several cones, and the same cone several times, they should decide whether a cone rolls according to a particular pattern. You might use this activity to lead into the lesson on circles, which follows.

Resources for Active Learning

Activities in Geometry, Activity 18, Addison-Wesley.

Developmental Math Cards, G³18, H³3, Addison-Wesley.

Freedom to Learn, Question 7 (making cones), p. 98, Addison-Wesley.

PAGES 208–209

Objective

Given a circle and its centre, the child will show his understanding of the terms circle, radius, *and* diameter *by identifying them on the figure.*

Preparation

Materials

heavy paper; ruler; scissors

To prepare for this lesson, make sure the children remember what a *simple closed curve* is. You might draw figures such as the following on the chalkboard.

Curves



(Closed but not simple)



(Simple closed curve)



(Simple closed curve)



(Not simple, not closed)

Ask the children which of these are simple closed curves. Keep this review brief and begin the investigation as quickly as possible.

Investigation

If the children line up a corner of their heavy paper with the left corner of the point finder, they can easily mark guidelines for cutting. However, stress that they are to make the length 8 centimetres. The measurement in the text is not exact. If you choose, you might pass out previously prepared strips of heavy paper (10 cm by 2 cm) which the children could easily make into the point finder. Give as much guidance as necessary in order for the children to make the point finder correctly.

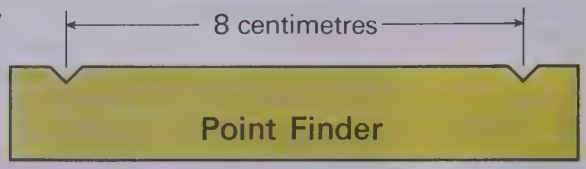
When the children begin to use the point finder according to the directions in the text, emphasize that one end of the point finder must remain in position with the red dot; it is only the other end of the point finder which should move. If a child starts to poke holes in his point finder and draw the circle as he would with a compass, do not prohibit this. However, do not encourage this practice, because it obscures the idea of finding a large set of points for the circle.



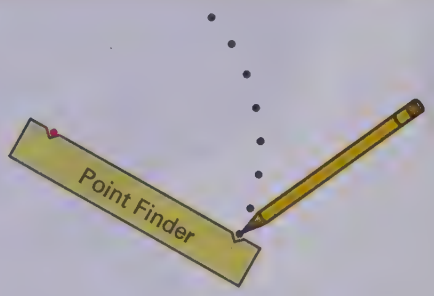
Let's explore circles.

Investigating the Ideas

Cut a "Point Finder" out of heavy paper.



Use your Point Finder to mark **many** black dots all the way around a red dot.

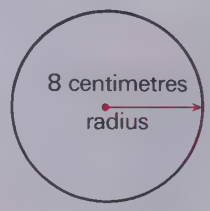


? Can you connect the black dots so that you have a **simple closed curve** that is always 8 centimetres from the red dot ?

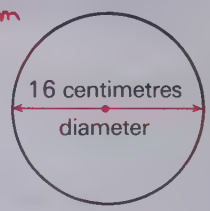
See Investigation.

Discussing the Ideas

1. In the Investigation you drew a **circle** with a **radius** of 8 centimetres. The red dot is the centre of the circle. What would the radius have been for a 10-centimetre Point Finder ? 10 cm



2. Your circle has a **diameter** of 16 centimetres. What would the diameter have been for a 10-centimetre Point Finder ? 20 cm



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Discussion

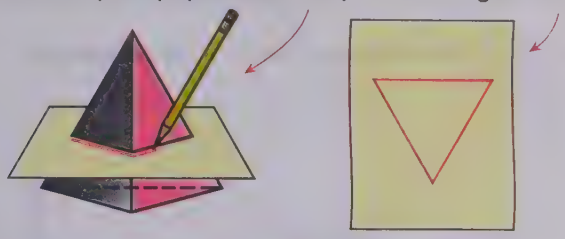
In this lesson, the child not only identifies a circle as a simple closed curve, but also reviews the meaning of radius and diameter. Discuss the simple closed curve drawn in the investigation. Help children see that the circle is drawn more easily when many dots have been drawn than when only a few dots have been drawn.

As you discuss exercises 1 and 2, point out that the centre is not part of the circle but is important in defining a circle and its radius and diameter. Speak of the *radius* as *any segment from the centre point to a point on the circle.* Speak of the *diameter* as *a line segment that passes through the centre point of the circle and has its endpoints on the circle.*

During this discussion of circle, it would be helpful to develop informally the concepts of the interior and the exterior of a circle. For example, display a circle and speak of some points inside the circle and some points outside the circle. Help the children realize that any single point must be inside the circle, on the circle, or outside the circle. Such a discussion should help children become aware that we think of the circle as the black outline only, which is quite distinct from the interior and the exterior of the circle.

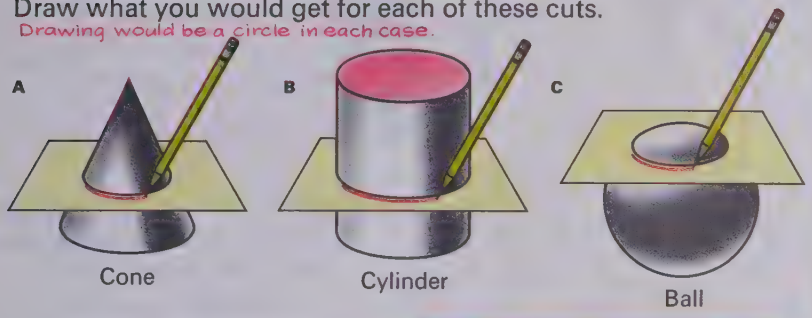
Using the Ideas

1. If you could cut the pyramid and mark your paper like this, you would get this.



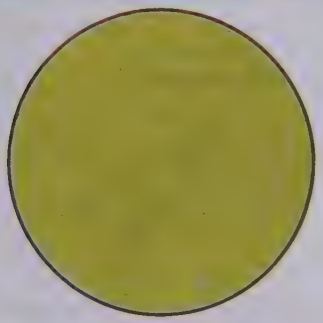
Draw what you would get for each of these cuts.

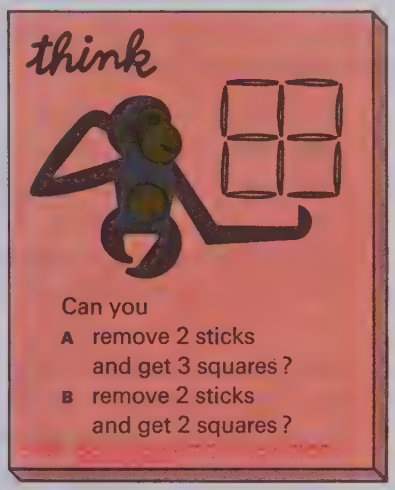
Drawing would be a circle in each case.



2. Trace and cut out this circle. See if you can find the centre by folding. Find the radius and the diameter.

Radius 2½ cm, Diameter 5 cm



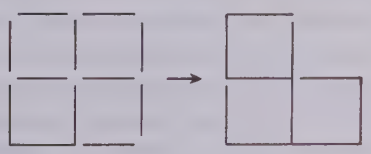


Using the Exercises

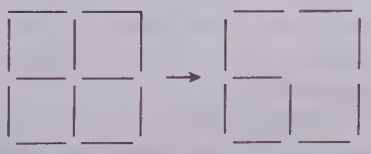
Have the children work the exercises on page 209 independently before discussing them. When the children finish, if feasible, have them cut models and actually place a piece of paper between the sections and trace the outline around the model, to prove to themselves whether their drawings for exercise 1 are correct.

Solutions, Think, page 209

a



b



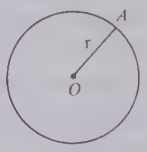
Assignments (page 209)

Minimum: 1–2. Average: 1–2. Maximum: 1–2.

Mathematics

The following are mathematical definitions for the circle, the radius, the diameter, the interior of the circle, and the exterior of the circle.

A *circle* with centre O and radius r is the set of all points A in a plane such that length OA is equal to r.

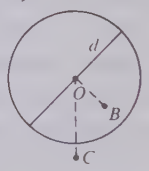


The following two definitions of radius demonstrate how this term is used from different viewpoints.

A *radius* is (1) any line segment from the centre point to a point on the circle; (2) the distance from the centre point to any point on the circle.

Notice that from one viewpoint the radius is a geometric abstraction (segment) and from another it is a number as used in the formula for circumference, $2\pi r$.

A *diameter*, d, is a chord that passes through the centre point of the circle. A *chord* is a line segment that has its endpoints on a given circle. (Sometimes the diameter is also referred to as the length of this chord.)



The *interior of the circle* is the set of all points B such that length OB is less than r.

The *exterior of the circle* is the set of all points C such that length OC is greater than r.

Resources for Active Learning

Activities in Geometry, Activities 7, 9, 10, 27, 48, Addison-Wesley.

Franklin Series: *Pencil and Paper Geometry*, "Circles," pp. 11–14, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Measuring Round Things," pp. 127–128, Addison-Wesley.

Geometry in the Classroom, pp. 61-62, Holt, Rinehart and Winston.

Math Activity Cards, "Circles," C7, Macmillan.

Workbook, page 63

PAGES 210–211

Objective

Given appropriate materials, the child will be able to draw a circle, using one of three different methods.

Preparation

Materials

compass (1 per child); paper clips; circular objects or models of a circle

Since this lesson deals mainly with the construction of circles, it would be appropriate to begin immediately with the investigation.

Investigation

Most children will be able to interpret from the illustrations the three methods suggested for drawing a circle. However, many will need guidance in the proper use of the compass. If possible, demonstrate with a large chalkboard compass how the compass may be turned by a dual motion: while the fingers turn the tip as the arrow indicates, the wrist moves in a circular motion, all of which is done by one hand while the second hand simply holds the paper. Once this technique is discovered, the compass ceases to be as awkward as some children at first find it to be.

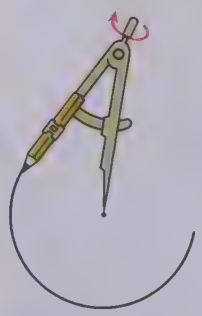
Encourage children to try to think of other objects in the classroom which they could use to draw a circle. Also suggest that they try any other method they think they would be able to use to draw a circle.



How can you draw circles?

Investigating the Ideas

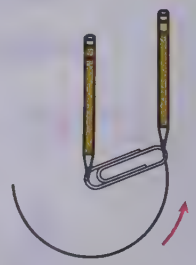
Can you draw a circle by using each of these methods?
See Investigation.



A compass



A round object



Two pencils and a paper clip

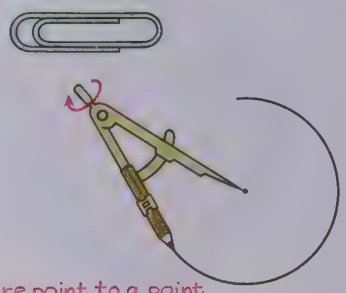
Discussing the Ideas

1. Which method above does not find the centre of the circle? Using a round object

2. How is the paper-clip method like using your Point Finder?
Answers will vary.

3. Use your compass or a paper clip to draw a circle. Explain how to find the radius and the diameter in centimetres.
Sample answer: Measure from the centre point to a point on the circle to find radius; measure the length of a segment which passes through the centre point and whose endpoints are on the circle to find diameter.

Point Finder



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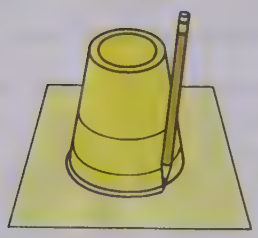
Discussion

If you have the children work in groups, you might pose the discussion questions to them as they work. Exercise 1, which relates to the use of a round object, stresses the concept that the centre point of a circle is not a part of the circle. Exercise 3 emphasizes that the centre of the circle must be found before the length of the radius or diameter can be measured.

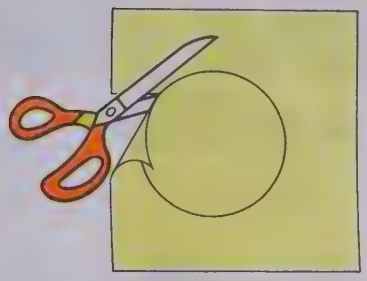
Encourage discussion of any methods for drawing circles that children suggest. Let the class decide if the results of such methods are indeed circles, and if the methods are convenient.

Using the Ideas

1. A Draw a circle by using a round object.



B Cut out the circle.
Constructions will vary.

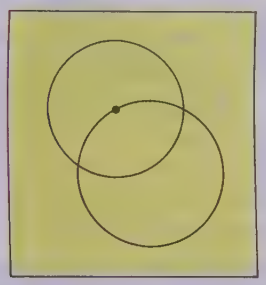


C Can you find the centre of the circle by folding your cutout? Yes
(Centre will be the point of intersection of any two fold lines that divide the circle into halves.)

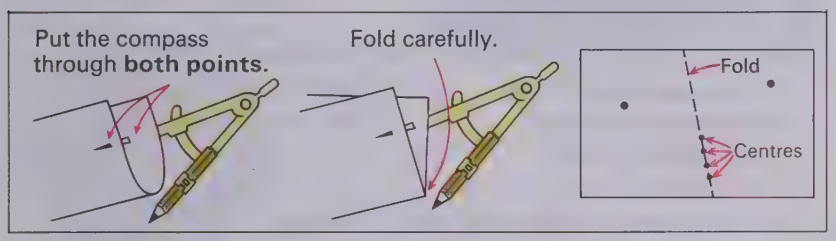
2. Mark a dot on your paper.

A Draw one circle so that the dot is the centre.

B Draw another circle so that the dot is on the circle.
See Using the Exercises.



★ **3.** Mark 2 points on your paper. Draw a circle through your 2 points by using this method.



The centre of any circle through these two points is on the fold.



Using the Exercises

You might treat the exercises on page 211 as extensions of the investigation, or simply as construction activities. Point out that the illustrations are provided to help them follow the directions for exercise 1. For exercise 2, some children will discover that the centre of the inscribed circle may be found by drawing both diagonals of the square and then marking their point of intersection.

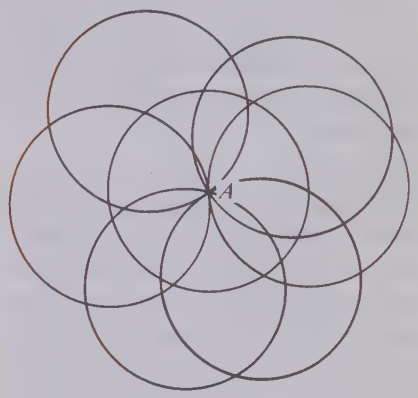
Suggestions for other activities are given in the follow-up section.

Assignments (page 211)
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

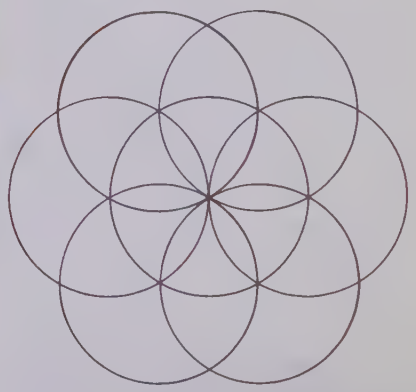
Follow-up

Encourage children to try some of these activities:

1. Use your compass and draw 3 circles which have the same centre but radii of different length.
2. A. Mark a point on your paper. Use your compass to draw a circle with this point as the centre.
 B. Now draw 6 circles through this point. Do this by putting the tip of your compass at different places on the circle.
 C. Are all the circles the same size? (Yes)



3. A. Mark a point on your paper. Use your compass to draw a circle with this point as the centre.
 B. Again draw 6 circles through this point. Draw the first of these 6 by putting the tip of your compass at any point on the original circle. For the other circles, put the tip of your compass at the point where the previously drawn circles meet.



C. Color a pattern that you see. Try to make other patterns with circles.

Resources for Active Learning

Activities in Geometry, Activity 27, Addison-Wesley.

Geometry in the Classroom, pp. 123-126, Holt, Rinehart and Winston.

PAGES 212–213

Objective

Given suitable materials, the child will be able to construct a line that is tangent to a circle and inscribe a circle in a triangle.

Preparation

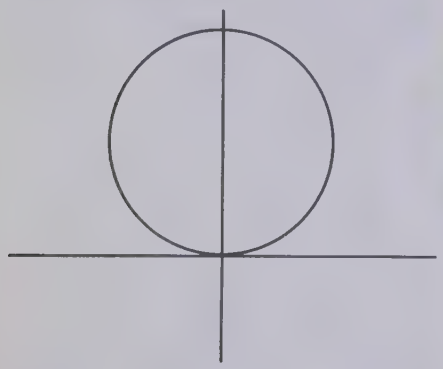
Materials

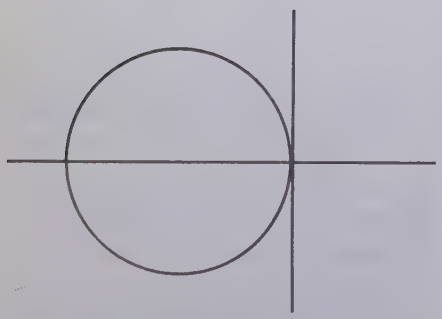
compass; paper suitable for folding

The only preparation essential to this investigation is a review of the term *right angle*. However, the illustration in the text is clear enough to enable most children to construct the folded lines without further guidance. For this reason, you might choose to begin immediately with the investigation.

Investigation

You may choose to guide the children in folding their papers. However, it is intended that they work on the investigation question independently. If necessary, help children interpret the word "touches"; explain that a circle "touches" a line if there is a point that is on both the line and the circle. Do not be overly concerned with any errors the children make; it is often in making and correcting mistakes that much of learning takes place.

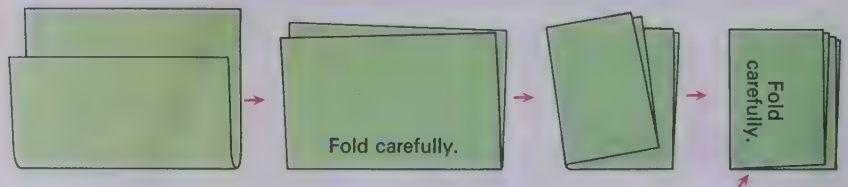




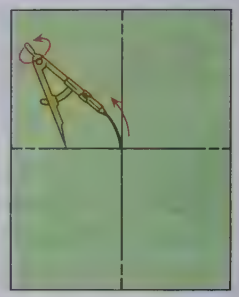
When is a line tangent to a circle?

Investigating the Ideas

Follow the steps below to show lines that cross at right angles.



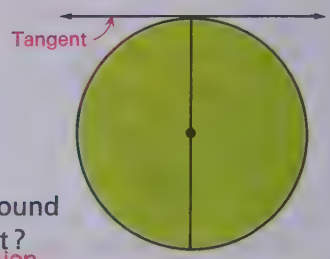
Make sure these two edges are together.



? Can you draw a circle that crosses one of the creases twice and touches the other at only one place?
See Investigation.

Discussing the Ideas

A line that touches a circle in exactly one point is a **tangent** to the circle.



1. Which crease is tangent to the circle you drew in the Investigation?
See Discussion.
2. What are some things in the world around you that suggest the idea of a tangent?
See Discussion.

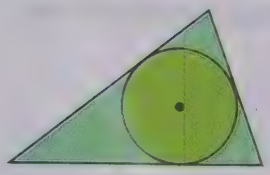
212

Discussion

Have several children show the circle they drew in the investigation, so that all will recognize the correct position even if they did not draw it themselves. Use the diagram and explanation in the text to present the meaning of the word *tangent*. Note that, in order to be tangent to a circle, a line must touch the circle in *one* and *only one* point. As you discuss exercise 1, children will find that circles in four different positions satisfy the requirements. In response to the second question, children might mention a basketball on a gym floor, a ball on top of a flagpole, an orange on a plate, etc.

Using the Ideas

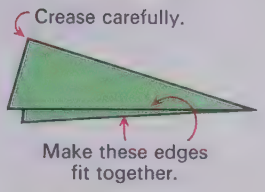
1. A circle is **inscribed** in a triangle if each side of the triangle is tangent to the circle.



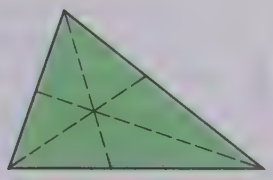
You can find the circle inscribed in a triangle in the following way. See Using the Exercises.

A Draw and cut out a large triangle. The sides of your triangle should be at least 12 centimetres long.

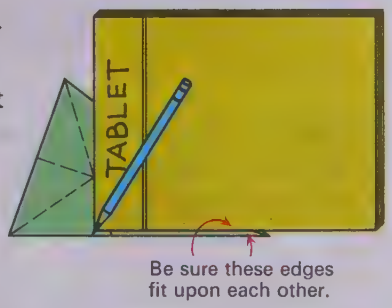
B Fold one corner of your triangle as shown in the figure.



C Now fold each of the other 2 corners as you did the first. If you did your work carefully, your triangle should look something like this. (The three creases should pass through one point.)



D Paste your triangle on another sheet of paper. Then use your tablet and mark the point right below the point where the folds intersect.



E Decide where to place your compass, and draw the circle inscribed in the triangle.

★2. Draw a square. Figure out a way to draw the circle that is inscribed in the square. See Using the Exercises.



Using the Exercises

Many children may need your guidance in following the directions for exercise 1 on page 213. Study with the children the explanation and diagram of the inscribed circle. If you have the children work the rest of the page independently, give help where needed. The sides of the triangle need not have the same length; each side should be at least 12 centimetres simply for purposes of manipulation. Make sure the children fold the triangle properly; they are not to fold the angles corner to corner but rather to fold the sides edge to edge. The radius of their inscribed circle should be the distance from the point where the folds intersect to the point they marked using the tablet (part D).

If time permits, encourage all the children to try the starred exercise. You might hint to some that they begin by searching for the centre of the square, for example, by folding the square into fourths.

Assignments (page 213)*

Minimum: 1. Average: 1.
Maximum: 1–2.

Mathematics

A tangent to a circle is a line which has exactly one point in common with the circle. There are several interesting mathematical relationships concerning tangents. For example, a line that contains both the centre and the point of tangency is at right angles to or is perpendicular to the tangent. The converse is also true: A line that is perpendicular to a tangent at the point of tangency passes through the centre of the circle.

The basic mathematical theorems for this lesson follow.

> A tangent to a circle has exactly one point in common with the circle.
>
> The bisectors of the three angles of a triangle intersect in one point, and this point is the centre of the inscribed circle of the triangle.
>
> The diagonals of a square intersect in a point that is the centre of the inscribed circle of the square.

Follow-up

Some children may enjoy making task cards by constructing a circle, drawing a quadrilateral around the circle, and transferring that quadrilateral to another card or erasing the circle. Another child may then be challenged to find the inscribed circle of that quadrilateral.

You might also suggest that children work with circles circumscribed about a triangle. A circumscribed circle may be found by folding the corners of a triangle one upon another two at a time. Such a folding will form creases along the perpendicular bisectors of the sides of the triangle. Their point of intersection is the centre of the circumscribed circle.

Resources for Active Learning

Franklin Series: *Learn to Fold . . .*, "Some Same Shapes and Sizes," pp. 31–42, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Franklin Series: *Mirror Magic,* "Congruence," pp. 34–44, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathematics in Modules, SK7, Addison-Wesley.

PAGES 214–215

Objective

Given sets of geometric figures, some of which are congruent, the child will be able to identify the congruent figures.

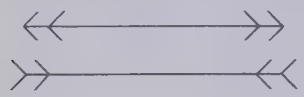
Preparation

Materials

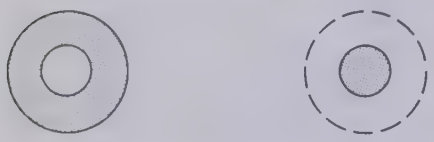
scissors

To create enthusiasm for this lesson, display and discuss a few optical illusions, such as those below.

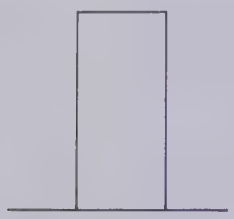
Which line segment is longer?



Which inside circle is larger?



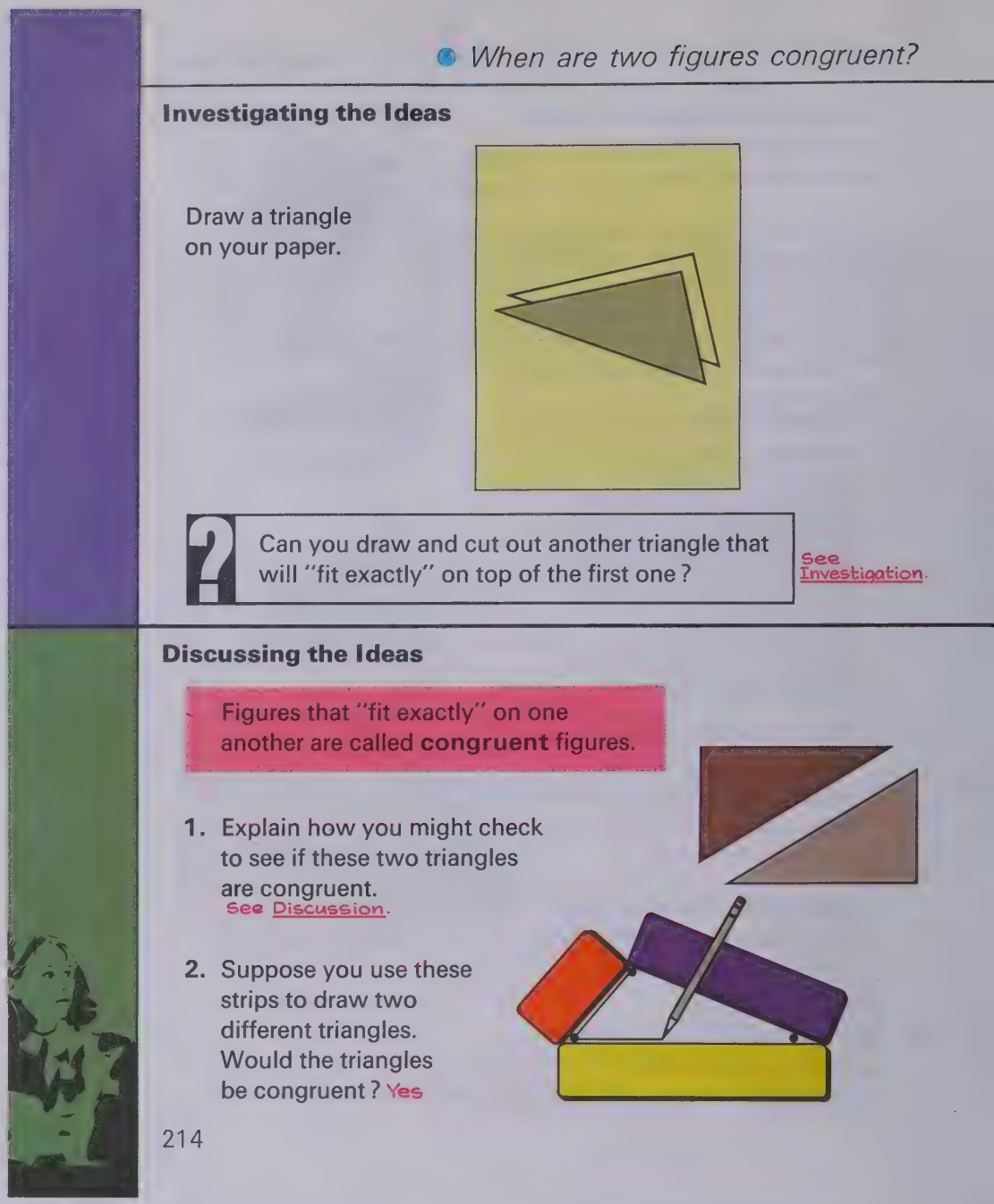
Is this hat higher than it is wide?



These illusions, or others, should help children realize that we cannot always determine the comparative sizes of things simply by observation. Explain that in this lesson they will investigate ways of finding out about figures that may look alike.

Investigation

To meet the requirements of this investigation, many children will choose the obvious procedure of tracing the triangle in the text and cutting it out. However, it is hoped that some children will be motivated to search for less obvious, more creative ways to find a triangle which would "fit exactly" on top of the one in the text. For example, some may measure the sides and try to draw a triangle with sides of the same lengths. This is often a difficult task, but not impossible. Others may figure out a device for measuring the angles and try to draw the matching triangle by using the same angle measures. Do not suggest these methods yourself; allow children the opportunity to share their own ideas.



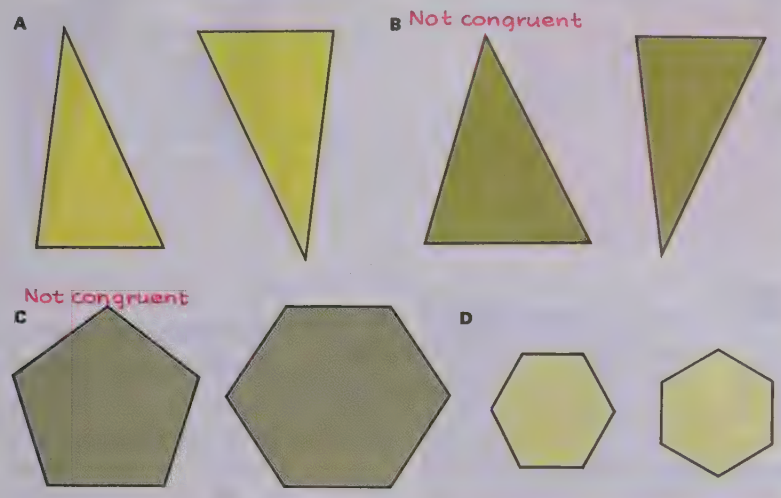
Discussion

Explain the meaning of the term *congruent*, and use the triangles studied in the investigation as examples. Exercise 1 gives children a chance to try out any method they or their classmates might have discovered in the investigation. Although measurement of the sides may show the sides to be of equal length, some children may not be convinced that this fact means the triangles are congruent. Have any such children trace the triangles and place one on top of the other. Remind them that congruent figures may look different because they may be not only turned from the first position but also "flipped" from the first position. Have different sets of congruent triangles cut from cardboard for the children to manipulate.

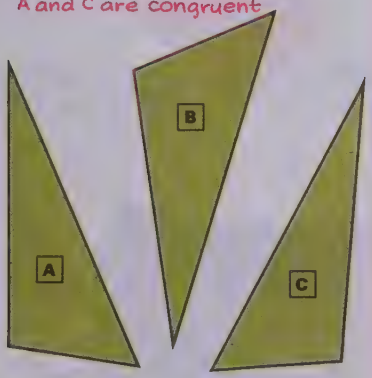
Exercise 2 demonstrates the mathematical theorem that, if all sides of a triangle correspond equally in length to the sides of another triangle, the triangles are congruent. Some children may want to work with several sets of strips before accepting this.

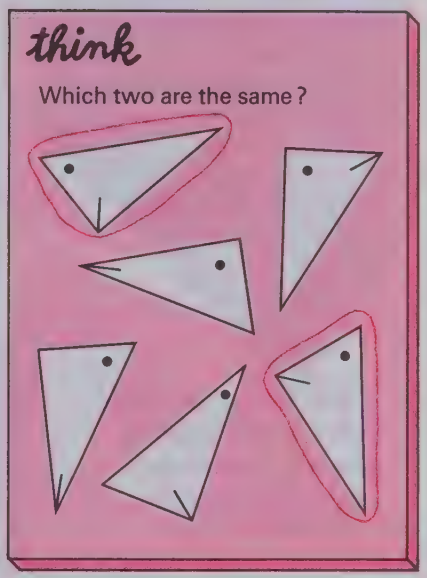
Using the Ideas

1. Tell "just by looking" which pairs of figures are not congruent.



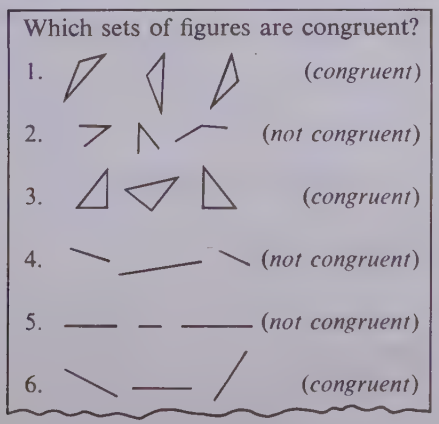
2. Use tracing to tell which two triangles are congruent.





Using the Exercises

Assign the exercises on page 215 as independent work. If you wish to extend these exercises, duplicate the following type of worksheet:



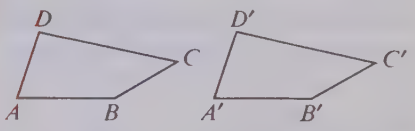
Encourage all the children to try the *Think* problem. Some may want to trace a figure or two and place it upon another. However, most will recognize which two figures are the same after a little visual study.

Assignments (page 215)
Minimum: 1–2. Average: 1–2.
Maximum: 1–2.

Mathematics

The relation between pairs of geometric figures that is known as *congruence* is one of the more important relations studied in elementary mathematics. The definition of congruent polygons is as follows:

> Two polygons are congruent if there is a one-to-one correspondence between the vertices of the polygons such that corresponding angles are congruent and corresponding sides are congruent.



In the pair of quadrilaterals illustrated above, we can establish the one-to-one correspondence between the vertices given by $A \leftrightarrow A'$, $B \leftrightarrow B'$, $C \leftrightarrow C'$, and $D \leftrightarrow D'$. Then the two quadrilaterals will be congruent if

$$\left.\begin{aligned} \angle A &\cong \angle A' \\ \angle B &\cong \angle B' \\ \angle C &\cong \angle C' \\ \angle D &\cong \angle D' \end{aligned}\right\} \text{ and } \left\{\begin{aligned} AB &\cong A'B' \\ BC &\cong B'C' \\ CD &\cong C'D' \\ DA &\cong D'A' \end{aligned}\right.$$

In this lesson children use informal methods of tracing and cutting out regions in order to compare two figures to see whether they are congruent. Notice that when a child places a figure on top of another to check for congruence he is establishing a correspondence between the vertices of the figures which allows him to compare the sizes of the various angles and the sides.

Duplicator Masters, page 41
Workbook, page 64

PAGES 216–217

Objective

Given a symmetric figure, the child will be able to recognize that the two halves of the figure are congruent to each other.

Preparation

Materials

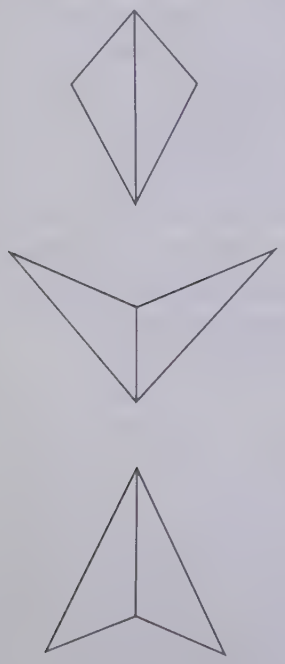
scissors

To prepare for this investigation, review the meaning of the term *symmetry*. Display several figures which are symmetric and have the children recall how the halves of a symmetric figure match. If necessary, refer to page 112 to review the concept of symmetry.

Investigation

Some children will realize that an easy way to cut two congruent triangles is to cut through two pieces of paper simultaneously. However, if children draw and cut one triangle and then trace and cut the other, do not correct them. Emphasize that the triangles may be any shape or size.

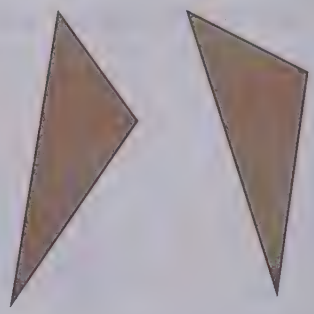
Following are the symmetric figures that can be formed for a given pair of congruent scalene triangles.



Let's explore congruent figures and symmetry.

Investigating the Ideas

Cut out two triangles that are congruent.

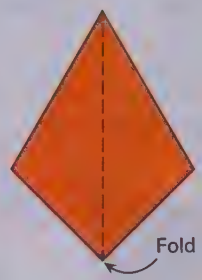


? How many ways can you place the sides of your triangles together so that you get a symmetric figure?
See Investigation.

Draw pictures to record your results.

Discussing the Ideas

1. Suppose this figure is symmetric. Why do you think the two triangles are congruent?
See Discussion.



2. Explain how you could use a folded piece of paper to cut out two congruent figures.
See Discussion.



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Discussion

Have the children use the results of the investigation as you discuss the explanation of *symmetric figures*. They might note that in each figure found in the investigation the line of symmetry is the side common to both triangles. Children may suggest a variety of reasons as you discuss exercise 1, such as, "We can place them one on top of the other to fit exactly," or, "They share a common side as did the congruent triangles in the investigation," or, "We can see that the sides are all equal," etc. Accept any statement which is a reasonable conclusion based on the previous concept development.

Exercise 2 gives some children an opportunity to explain how they cut out the triangles in the investigation. Others will be able to compare the method they used to the one suggested here.

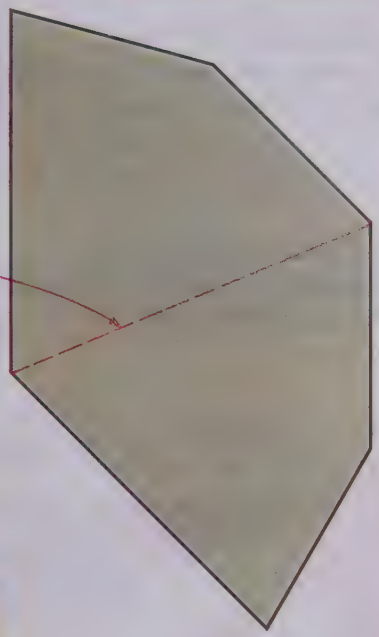
Using the Ideas

1. Trace this square and cut it out. Can you fold and cut so that you get
- A two congruent triangles?
- B two congruent rectangles?

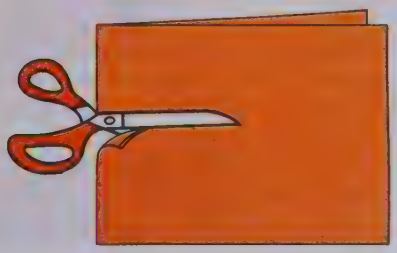
A Fold along either diagonal.
B Fold into halves vertically or horizontally.



2. Trace and cut out this figure. Can you find a way to fold and cut it so that you get two congruent quadrilaterals (4-sided figures)? Fold along this diagonal



★ **3.** Make two congruent pentagons (5-sided figures) by folding and cutting paper.
See Using the Exercises.



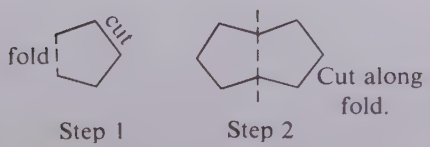


Using the Exercises

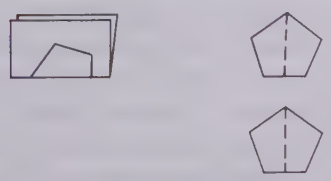
You might assign the exercises on page 217 as activities and add to them either from the activity cards in the back of the book or from suggestions in the follow-up section. Note that in exercises 1 and 2, children must actually cut along a line of symmetry to form two congruent figures.

Two possibilities for exercise 3 are as follows:

1. Fold and cut 1 piece of paper.



2. Fold and cut 2 pieces of paper. (Note that one cut is perpendicular to the fold.)



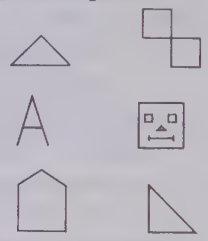
Assignments (page 217)
Minimum: 1. Average: 1–2.
Maximum: 1–3.

Follow-up

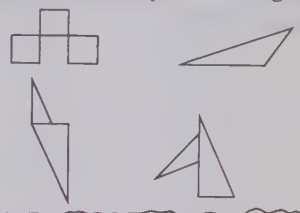
The use of a geoboard would give children further opportunity to investigate congruent and symmetric figures.

You might also distribute a page of congruent and symmetric figures such as the sample below.

1. Draw a line through each symmetric figure to form congruent figures.



2. Trace figures congruent to each of the following figures and use them to make symmetric figures.



Resources for Active Learning

Activities in Geometry, Activities 36, 37, Addison-Wesley.

Developmental Math Cards, D[3]3, Addison-Wesley.

Inquiry in Mathematics via the Geoboard, "Congruent Triangles," Geo-Card 22; "Similar Triangles," Geo-Cards 23/1–3, Walker. (Available from Fitzhenry & Whiteside)

Workbook, page 65

PAGES 218–219

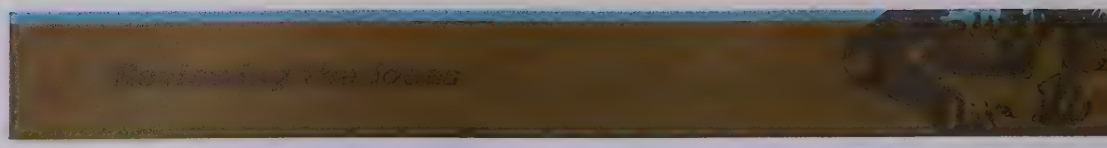
Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

If you choose to prepare for the review exercises, briefly discuss the principal topics and terms treated in this chapter, such as cylinders, cones, circles, radius, diameter, tangent, congruent, and symmetric figure. If you choose to prepare for the Keeping in Touch exercises before assigning page 219, use a short oral review of basic facts for both addition and multiplication. However, in either case, remember to keep this part of the lesson brief.



1. Answer **cone** or **cylinder** for each shape.

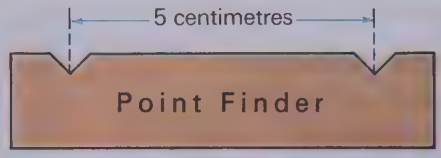
A Cone 

B Cylinder 

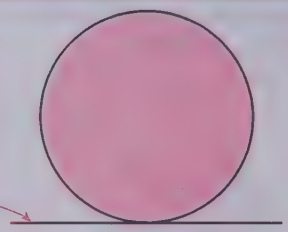
C Cylinder 

D Cone 

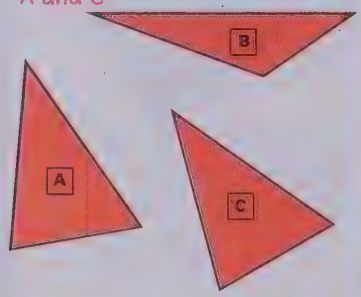
2. What would be the radius of a circle made with this Point Finder? 5cm

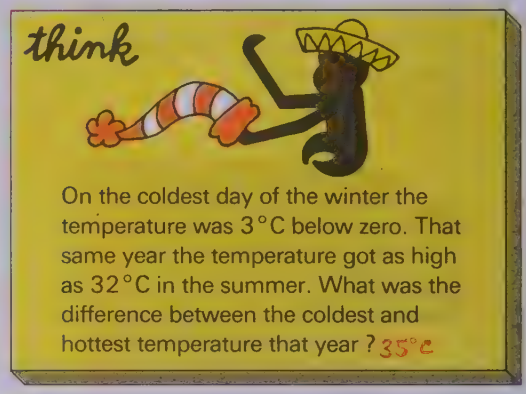


3. Use any method you want to draw a circle. Draw a line that is tangent to your circle.
Constructions will vary.



4. Which two triangles appear to be congruent?
A and C





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Discussion

Page 218 may be used as an evaluative instrument, or you may use it as a review lesson. Whether you work through the exercises with the children or discuss them after the children have completed them, include in your discussion the other topics treated in this chapter. Make sure children realize the distinction between cone and cylinder and between radius and diameter.

Keeping in Touch with Addition Subtraction Multiplication Measurement

1. Find the sums and differences.

A 35 +24 = 59	B 78 −46 = 32	C 39 +25 = 64	D 62 −48 = 14	E 75 −36 = 39	F 57 +9 = 66
G 65 +72 = 137	H 134 −91 = 43	I 84 +77 = 161	J 126 −58 = 68	K 95 +67 = 162	L 160 −84 = 76

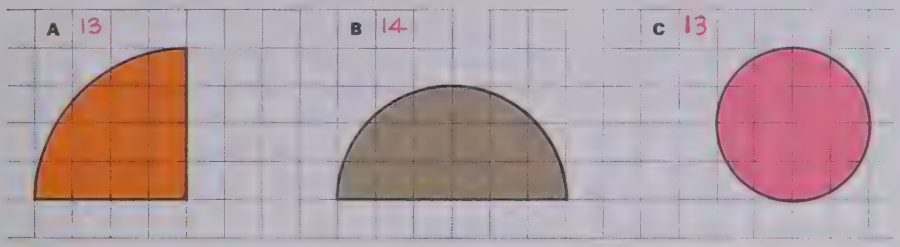
2. Find the products.

A	5 × 5 = 25	E	7 × 4 = 28	I	3 × 9 = 27	M	4 × 4 = 16	Q	6 × 2 = 12	U	5 × 4 = 20
B	2 × 8 = 16	F	2 × 9 = 18	J	6 × 6 = 36	N	6 × 5 = 30	R	8 × 9 = 72	V	8 × 5 = 40
C	9 × 6 = 54	G	5 × 7 = 35	K	0 × 7 = 0	O	8 × 4 = 32	S	9 × 1 = 9	W	5 × 9 = 45
D	9 × 4 = 36	H	7 × 2 = 14	L	4 × 6 = 24	P	6 × 8 = 48	T	7 × 7 = 49	X	8 × 8 = 64

3. Solve.

A $3 \times 100 = n$ (300)	C $100 \times 17 = b$ (1700)	E $30 \times 40 = t$ (1200)	G $200 \times 7 = e$ (1400)
B $10 \times 56 = r$ (560)	D $28 \times 10 = g$ (280)	F $20 \times 70 = h$ (1400)	H $8 \times 300 = m$ (2400)

4. Estimate the area of each region.





You are invited to explore

ACTIVITY CARD 10 Page 352

Using the Exercises

Assign the exercises on page 219 as independent work. When the children have finished, check the papers and clarify concepts as needed. If exercise 2 indicates that some children lack knowledge of a few of the basic facts, encourage them to devise a method of study to learn the ones they missed.

Follow-up

To give children an opportunity to work with all of the shapes and figures treated in this chapter, you might have the class use these shapes in making mobiles. Wire coat hangers, string, and stiff paper are readily available materials which can be used to make attractive mobiles of symmetric figures and congruent triangles and quadrilaterals. Tiny cones and cylinders may be constructed and covered with various colors and designs.

Workbook, page 66 (Use with page 218.)

General Objectives

To maintain understanding of division concepts

To develop the division algorithm for 1-digit divisors

To develop the division algorithm for certain 2-digit divisors

To provide a variety of word-problem experiences

To provide experiences in estimation

Much of this chapter's material is directed toward developing a complete understanding of the long-division algorithm. In order to keep the computation at a level that is reasonable for fourth-grade children, the algorithm is not carried beyond simple 2-digit divisors. This restraint insures that children will not be overburdened by computational complexities while mastering the essential concepts that are developed in this comprehensive treatment of the long-division algorithm.

The opening pages of the chapter are devoted to estimation leading to divisions involving only the basic division facts and remainders. Following this, work is provided to lead toward an understanding of division with 1-digit divisors and 2-digit quotients. The role of repeated subtraction in the development of this algorithm is strongly emphasized. The following lessons concentrate on multiples of ten in estimation to assist children in making the best guess when they attempt to divide using 1-digit divisors and 2-digit quotients. Again, much attention is paid to the role of repeated subtraction in the division algorithm. A lesson on averages is introduced as a useful application for the division material developed thus far. Then exercises involving 1-digit divisors and 3-digit quotients extend the concepts developed previously and utilize more material on estimation—this time with multiples of 100. Following a cumulative review, the children are given material leading to an understanding of two-digit divisors. Use of inequalities continues to play a role in making first quotient estimates, and two-digit division is introduced by a study of multiples of ten.

A key feature of the content organization of this chapter is that, at each new stage of development, material is provided to assist the child in making accurate estimates when he reaches a particular phase of the division algorithm. However, as you go through the material, you will note that the door is left open for the child who does not make the best estimate on his initial try. More of this material is provided later in the text.

Mathematics

Our basic definition for division (for whole numbers a, b, and c, if $a \times b = c$ and $b \neq 0$, then $a = c \div b$) must be extended to allow for remainders in division. That is, we would like to define $c \div b$ for the case when there is no whole number a such that $a = c \div b$. There are two choices: we can reconsider the definition above without a restriction that a, b, and c be whole numbers, or we can redefine division to allow for nonzero remainders. Consider the following definition.

> If a, b, c, and d are whole numbers such that $(a \times b) + d = c$ and $b \neq 0$ and $d < b$,
> then for $c \div b$, a is the quotient and d is the remainder.

Notice in this definition that, if $d = 0$, we have the original definition. That is, $a \times b = c$ simply implies $c \div b = a$. Of course, if we wanted to make a statement of equality with regard to $c \div b$ when $d \neq 0$, we would be involved with fractional numbers. That is, we would have $c \div b = a + \frac{d}{b}$. An example of this would be $14 \div 3$.

$$\begin{array}{r} 4 \\ 3\overline{)14} \\ \underline{12} \\ 2 \end{array} \qquad 14 \div 3 = 4 + \tfrac{2}{3} \text{ or } 4\tfrac{2}{3}$$

When we develop fractional numbers in future work, we can return to this idea and establish that nonzero remainders give a quotient which is not a whole number.

You will note that we are considering two different ways of thinking about quotients: as whole numbers with a number called the remainder related to this quotient and as rational numbers such that the quotient times the divisor equals the dividend in each case. This latter case will be explored in Book 6.

Teaching the Chapter

Materials

Almanacs, atlases, encyclopedia, etc.
Celsius thermometers, alcohol and/or mercury (also, if possible, clinical thermometers)
Demonstration clock
Map of Canada
Metre stick
Overhead projector (if available) and transparencies

Vocabulary

average dividend remainder
degree divisor

The materials for this chapter are listed primarily to help children with word-problem sets rather than to help conceptually with the mathematics of the chapter. Use of materials to demonstrate concepts is not an important part of the development in this chapter, since the children have worked carefully with the division concept in a previous chapter. The chief objective is to help the children understand the common skills used for determining quotients in various types of divisions, and little can be done with sets and physical objects to aid this computational work.

Lesson Schedule

This chapter is designed to be covered in about three and one half to four weeks. You should modify this time schedule to fit the needs and abilities of your class. If the schedule is flexible, provide several changes of

pace, since some children may become bogged down by the heavy concentration on long division.

Evaluation of Progress

Generally, the children's progress in mastering algorithmic skills is easily tested and evaluated. A good way to evaluate their understanding of the algorithm is to observe daily how quickly the children grasp the ideas involved. We recommend that you do not penalize children who find quotients by making many guesses and subtracting each time. This method of finding the correct quotient is, in many ways, superior to memorizing an algorithm without fully understanding the process.

The following general statements describe our overall approach to the division algorithm.

A careful development of concepts precedes development of the algorithm.

Numerous experiences with estimation involving sums, differences, products, and quotients prepare for the work in dividing.

Repeated subtraction plays a major role in the dividing process. The quotient is thought of as the greatest multiple of the divisor that can be subtracted from the dividend.

Initially, the children are encouraged to think about the entire divisor and use it to estimate the greatest number of times the divisor is contained in the entire dividend.

Some children may discover the shortcuts for using part of the divisor and part of the dividend for estimating before they are taught the standard algorithm. But keep in mind that you want them to think of numbers before they learn shortcuts using digits. One goal is to encourage students to use estimation effectively in arriving at quotients. To this end, we give considerable attention to exercises that help children estimate accurately and quickly.

This last statement requires further discussion. An important aspect of the program is that when using this approach, children may make less accurate estimates, subtract, and then estimate again. We encourage them to develop an efficient method of estimating by making careful estimates of the quotient the first time, when 1-digit divisors are involved. Though the efficient method can be mastered by almost all pupils, they still may make a less careful guess, subtract, and then have to guess again. Dividing with 2-digit divisors is much more complicated, but those pupils who are weak in performing this algorithm can still succeed by making a series of estimations and subtractions. Obviously, if a child's estimate is too large, he must start over, as he would with most approaches to the division algorithm. In this flexible context, the most efficient way to find quotients by making good initial estimates is clearly stressed.

Resources for Active Learning

General Activities

[Refer to Chapter 7 for estimation activities.]

Manipulative Devices

Thermometers (Edmund Scientific; school supplier)

Commercial Games

Orbiting the Earth: Division (Scott Foresman)

TUF (Creative Publications; Cuisenaire Co.; TUF)

PAGES 220–221

Objective

Given a division problem, the child will be able to find the quotient by repeated subtraction.

Preparation

To begin this chapter, use an oral game or drill to review basic multiplication and division facts. Include a drill of multiples of 10 and 100. For example, use the "What's My Rule" game, and say, "When I think of 240, my answer is 30; when I think of 720, my answer is 90; if I think of 320, can you give my answer?" (See page 65 for a description of the "What's My Rule" game.)

Investigation

Have the children work on this investigation independently. If children begin by repeatedly subtracting 7, they will probably soon realize that subtracting by sevens would be a tedious task. Of course, the simplest way would be to subtract a group of 10 sevens, or a group of 20 sevens and then smaller groups of sevens. Keep in mind that one of the basic purposes of this lesson is to help the child recognize the need for more efficient ways of finding quotients than by repeated subtraction of the divisor. In any case, remind them to be very careful in performing the subtraction, regrouping when necessary.

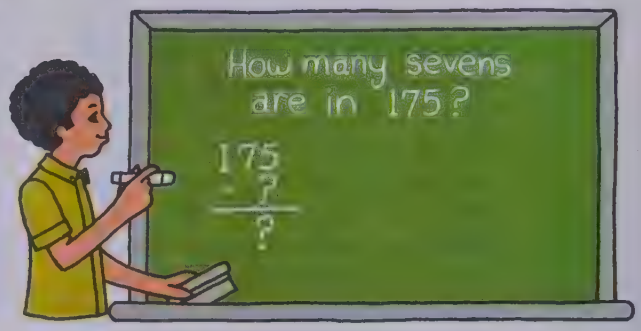
For those who finish quickly, you might suggest a few other exercises to investigate, such as:

How many nines are in 171?
How many fives are in 105?
How many sixteens are in 112?

10 Dividing

Can you find quotients by subtracting?

Investigating the Ideas



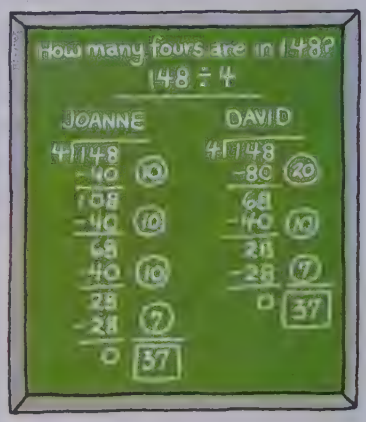
? Can you use subtraction to find how many sevens are in 175? See Investigation.

Discussing the Ideas

1. A How many sevens are in 175?
 B Solve: $175 \div 7 = n$ 25

2. A Explain the steps Joanne used to find how many fours are in 148. See Discussion.
 B How is David's work shorter than Joanne's? 37
 C Solve: $148 \div 4 = n$

3. Try these on your own.
 A $138 \div 6$ 23 B $204 \div 6$ 34





Discussion

Use discussion exercise 1 to relate the subtraction in the investigation to division. You may want to use a set demonstration and have children separate 175 counters into groups of sevens. Write on the chalkboard both the related division equation, $175 \div 7 = n$, and the division symbol used in the algorithm, $7\overline{)175}$. Help children realize that we use the symbol $\overline{)}$ to indicate division.

An important point in this lesson is to demonstrate the *need* for a division method which is more convenient than subtraction. Stress this as you discuss exercise 2. Encourage children to find a way that is even shorter than David's as you discuss exercise 2C; for example:

$$\begin{array}{rr} 4\overline{)148} & \\ 120 & (30) \\ \hline 28 & \\ 28 & (7) \\ \hline 0 & 37 \end{array}$$

Develop exercise 3 similarly, comparing ways different children used, emphasizing those which are the shortest. However, praise all those who do correct work, no matter how many subtractions they use.

Using the Ideas

1. Find the quotients. The numbers in the rings tell how many fives were subtracted each time.

A $115 \div 5$

23
5)115
−50 (10)
65
−50 (10)
15
−15 (3)
0

B $230 \div 5$

46
5) 230
−150 (30)
80
−50 (10)
30
−30 (6)
0

C $185 \div 5$

37
5) 185
−100 (20)
85
−50 (10)
35
−35 (7)
0

2. Copy each exercise. Give the number of threes for each ring. Then give the quotient.

A $105 \div 3$

35
3)105
−60 (20)
45
−30 (10)
15
−15 (5)
0

B $168 \div 3$

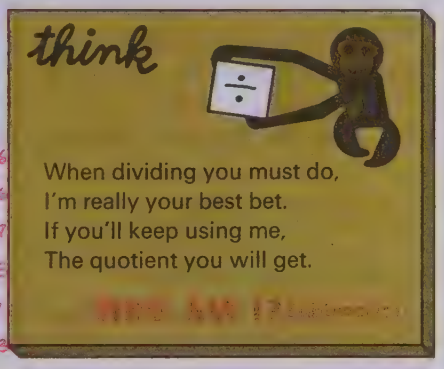
56
3)168
−90 (30)
78
−60 (20)
18
−18 (6)
0

C $144 \div 3$

48
3)144
−60 (20)
84
−60 (20)
24
−24 (8)
0

3. Use subtraction as in exercises 1 and 2 to help you find the quotient.

A $45 \div 3$ 15
B $104 \div 4$ 26
C $85 \div 5$ 17
D $144 \div 6$ 24
E $58 \div 2$ 29
F $84 \div 4$ 21
G $108 \div 3$ 36
H $180 \div 5$ 36
I $162 \div 6$ 27
J $140 \div 4$ 35
K $136 \div 8$ 17
L $207 \div 9$ 23



Using the Exercises

Explain to the children that in the exercises on page 221 some of the work has already been done for them. Then give them an opportunity to go ahead and do the exercises on their own. These exercises are intended to lead the children toward an understanding of the role of subtraction in this simple form of the division algorithm.

The children should be able to solve the *Think* problem if they understand the day's work in using successive subtraction to develop the concept of division.

Assignments (page 221)

Minimum: 1–3. Average: 1–3.
Maximum: 1–3.

Follow-up

Exercises involving multiples of 10 and 100 will continue to prepare the children to make better estimates of quotients. For this purpose, you might prepare a worksheet similar to the following.

1. For each set of numbers, write a multiplication and a division equation.
 A. 2, 180, 90
 B. 50, 3500, 70
 C. 320, 80, 4
 D. 9, 630, 70
 E. 6, 240, 40
 F. 4200, 600, 7
2. For each pair of numbers, write a multiplication and a division equation. Supply the missing number for each equation yourself.
 A. 2700, 90
 B. 8, 560
 C. 4, 1600
 D. 5, 400
 E. 70, 80
 F. 6, 4800

Workbook, pages 67–68

PAGES 222–223

Objective

Given division exercises which have remainders, the child will be able to find the quotient by subtracting and will recognize that the remainder must be less than the divisor.

Preparation

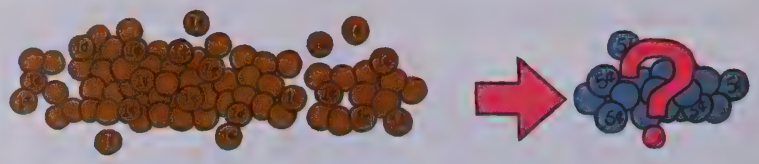
You might find it appropriate to begin immediately with the investigation and let it function as the preparation for the discussion. However, if you prefer, conduct a short oral practice session on the basic multiplication facts. Continue to include facts which use multiples of 10 and 100.

Investigation

Have children work on this investigation independently. Encourage them to recall what they learned about division and subtraction in the previous lesson but to use any method they wish to answer the question. Upon subtracting fives from 83, they should find that 3 pennies will be left over after they have gotten 16 nickels for the 80 pennies. This is the main point of the investigation, namely, that some numbers cannot be divided evenly, so they have a remainder.

● *Let's explore quotients and remainders.*

Investigating the Ideas



Can you find the number of nickels you can get for 83 pennies?
16 nickels, with 3 pennies left over

See Investigation.

Discussing the Ideas

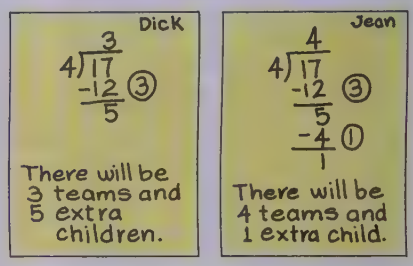
Study this example to review the meaning of the words **divisor**, **quotient**, **dividend**, and **remainder**.

$$\begin{array}{r} 8 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow 6\overline{)50} \leftarrow \text{dividend} \\ -48 \\ \hline 2 \leftarrow \text{remainder} \end{array}$$

1. In the Investigation you found how many fives are in 83. Was this number the quotient or the remainder? Quotient

2. Is the number of extra pennies the quotient or the remainder? Remainder

3. Dick and Jean were asked to find how many teams of 4 could be formed from 17 children.



 A. Whose paper do you think is correct? Jean's

 B. Name the divisor, quotient, dividend, and remainder on Jean's paper. Divisor, 4; quotient, 4; dividend, 17; remainder, 1

4. Give the missing word.
 If the dividing has been completed correctly, then the remainder is less than the __?__. divisor

222

Discussion

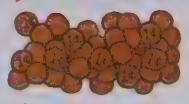
As you discuss the children's investigation results, you may want to use sets to demonstrate other examples of division with a remainder. Write a division algorithm on the chalkboard for each example you demonstrate and explain the terms *divisor*, *quotient*, *dividend*, and *remainder*. As you discuss exercises 1 and 2, point out that the remainder is less than the divisor. With exercise 3A ask the children to explain why Dick's work is incorrect, and how he could correct it. They might recognize that Jean's work shows how Dick could correct his. Also, ask if anyone can suggest an easier method, that is, guessing 4 as the quotient the first time. Point out that the remainder is always less than the divisor, if the division has been properly performed. During your discussion, explain to the children the following form for writing quotient and remainder.

$$\begin{array}{r} 3, \text{ R } 2 \\ \hline 4)14 \end{array}$$

Explain that this is read as "14 divided by 4 is 3 with remainder 2."

Using the Ideas

1. A How many nickels can you get for 47 pennies? 9
 B How many pennies will be left? 2



2. There were 49 boys at the park. If they were divided into baseball teams with 9 on a team,
 A how many teams could be made? 5
 B how many extras would there be? 4



3. A carton holds 6 bottles of soda.
 A How many cartons can you fill if you have 52 bottles? 8
 B How many extra bottles will there be? 4

4. The coach asked 45 boys to line up in rows of 7.
 A How many full rows did they make? 6
 B How many boys were left? 3



5. Kay has 59 stamps. She pasted 8 stamps in each row.
 A How many rows of 8 could she make? 7
 B How many extra stamps would she have to start another row? 3

6. Copy each exercise. Find the quotient and the remainder.

A $6\overline{)40}$ 6R4	B $3\overline{)23}$ 7R2	C $2\overline{)17}$ 8R1	D $5\overline{)27}$ 5R2	E $4\overline{)33}$ 8R1	F $6\overline{)39}$ 6R3
G $8\overline{)62}$ 7R6	H $7\overline{)41}$ 5R6	I $9\overline{)58}$ 6R4	J $4\overline{)31}$ 7R3	K $8\overline{)55}$ 6R7	L $9\overline{)63}$ 7

7. Divide these numbers by 3 and list the remainders. 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30
 See Answers, T.E. page 223.

8. List the possible remainders when any number is divided by 4. 0, 1, 2, 3



Follow-up

Exercises 7 and 8 may be used as an extension in the understanding of the division algorithm. The discussion may take this form.

"We agree that

Dividend equals divisor times quotient plus remainder."

In equation form,

$$D = (d \times q) + r$$

Using this equation, look at the pattern for dividing by 4.

$4 = (4 \times 1) + 0$ $\quad 8 = (4 \times 2) + 0$
$5 = (4 \times 1) + 1$ $\quad 9 = (4 \times 2) + 1$
$6 = (4 \times 1) + 2$ $\quad 10 = (4 \times 2) + 2$
$7 = (4 \times 1) + 3$ $\quad 11 = (4 \times 2) + 3$

What will the pattern look like if 12 is the dividend? Complete these equations.

$12 = (4 \times 3) + 0$ $\quad 16 = (4 \times 4) + 0$
$13 = (4 \times 3) + 1$ $\quad 17 = (4 \times 4) + ?$
$14 = (4 \times 3) + 2$ $\quad 18 = (4 \times 4) + ?$
$15 = (4 \times 3) + 3$ $\quad 19 = (4 \times 4) + ?$

What are the remainders when dividing by 4? Build a pattern to show remainders when dividing by 6.

Workbook, page 69

Using the Exercises

On page 223, children may need guidance in writing the division problems in the standard algorithmic form. If necessary, work through the first few with them. Suggest that they think of missing factors in order to find the quotient. For example, in exercise 1 a child may think "5 nines are 45, so I can get nine nickels for 47 cents and have two pennies left over." He would write

$$\begin{array}{r} 9,\ \text{R2} \\ 5\overline{)47} \\ 45 \\ \hline 2 \end{array}$$

Answers, exercise 7, page 223

7. $12 \div 3 = 4$
$13 \div 3 = 4,\ \text{R1}$
$14 \div 3 = 4,\ \text{R2}$
$15 \div 3 = 5$
$16 \div 3 = 5,\ \text{R1}$
$17 \div 3 = 5,\ \text{R2}$
$18 \div 3 = 6$
$19 \div 3 = 6,\ \text{R1}$
$20 \div 3 = 6,\ \text{R2}$
$21 \div 3 = 7$
$22 \div 3 = 7,\ \text{R1}$
$23 \div 3 = 7,\ \text{R2}$
$24 \div 3 = 8$
$25 \div 3 = 8,\ \text{R1}$
$26 \div 3 = 8,\ \text{R2}$
$27 \div 3 = 9$
$28 \div 3 = 9,\ \text{R1}$
$29 \div 3 = 9,\ \text{R2}$
$30 \div 3 = 10$

Assignments (page 223)

Minimum: 1–8, oral. Average: 1–8.
Maximum: 1–8.

PAGES 224–225

Objective

Given division problems with single-digit divisors, the child will be able to give estimates of the quotients.

Preparation

To prepare for this lesson, review the meaning of the inequality symbols, $>$ (greater than), $<$ (less than). For example, ask someone to write with mathematical symbols the statement "seven is less than twenty" ($7 < 20$). Continue with several more examples, including other symbols if you think your class would benefit:

$=$ (equals) $\div$ (divided by) $\times$ (times)

$\leq$ (less than or equal to)

$\geq$ (greater than or equal to)

Investigation

This investigation is intended to be completed after a few minutes of independent work. Instruct the children to list only those numbers which make the inequality a true statement. Some children may immediately realize that, since 5 may not be used, neither may 6, 7, 8, or 9. Others may try each of the numbers to see if they might be used. For those who finish quickly, suggest a similar exercise with the following numbers:

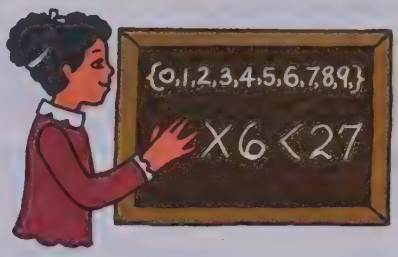
{0,1,2,3,4,5,6,7,8,9} ? $\times 4 < 35$

or

{6,7,8,9,10,11,12,13,14,15} ? $\times 3 < 34$

How can estimation help you find quotients?

Investigating the Ideas



Gail

? Which numbers from the set do you think Gail might be covering? 0,1,2,3,4

See Investigation.

Discussing the Ideas

1. A Which numbers were too large for Gail to be covering? 5,6,7,8,9

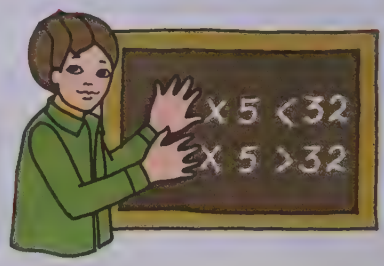
 B What is the largest number she could be covering? 4

 Is it the quotient for $6\overline{)27}$? Yes

2. Dan is covering one of these pairs of numbers. $\begin{matrix}5 & 6\\ 6 & 7\end{matrix}$

 A Which pair do you think Dan is covering? $\begin{matrix}6\\7\end{matrix}$

 B What is the quotient for $5\overline{)32}$? 6



Dan

3. A Which pair $\begin{matrix}3\\4\end{matrix}$ or $\begin{matrix}4\\5\end{matrix}$ is correct for $\begin{matrix}\times 4 < 18\\ \times 4 > 18\end{matrix}$? $\begin{matrix}4\\5\end{matrix}$

 B What is the quotient for $4\overline{)18}$? 4

224



Discussion

In order for the investigation to be pertinent, it is essential that the answers the children listed (0,1,2,3,4) be related to the quotient $6\overline{)27}$. Discussion exercise 1 should help them realize that the greatest number which makes the inequality true is the best guess for the quotient. Discuss exercise 1 in relation to any additional exercises used in the investigation. For both exercises 2 and 3, have children show on the chalkboard the two inequalities that result from substituting the correct pair. Then have children give the correct products so that everyone will see that the inequality signs are correct. Again point out that the largest number which makes the "less than" inequality true is the best guess for the quotient.

Using the Ideas

1. For each of parts A through H, copy the two inequalities. Use the correct number pair from those which follow. Then find the quotient and remainder.

Number pairs:	1	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9	10

Example:

$\square \times 4 < 17$
$\times 4 > 17$ → $4\overline{)17}$

Answer:

$4 \times 4 < 17$
$5 \times 4 > 17$ → $4\overline{)17}$ quotient 4, 16, remainder 1

A
$9 \times 5 < 47$
$10 \times 5 > 47$ → $5\overline{)47}$ 9 R2

B
$5 \times 2 < 11$
$6 \times 2 > 11$ → $2\overline{)11}$ 5 R1

C
$8 \times 3 < 26$
$9 \times 3 > 26$ → $3\overline{)26}$ 8 R2

D
$7 \times 4 < 30$
$8 \times 4 > 30$ → $4\overline{)30}$ 7 R2

E
$4 \times 6 < 27$
$5 \times 6 > 27$ → $6\overline{)27}$ 4 R3

F
$9 \times 3 < 29$
$10 \times 3 > 29$ → $3\overline{)29}$ 9 R2

G
$6 \times 7 < 46$
$7 \times 7 > 46$ → $7\overline{)46}$ 6 R4

H
$5 \times 8 < 45$
$6 \times 8 > 45$ → $8\overline{)45}$ 5 R5

2. Find the **largest** whole number that will make the sentence true. Then copy the completed sentence. Find the quotient and remainder.

Example:

$n \times 5 < 37$ → $5\overline{)37}$

Answer:

$7 \times 5 < 37$ → $5\overline{)37}$ quotient 7, 35, remainder 2

A $n8 \times 5 < 43$ → $5\overline{)43}$ 8 R3

B $n7 \times 3 < 22$ → $3\overline{)22}$ 7 R1

C $n8 \times 4 < 35$ → $4\overline{)35}$ 8 R3

D $n5 \times 8 < 47$ → $8\overline{)47}$ 5 R7

E $n6 \times 3 < 20$ → $3\overline{)20}$ 6 R2

F $n6 \times 6 < 38$ → $6\overline{)38}$ 6 R2

G $n7 \times 7 < 50$ → $7\overline{)50}$ 7 R1

H $n9 \times 7 < 65$ → $7\overline{)65}$ 9 R2

More practice, page A-21, Set 38



Using the Exercises

Since exercise 1 on page 225 is similar to exercises 2 and 3 of the discussion section, work through the directions and example with the children and then assign the remaining exercises as independent work. You might observe with the children that in exercise 2 the inequality is always "less than," so they must find the largest whole number that will make the product less than the given number.

Assignments (page 225)
Minimum: 1–2. Average: 1–2.
Maximum: 1–2.

Follow-up

To help children better relate the use of estimation and inequalities to finding quotients, write on the chalkboard or prepare on a duplicated worksheet exercises similar to the ones below.

For each of the following, give an inequality that will help show the best guess for each quotient. Then find the quotients and remainders.
Example:

$4 \times 6 < 26$ → $4\overline{)26}$ quotient 6 R2, -24, 2 ⑥

$7\overline{)34}$
$8\overline{)49}$
$4\overline{)37}$
$6\overline{)40}$
$5\overline{)28}$
$3\overline{)14}$
$9\overline{)80}$

PAGES 226–227

Objective

Given a division problem with a single-digit divisor and a quotient greater than 10, the child will be able to find the quotient by subtracting multiples of 10.

Preparation

Since this lesson deals with multiples of 10, you might want to review products of multiples of 10 and 100. For variety, you might use the written form of the "What's My Rule" game and prepare a grid something like the following:

e	630	450	?	?	?	?
d	140	100	?	?	?	?
a	7	5	4	3	8	6
b	210	150	?	?	?	?
c	560	400	?	?	?	?

Notice that each new row is formed by multiplying row *a* by some multiple of 10. (See page 65 for a more detailed description of this game.)



Let's organize our work.

Discussing the Ideas

Copy the part of the example that says "write" as you work through the steps for finding 136 ÷ 4.

Step 1

Think: I can subtract 20 fours from 136.

Think	Write
4)136	4)136
−80 ← 20 × 4	−80 (20)
56	56

Step 2

Think: I can subtract 10 more fours.

Think	Write
4)136	4)136
−80 ← 20 × 4	−80 (20)
56	56
−40 ← 10 × 4	−40 (10)
16	16

Step 3

Think: I can subtract 4 more fours.

Think	Write
34 ←	34
4)136	4)136
−80 ← 20 × 4	−80 (20)
56	56
−40 ← 10 × 4	−40 (10)
16	16
−16 ← 4 × 4	−16 (4)
0	0

1. Can you find the quotient 136 ÷ 4 by using subtractions different from those above? Yes; for example, subtract 30 fours and then 4 fours.

2. Find the quotient 215 ÷ 5. Check your work with your teacher. 43 See Discussion.

Discussion

The diagrams on page 226 show one way of organizing the long-division process by using subtraction. Discuss each step of the process and ask the children to answer discussion exercises 1 and 2.

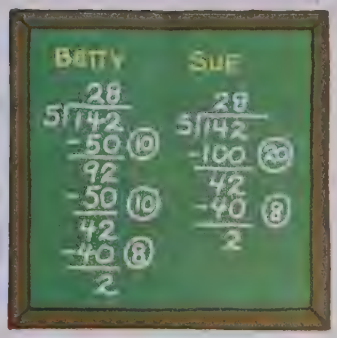
As you discuss exercise 1, help children see that 30 fours may have been subtracted as a first step, thus reducing the number of subtractions needed to find the quotient. Have volunteers write their work for exercise 2 on the chalkboard. Emphasize that the best first step is that which subtracts the highest multiple of ten times the divisor, which here is the highest multiple of ten times five.

5)215	
200	(40)
15	
15	(3)
0	

However, take time to work through other steps with this same problem so that children will know how to work with a problem when their first guess is not accurate.

Using the Ideas

1. Betty and Sue each found the quotient and remainder for 142 ÷ 5.
 - A Did both girls get the same quotient? Yes
 - B How many fives did Betty subtract the first time? 10
 - C How many fives did Sue subtract the first time? 20
 - D Whose work is shorter? Sue's



2. Copy each exercise and give the missing numbers.

A
38
4)152
−40 (10)
112
−40 (10)
72
−40 (10)
32
−32 (8)
0

B
38
4)152
−80 (20)
72
−40 (10)
32
−32 (8)
0

C
38
4) 152
−120 (30)
32
−32 (8)
0

3. Did you get the same quotient for each part of exercise 2? Yes

4. Find the quotients.

A 13 3)39	B 15 5)75	C 22 4)88	D 13 R1 6)79
E 23 R2 7)163	F 34 4)136	G 27 R1 6)163	H 35 8)280
I 39 8)312	J 26 9)234	K 84 5)420	L 54 6)324
M 63 4)252	N 34 R5 7)243	O 64 R2 7)450	P 29 R3 8)235



Using the Exercises

On page 227, exercise 1 may be used as a basis for discussion. For example, ask the children to study the two problems in the illustration of the chalkboard and work through the exercise questions. Assign exercises 2 and 3 and selected parts of exercise 4. Encourage the class to make their work efficient by trying to subtract the largest possible multiple the first time in each problem.

Assignments (page 227)
Minimum: 1, oral; 4. Average: 1–4.
Maximum: 1–4.

Follow-up

For additional practice in finding the largest possible multiple and the smallest possible remainder, duplicate a worksheet of problems like the following ones.

DIVIDEND EQUALS DIVISOR times QUOTIENT plus REMAINDER
Now study the examples and find the missing numbers.
60 = (2 × 30) + 0
64 = (2 × 30) + 4
150 = (___ × 50) + ___
152 = (___ × 50) + ___
240 = (___ × 60) + ___
243 = (___ × 60) + ___
86 = (8 × ___) + ___
127 = (___ × 60) + ___
146 = (___ × 20) + ___
354 = (___ × 50) + ___
285 = (4 × ___) + ___
636 = (___ × 90) + ___

Duplicator Masters, page 42
Workbook, page 70
Skill Masters, page 42

PAGES 228–229

Objective

Given division exercises with 1-digit divisors, the child will estimate the quotients to the nearest multiple of ten that is less than the dividend.

Preparation

Conduct a brief oral practice session in which the children find products such as 7×30, 6×50, 8×70, 9×20, 4×90, and 2×80. After this, write equations such as $n \times 80 = 560$, $n \times 6 = 420$, $n \times 4 = 320$, $n \times 7 = 350$, and so on, on the board, and ask the children to give the missing factor. Here they must find the multiple of ten which, when multiplied by a 1-digit number, gives a certain product.

Investigation

One of the most important aims of this lesson is to help children develop the ability to think quickly and accurately about finding the product of a given number and a multiple of ten and then relate this skill to finding quotients. For this investigation, have children work independently to verify the correctness of the inequalities by finding the product of each pair of factors. Then let them proceed to try to use the inequalities as an aid in finding the quotient. You might suggest that children who finish quickly try the same approach with another pair of inequalities and quotient, such as:

$$\left.\begin{array}{l} 60 \times 7 < 448 \\ 70 \times 7 > 448 \end{array}\right\} 7\overline{)448}$$

● *Let's improve our estimated quotients.*

Investigating the Ideas

Are these inequalities correct? Yes

$$\left\{\begin{array}{l} 40 \times 6 < 282 \\ 50 \times 6 > 282 \end{array}\right.$$



? Can you use the inequalities to help you find this quotient in the fewest number of steps?

$$6\overline{)282} \quad 47$$

See Investigation.

Discussing the Ideas

Give the number pair for each gray space. Then explain how to use your answer for your first quotient estimate. See Discussion.

Number pairs:

10	20	30	40	50	60	70	80	90
20	30	40	50	60	70	80	90	100

1. 30 $\times$ 4 $<$ 128 / 40 $\times$ 4 $>$ 128 $\rightarrow 4\overline{)128}$ 32
2. 60 $\times$ 5 $<$ 325 / 70 $\times$ 5 $>$ 325 $\rightarrow 5\overline{)325}$ 65
3. 80 $\times$ 3 $<$ 264 / 90 $\times$ 3 $>$ 264 $\rightarrow 3\overline{)264}$ 88
4. 30 $\times$ 4 $<$ 132 / 40 $\times$ 4 $>$ 132 $\rightarrow 4\overline{)132}$ 33
5. 10 $\times$ 9 $<$ 162 / 20 $\times$ 9 $>$ 162 $\rightarrow 9\overline{)162}$ 18
6. 70 $\times$ 8 $<$ 608 / 80 $\times$ 8 $>$ 608 $\rightarrow 8\overline{)608}$ 76

Discussion

Allow volunteers to show on the chalkboard their work for the investigation. Be sure the children recognize why it is better for them to use the factor in the "less than" statement as their first estimate of the quotient.

The discussion exercises continue this idea of relating inequalities to finding quotients. Have the children notice again in these exercises that the best estimate of the quotient is the factor in the "less than" statement. In general, they should look for the largest possible multiple of ten that will make the product less than the given number.

It would be helpful to demonstrate the use of this skill of estimating quotients with the quotients in exercises 1 through 6. Show the children how to subtract the product of the divisor and their estimated quotient from the dividend as the first step in division.

$30 \times 4 < 128$

$$\begin{array}{rl} & 32 \\ 4\overline{)128} & \\ 120 & (30) \\ \hline 8 & \\ 8 & (2) \\ \hline \end{array}$$

Work through several exercises, stressing both the skill of estimating and the steps in the division method.

Using the Ideas

1. From the set {10, 20, 30, 40, 50, 60, 70, 80, 90}, find the **largest** number that will make each sentence true.

A $n \times 4 < 128$ (Answer: $30 \times 4 < 128$)
B $n \times 4 < 92$ 20
C $n \times 3 < 96$ 30
D $n \times 7 < 189$ 20
E $n \times 6 < 258$ 40
F $n \times 7 < 448$ 60
G $n \times 8 < 272$ 30
H $n \times 9 < 387$ 40
I $n \times 5 < 390$ 70
J $n \times 7 < 546$ 70
K $n \times 8 < 456$ 50
L $n \times 7 < 294$ 40
M $n \times 6 < 510$ 80

2. Copy the problem. Complete the work of finding the quotient and remainder.

A 38: $8\overline{)304}$, -240 (30)
B 65 R2: $5\overline{)327}$, -300 (60)
C 57: $9\overline{)513}$, -450 (50)
D 49: $6\overline{)294}$, -240 (40)

3. Copy each exercise and write the correct digits instead of the ▮.

A 42: $5\overline{)213}$, $200 - ▮▮0$ (40), $13\,▮▮$, $10 - ▮▮$ (▮2), $3▮$
B 87: $7\overline{)612}$, $560 - ▮▮0$ (80), $52\,▮▮$, $49 - ▮▮$ (▮7), $3▮$
C 39: $4\overline{)156}$, $120 - ▮▮0$ (30), $36\,▮▮$, $36 - ▮▮$ (▮9), $0▮$
D 54: $8\overline{)435}$, $400 - ▮▮0$ (50), $35\,▮▮$, $32 - ▮▮$ (▮4), $3▮$

4. Find the quotients and remainders.

A 22: $3\overline{)66}$
B 23 R2: $4\overline{)94}$
C 27: $7\overline{)189}$
D 24 R1: $4\overline{)97}$
E 53: $8\overline{)424}$
F 98 R4: $5\overline{)494}$
G 66: $7\overline{)462}$
H 85 R3: $6\overline{)513}$
I 26: $9\overline{)234}$
J 92 R2: $4\overline{)370}$

think

In the A figure below, the ▮ covers the same digit. What is this digit? What digit is covered by the ▮ in B?



A: 5, ▮1 × ▮5 = 2▮5, 55
B: 6, ▮1 × ▮6 = 3▮6, 66

More practice, page A-21, Set 39



Follow-up

If children would benefit from more practice, you might write several exercises on the chalkboard. Children might work through the exercises on their own, or you might have them work with partners and write an inequality to determine the estimate for each quotient. A worksheet similar to the following might also be helpful.

Find the largest multiple of ten that will make the inequality true. Then use that number to help you find the quotient.

$n \times 9 < 612$	$9\overline{)612}$
$n \times 8 < 684$	$8\overline{)684}$
$n \times 9 < 747$	$9\overline{)747}$
$n \times 6 < 534$	$6\overline{)534}$
$n \times 3 < 252$	$3\overline{)252}$

Duplicator Masters, page 43
Workbook, page 71
Skill Masters, page 43

Using the Exercises

Following the demonstrations suggested in the discussion section, assign selected exercises on page 229 as independent work. When the children have finished, ask volunteers to work through some of the exercises on the chalkboard, explaining their steps as they proceed.

The quick solution to the *Think* problem hinges upon the clue that, within each problem, the missing digit is the same for all the screens. By looking at the third digit in the answer, which gives the limits of the second product, the proper doubles combination can be quickly located. For example in part A, the child might explain his thinking by saying, "Try four. Four times one is four, and four times forty is one hundred sixty. Four is too small. Try five" Some children may wish to try 6 after finding that 5 will work, just to prove to themselves that 5 is the only logical choice.

Assignments (page 229)
Minimum: 1, 4. Average: 1–4.
Maximum: 1–4.

PAGES 230–231

Objective

Given division exercises with single-digit divisors, the child will be able to find the quotients and remainders and check each exercise by multiplying the divisor by the quotient and adding the remainder.

Preparation

Conduct a short oral drill of basic multiplication facts to review the relation between multiplication and division. For example, name three numbers and ask children to give both a multiplication and division equation using them, such as 7, 8, 56 and 56 ÷ 7 = 8. Include multiples of ten in the numbers you give them, but stress the relation between the multiplication and division equations.

Investigation

You might suggest that the children work in groups of two or three for this investigation. Explain that they are to try to find a way of showing whether or not Matt's answers are correct, and they are to do so by using multiplication, not division. Circulate around the room as children are working. If some seem to be having difficulty, you might give them a hint by reminding them that they know 48 ÷ 6 = 8 because 8 × 6 = 48. If children apply this relationship and multiply the quotient by the divisor, they will find that this product equals the dividend in exercises 1, 2, and 4. They should find that exercise 3 is incorrect; the answer should be 36, R 0. Exercise 5 requires that the remainder 2 be added to the product 3 × 26. If any children finish quickly, suggest similar exercises for them to check.

How can we check answers in dividing?

Investigating the Ideas

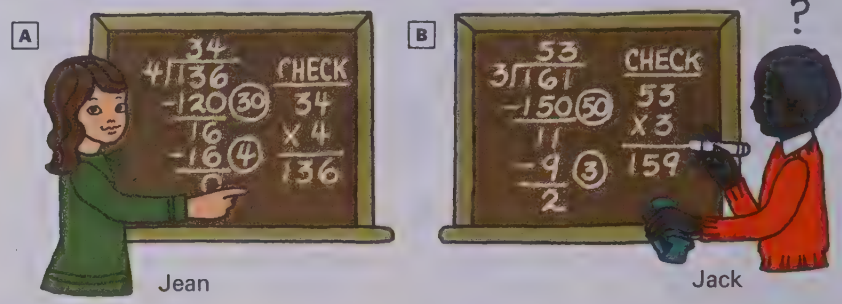


Matt	answers
1. 65 ÷ 5	1. 13, R0
2. 52 ÷ 2	2. 26, R0
3. 108 ÷ 3	3. 38, R0
4. 108 ÷ 4	4. 27, R0
5. 80 ÷ 3	5. 26, R2

Can you use multiplication to help you grade Matt's paper? (R stands for remainder.)
See Investigation.

All correct except exercise 3, which should be 36.

Discussing the Ideas



1. Jean checked problem A by multiplying. How did she know her quotient was correct?
The product of the quotient and the divisor was equal to the dividend.

2. Jack checked problem B. His product was 159. He thought it should be 161. Explain how Jack can finish checking.
Add the remainder to 159.

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Discussion

Have children explain how they used multiplication to check Matt's answers. Develop children's understanding of the idea that *dividend equals divisor times quotient plus remainder.*

Work through the two discussion exercises. Observe that in problem A, the check worked out correctly (that is, there is a divisor of 4, a dividend of 136). However, when Jack checked problem B, he found that his divisor times his quotient (3 × 53) was 159, and he wanted to get 161. Have the children explain why Jack did not get 161. They should understand that, unless the remainder is zero, they must add the remainder to the product of the divisor and the quotient to get the dividend.

Using the Ideas

1. From {10, 20, 30, 40, 50, 60, 70, 80, 90}, find the largest number that will make each sentence true.
 - A Since $6 \times 4 < 25$, we know that $n \times 4 < 256$. 60
 - B Since $5 \times 3 < 16$, we know that $n \times 3 < 164$. 50
 - C Since $8 \times 7 < 59$, we know that $n \times 7 < 595$. 80
 - D Since $7 \times 6 < 43$, we know that $n \times 6 < 439$. 70
 - E Since $9 \times 6 < 55$, we know that $n \times 6 < 554$. 90
 - F Since $9 \times 8 < 74$, we know that $n \times 8 < 748$. 90
 - G Since $9 \times 9 < 83$, we know that $n \times 9 < 837$. 90

2. Find the quotients and remainders. Check each exercise. Exercise 1 should help you.

A $4\overline{)256}$ 64	B $3\overline{)164}$ 54 R2	C $7\overline{)595}$ 85	D $6\overline{)439}$ 73 R1	E $2\overline{)92}$ 46
F $5\overline{)135}$ 27	G $6\overline{)554}$ 92 R2	H $8\overline{)748}$ 93 R4	I $9\overline{)837}$ 93	J $7\overline{)656}$ 93 R5

3. Find the quotients and remainders. Check each exercise.

A $5\overline{)305}$ 61	B $3\overline{)249}$ 83	C $4\overline{)85}$ 21 R1	D $7\overline{)327}$ 46 R5	E $8\overline{)363}$ 45 R3
F $9\overline{)605}$ 67 R2	G $3\overline{)288}$ 96	H $7\overline{)620}$ 88 R4	I $2\overline{)179}$ 89 R1	J $5\overline{)385}$ 77
K $8\overline{)603}$ 75 R3	L $4\overline{)312}$ 78	M $9\overline{)506}$ 56 R2	N $7\overline{)523}$ 74 R5	O $8\overline{)536}$ 67

think

Find the digit for each ▯. Find as many of the other digits as you can.

1.
$$\begin{array}{r} x \\ 8\overline{)\,2xx} \\ -xxx \\ \hline 1x \\ -xx \\ \hline 0 \end{array}$$

2.
$$\begin{array}{r} 3x \\ x\overline{)\,31x} \\ -\,7x \\ \hline xx \\ -xx \\ \hline 2 \end{array}$$



More practice, page A-22, Set 40



Follow-up

You might challenge the children with other exercises similar to those in the *Think* problem. Here are a few examples.

x1	60	xx
$9\overline{)6xx}$	$x\overline{)x00}$	$4\overline{)x12}$
$-x3x$	$-x00$	$-2x0$
9	0	3x
$-x$		$-xx$
0		0

x2	xx	4x
$6\overline{)4xx}$	$7\overline{)x95}$	$x\overline{)33x}$
$-x8x$	$-56x$	$-x2x$
1x	xx	xx
$-xx$	$-xx$	$-xx$
0	0	7

Solutions

71	60	78
$9\overline{)639}$	$5\overline{)300}$	$4\overline{)312}$
-630	-300	-280
9	0	32
-9		-32
0		0

82	85	41
$6\overline{)492}$	$7\overline{)595}$	$8\overline{)335}$
-480	-560	-320
12	35	15
-12	-35	-8
0	0	7

Resources for Active Learning

Mathematics in Modules, WN21, Addison-Wesley.

Workbook, page 72

Using the Exercises

Before assigning the exercises on page 231, you may want to help children see the relationship between exercises 1 and 2. Observe with the children that, since $6 \times 4 < 25$, they should be able to reason immediately that $60 \times 4 < 256$. Although such reasoning encourages the children to consider the possible shortcuts for finding the number of tens in the quotient, it would be better not to make an issue of the connection between this and finding quotients and remainders. Keep in mind that the *Investigating School Mathematics* approach to division concentrates primarily on numbers and not on a breakdown of digits, in either the dividend, the quotient, or the divisor. At this time, we prefer to avoid making a special point of thinking "8 into 57" for the division $8\overline{)572}$. We would rather have the children think of the largest multiple of ten that they can multiply by eight to obtain a product less than 580, and then discover their own shortcuts from this.

Assignments (page 231)

Minimum: 1–3F. Average: 1–3. Maximum: 1–3.

PAGES 232–233

Objective

Given a set of numbers (containing not more than 6 or 7 elements), the child will be able to find the average of the numbers in the set.

Preparation

Because of the nature of both the investigation and the concept developed, you might find it appropriate to omit specific preparation and begin immediately with the investigation. You might simply explain to the children that the concept treated in this lesson is one they should find particularly interesting and which will supply them with ideas for class projects when they sufficiently understand it.

Investigation

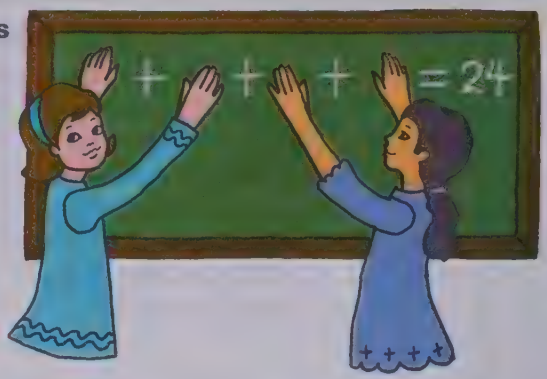
This investigation may be done by children working individually or by children working in groups of two or three. Suggest to the children that, after they have found the two sets of numbers called for in the text, they might try to list some other sets of 4 numbers which might be covered. You might also present a similar exercise on the chalkboard, such as

$$? + ? + ? = 18$$

Remind children to record their work carefully. When they have had sufficient time to work on these questions, ask volunteers to write on the chalkboard the 4 numbers which they thought the girls might be covering.

What is the average of the numbers in a set?

Investigating the Ideas



Pam and Julie are covering 4 addends. Find 4 numbers (not all the same) they **might** be covering.

Several combinations are possible. For example:
4+5+6+9
4+5+7+8

Can you find the hidden numbers if they are all the same? 6

Discussing the Ideas

1. A Will the numbers in this set ⟶ {7, 4, 8, 5} work in the example above? Yes

 B We say that the **average** of the numbers in this set of numbers is 6. What is special about the number 6 with respect to this set? Sample response: The sum of four 6's is the same as the sum 7+4+8+5.

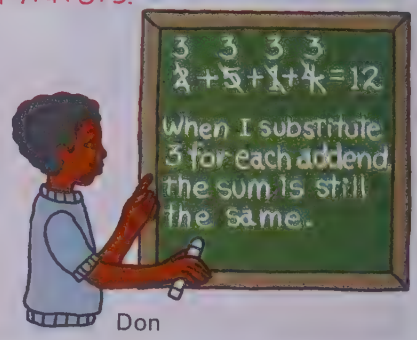
2. Look at Don's example. What is the average of the numbers in {2, 5, 1, 4}? 3

3. Can you find a "substitute" (the average) for the addends in each?

 A $4 + 5 + 9 = 18$ 6

 B $5 + 7 + 12 + 8 = 32$ 8

 C $10 + 5 + 6 = 21$ 7





232

Discussion

Use the results of the investigation, and discussion exercises 1 and 2, to explain the meaning of average. Have children note that 6 is the average of all the numbers in the sets they wrote on the chalkboard in relation to the investigation equation. Also, have them note that, if all the hidden numbers are the same, each would be 6. Work through the remaining discussion exercises.

It would be better not to give the children the usual rule for finding averages at this time. When the children have worked on several exercises, you might give them hints leading toward discovery of a rule if they have not already found one. For example, in exercise 2 have the children note that the average of 2, 5, 1, and 4 is 3, that there are four addends, and that $12 \div 4 = 3$. Similarly, in exercise 3 C, the average is 7, there are three addends, and $21 \div 3 = 7$. The intention here is to have children discover a rule for finding averages through experiences in working with concepts. Thus, you will want to discuss the rule only after children have worked several of the exercises on page 233.

Using the Ideas

1. Find the "substitute" for the addends.
Then give the average of the addends.

A $4 + 3 + 8 = 15$ (Answer: 5 + 5 + 5 = 15 The average is 5.)
B $10 + 4 = 14$ 7
C $6 + 4 + 5 = 15$ 5
D $3 + 7 + 6 + 4 = 20$ 5
E $12 + 18 = 30$ 15
F $2 + 3 + 10 + 9 + 11 = 35$ 7
G $12 + 14 + 7 = 33$ 11

2. Find the "substitute" for the numbers in each set.
Then give the average of the numbers in the set.

A {2, 10} (Answer: 6 + 6 = 12 The average is 6.)
B {1, 2, 6} 3
C {2, 5, 4, 5} 4
D {6, 13, 11} 10
E {6, 9, 13, 8} 9
F {7, 17} 12
G {60, 74} 67
H {73, 80, 93} 82
I {72, 53, 68, 71} 66

3. Here is a list of points Jim scored in each of 3 basketball games. Give the **average** number of points scored. 10

Game 1	Game 2	Game 3
12	7	11

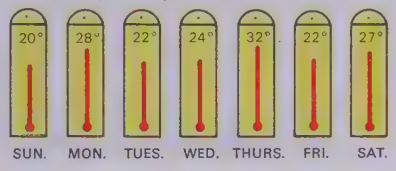
4. Here are Jane's spelling scores for one week. Find the average of her scores. 18



5. For each exercise, find the average of the numbers in the set.

A {3, 7, 5, 5} 5
B {8, 12, 7} 9
C {4, 3, 5, 4} 4
D {15, 21} 18
E {34, 42} 38
F {93, 97} 95
G {21, 34, 44} 33
H {62, 79, 87} 76
I {52, 56, 51} 53
J {9, 8, 7, 6, 5, 4, 3} 6
K {81, 82, 91, 78} 83
L {91, 68, 75, 84, 52} 74

6. The Celsius thermometers show the temperature for each day during one week. Give the average of these temperatures. 25°



More practice, page A-22, Set 41

Using the Exercises

As implied in the preceding section, these exercises should be assigned as independent work, to be done individually or in small groups.

After the children have worked through them, use them as a basis for discussion and give the standard rule for finding averages: *add the numbers and divide the total by the number of addends.* Encourage children to give any other rules they followed and discuss the merits of each.

Assignments (page 233)
Minimum: 1–2. Average: 1–6.
Maximum: 1–6.

Follow-up

The concept of average offers possibilities for class or group projects. For example, you might suggest the following projects and encourage children to make up their own.

1. Find the average daily high or low temperature of this month.
2. Find the average height of the children in this class.
3. Find the average daily attendance of this class for a week, or for two weeks.

Projects like these will give you an opportunity to stress development of such skills as collecting data, recording data, showing data on charts, etc.

Resources for Active Learning

Freedom to Learn, "Statistics," pp. 148–149, Addison-Wesley.
Mathematics in Modules, S2, Addison-Wesley.

Duplicating Masters, page 44
Workbook, page 73
Skill Masters, page 44

PAGES 234–235

Objective

Given division exercises with single-digit divisors and 3-digit quotients, the child will be able to find the quotient and remainder by using multiples of 100 for finding first quotient estimates.

Preparation

To prepare for this lesson, give the children two or three division problems with 2-digit quotients similar to the following. You might ask volunteers to work them on the chalkboard while others work at their places.

$7\overline{)595}$ $2\overline{)92}$ $9\overline{)837}$

If you prefer an oral warm-up activity, have the children find products like these: 6×500, 7×300, 4×200, 5×200, 9×300. You might also include practice with quotients and missing factors such as "What number times 8 equals 4800?" or "What number times 4 equals 3600?"



Let's explore 3-digit quotients.

Discussing the Ideas

1. Study this example.

Think:

Since $300 \times 6 < 1944$
$400 \times 6 > 1944$,

324			324
6)1944		we can subtract	6)1944
1800	← 300 × 6	**300** sixes from 1944.	1800 (300)
144	\|	Then we can	144
120	← 20 × 6	subtract **20** more sixes.	120 (20)
24	\|	Finally, we can	24
24	← 4 × 6	subtract **4** more sixes.	24 (4)
0		The quotient is 324.	0

Now find this quotient on your own.

$400 \times 3 < 1368$
$500 \times 3 > 1368$

456
$3\overline{)1368}$

2. Explain how you can use the inequalities to help you find your first quotient estimate.

The largest multiple of 100 which makes the less-than inequality true will be suitable as a first quotient estimate.

3. From {100, 200, 300, 400, . . .}, find the largest multiple of 100 for n. $n \times 4 < 1384$ 300
Then find this quotient. $4\overline{)1384}$ 346

4. Explain how the inequality helped you find your first quotient estimate. See exercise 2.

Discussion

Study the example in discussion exercise 1 with the children, noting with them the "thinking step" compared to the "writing step." It would be helpful to work through the written part step by step on the chalkboard.

Also, to clarify how the pair of inequalities shown at the top of the box applies to the problem and to making the best guess first, note that the inequalities show that there are as many as 300 sixes in 1944 but that there are not as many as 400 sixes in 1944. As children work on finding the quotient $3\overline{)1368}$, you might ask volunteers to prepare to present their work to the class. Relate exercise 2 to this example. If some children have difficulty understanding the role of the inequalities, do not belabor the point; such children may not be ready to make the "best guess" or to find "large" quotients right away. In exercise 3, point out that the three dots mean that the set continues beyond 400. Again ask children to explain their use of the inequalities, as directed in exercise 4.

Using the Ideas

1. Find the correct number pair.
Then find the quotient and remainder.

Number pairs:	100 200	200 300	300 400	400 500	500 600	600 700	700 800	800 900	900 1000

A 300 × 6 < 1944 / 400 × 6 > 1944 → 324 R 6)1944

B 200 × 3 < 711 / 300 × 3 > 711 → 237 R 3)711

C 100 × 5 < 643 / 200 × 5 > 643 → 128 R3 5)643

D 600 × 4 < 2437 / 700 × 4 > 2437 → 609 R1 4)2437

E 800 × 2 < 1715 / 900 × 2 > 1715 → 857 R1 2)1715

F 300 × 7 < 2462 / 400 × 7 > 2462 → 351 R5 7)2462

G 400 × 8 < 3847 / 500 × 8 > 3847 → 480 R7 8)3847

H 600 × 9 < 5964 / 700 × 9 > 5964 → 662 R6 9)5964

I 700 × 4 < 3143 / 800 × 4 > 3143 → 785 R3 4)3143

J 900 × 5 < 4987 / 1000 × 5 > 4987 → 997 R2 5)4987

2. Find the **largest** number from {100, 200, . . .}, that will make the sentence true. Then find the quotient and remainder.

A 300 $n \times 6 < 1944$ → 324 R 6)1944

B 500 $n \times 3 < 1624$ → 541 R1 3)1624

C 400 $n \times 6 < 2578$ → 429 R4 6)2578

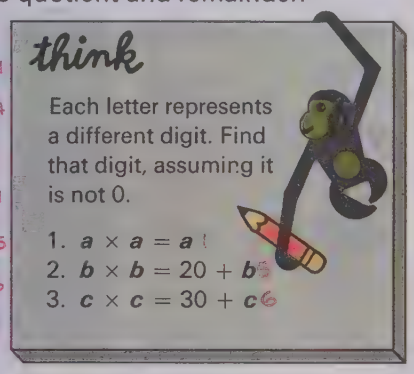
D 600 $n \times 4 < 2713$ → 678 R1 4)2713

E 800 $n \times 5 < 4316$ → 863 R1 5)4316

F 800 $n \times 8 < 6845$ → 855 R5 8)6845

G 900 $n \times 7 < 6425$ → 917 R6 7)6425

H 400 $n \times 3 < 1369$ → 456 R1 3)1369



More practice, page A-23, Set 42



Follow-up

To arouse enthusiasm for further practice in finding quotients, write the following code and exercises on the chalkboard and suggest that the children decode the "secret" message.

Code:

A	E	I	O	U	M	C
15	39	63	4	425	13	68
S	T	H	N	F		
59	35	34	320	132		

Exercises

1. $3\overline{)39}$ (*M*)
2. $5\overline{)75}$ (*A*)
3. $8\overline{)280}$ (*T*)
4. $4\overline{)136}$ (*H*)
5. $8\overline{)312}$ (*E*)
6. $5\overline{)65}$ (*M*)
7. $7\overline{)105}$ (*A*)
8. $4\overline{)140}$ (*T*)
9. $4\overline{)252}$ (*I*)
10. $5\overline{)340}$ (*C*)
11. $9\overline{)531}$ (*S*)
12. 7×9 (*I*)
13. $(4 \times 15) - 1$ (*S*)
14. $3\overline{)396}$ (*F*)
15. $7\overline{)2975}$ (*U*)
16. $5\overline{)1600}$ (*N*)

To find the message, children should match the answers of the exercises to the letters in the code. The message will begin to appear as the exercises are completed in order. The message is shown in italics for your convenience.

Duplicator Masters, page 45
Workbook, page 74
Skill Masters, page 45

Using the Exercises

Before assigning the exercises on page 235 as independent study, work through a few parts of each exercise to show the children how to organize their work. Remind them that they can look back at the "writing step" on page 234 if they need help in exercise 1 A.

In exercise 2, make sure children realize that the set 100, 200, 300, . . . does not stop at 300 but continues, as the three dots indicate, and includes all multiples of 100.

Assignments (page 235)
Minimum: 1. Average: 1–2.
Maximum: 1–2.

PAGES 236–237

Objective

Given division exercises with single-digit divisors, the child will practice finding quotients and remainders by making estimates based on inequalities.

Preparation

Conduct a short oral session to review all basic operations by giving the children some mental chain games.

Examples:

"Start with 240 . . . Divide by 8 . . . Divide by 6 . . . Multiply by 5. Your answer is ? " (25)

"Start with 81 . . . Divide by 9 . . . Multiply by 4 . . . Divide by 6 . . . Multiply by 4 . . . Divide by 3 . . . Add 2. Your answer is ? " (10)

"Start with 7 . . . Multiply by 5 . . . Subtract 3 . . . Divide by 4 . . . Add 1 . . . Multiply by 8 . . . Add 1. Your answer is ? " (73)

"Start with 2 . . . Multiply by 10 . . . Divide by 5 . . . Multiply by 10 . . . Add 5 . . . Divide by 5 . . . Add 1. Your answer is ? " (10)

Let's practice dividing.

1. Find the largest number from {100, 200, 300, 400, 500, 600, 700, 800, 900}, that will make each sentence true. Then find the quotient and remainder.

A Since $4 \times 3 < 14$, we know that $400 \times 3 < 1461$ → 3)1461 = 487

B Since $3 \times 9 < 29$, we know that $300 \times 9 < 2943$ → 9)2943 = 327

C Since $5 \times 8 < 42$, we know that $500 \times 8 < 4296$ → 8)4296 = 537

D Since $5 \times 2 < 11$, we know that $500 \times 2 < 1126$ → 2)1126 = 563

E Since $8 \times 6 < 51$, we know that $800 \times 6 < 5118$ → 6)5118 = 853

F Since $6 \times 4 < 27$, we know that $600 \times 4 < 2788$ → 4)2788 = 697

G Since $9 \times 7 < 67$, we know that $900 \times 7 < 6776$ → 7)6776 = 968

H Since $6 \times 8 < 55$, we know that $600 \times 8 < 5584$ → 8)5584 = 698

I Since $7 \times 9 < 65$, we know that $700 \times 9 < 6538$ → 9)6538 = 726R4

J Since $9 \times 4 < 39$, we know that $900 \times 4 < 3966$ → 4)3966 = 991R2

K Since $8 \times 3 < 25$, we know that $800 \times 3 < 2575$ → 3)2575 = 858R1

L Since $9 \times 5 < 46$, we know that $900 \times 5 < 4646$ → 5)4646 = 929R1

2. Find the quotients and remainders. Check each exercise.

A 3)396 = 132

B 2)1846 = 923

C 9)2727 = 303

D 5)1600 = 320

E 4)3243 = 810R3

F 6)5208 = 868

G 5)4008 = 801R3

H 9)3627 = 403

I 5)3360 = 672

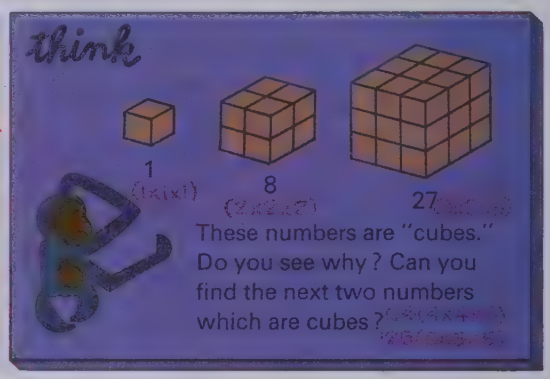
J 8)5002 = 625R2

K 7)2975 = 425

L 3)2862 = 954

M 7)4100 = 585R5

N 6)1459 = 243R1



236

Discussion

Before children begin page 236, note with them how the inequalities can help them make a good estimate of the first quotient. You may choose to work a few exercises together. Then assign selected parts from both 1 and 2. Since there is too much work here for some children, use discretion in determining individual assignments. When children have finished the exercises, allow time for further discussion and for checking papers.

Assignments (page 236)

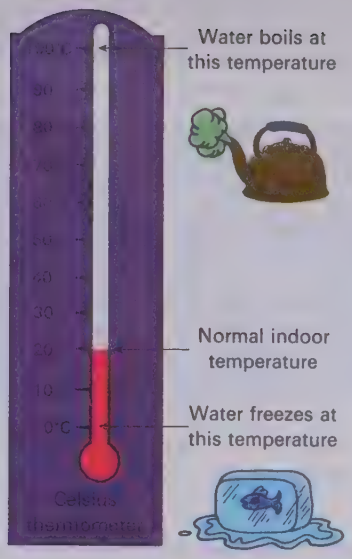
Minimum: 1A–F, 2A–F.

Average: 1–2. Maximum: 1–2.

Solving Story Problems

TEMPERATURE

A **thermometer** is used to measure temperature. The unit used in most of the world to measure how hot or cold it is is the **degree Celsius.** On the Celsius thermometer normal body temperature is 37°C (read 37 **degrees Celsius**); normal room temperature is 20°C.



1. On the Celsius thermometer what is the boiling point of water ? the freezing point of water ? 100° 0°

2. How many degrees Celsius greater is the boiling point of water than the freezing point of water ? 100°

3. Which of the following temperatures best describes a hot summer day in your town ? A 5°C (B 30°C) C 110°C

4. If it were snowing outside, which of the following would best describe the temperature ? (A 0°C) B 10°C C 20°C

5. If you were sick and had a fever, which of the following would best describe your temperature ? A 100° B 70°C (C 40°C)



6. Which of the following best describes the temperature in your classroom ? A 10°C (B 20°C) C 30°C

Using the Exercises

For page 237, study with the children the thermometer and the explanatory paragraph. Display actual thermometers if possible. Encourage them to discuss the various temperature facts that they may know already. Be sure to stress the boiling and freezing points of water, and typical indoor temperatures.

Following this introductory discussion, have the children do the exercises.

Assignments (page 237)* _____
Minimum: 1–3, oral. Average: 1–4G.
Maximum: 1–4.

Follow-up

Real thermometers—outdoor and indoor; alcohol or mercury; liquid or dry; house, clinical, or scientific—make excellent physical models for a wide variety of mathematical applications. Encourage the children to bring examples from home and use them for experiments such as making graphs to compare data and so on. Recording the outside temperature at the same time every day would involve all students. More advanced children could be introduced to the weather map in a daily paper.

Resources for Active Learning

Franklin Series: *Learning About Measurement*, pp. 86–90, Lyons and Carnahan. (Temperature measurement) (Available from McGraw-Hill Ryerson)

Measure and Find Out, Book 1, Activities 20, 21, 23; Book 2, pp. 21-22, Scott Foresman. (Available from Gage Educational)

Duplicator Masters, page 46
Workbook, pages 75–76
Skill Masters, page 46

PAGES 238–239

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review and with word problems which include the basic operations.

Preparation

To prepare for this lesson, provide a short review session of any topic the class has previously found particularly troublesome. For example, you might choose to have the children work through one or two problems of multiplication with 2- or 3-digit multipliers, while you review the steps on the chalkboard. You might also use this time to review briefly a few of the measurement concepts which were developed in Chapter 1.

Keeping in Touch with

Addition
Subtraction
Multiplication
Measurement
Place value

1. Give the correct number for each red digit.
In exercise A the 8 means 8000.

A 4 378 615 8000
B 64 124 752 60 000 000
C 8 389 741 6 000 000
D 3 026 518 0
E 4 250 697 0
F 97 643 287 7 000 000

2. Find the sums, differences, and products.

A	B	C	D	E
6728 + 9357 = 16 085	8436 − 2759 = 5677	432 × 65 = 28 080	8463 + 9287 = 17 750	7062 − 888 = 6174

F	G	H	I	J
745 × 63 = 46 935	8469 + 7531 = 16 000	8002 − 6457 = 1545	846 × 352 = 297 792	764 × 803 = 613 492

★ **3.** Look at the red segment and its length. Then use that length to estimate the lengths of the other segments.

5

A 6 _____ B 3 _____

C 11 _____

D 2 _____ E 4 _____

★ **4.** Look at the red figure and its area. Then use that area to estimate the areas of the other figures.



A 7



B 2





You are invited to explore

ACTIVITY CARD 11
Page 352

238

Discussion

Before assigning page 238, you might ask the children to do a few parts of exercise 3 orally. In this exercise the children observe that the unit length is given in terms of 5 units; from this they must judge the length of the other segments. The required estimates are not easy, but the children should find the challenge thought-provoking and enjoyable if you assure them that their estimates need not be very precise. For example, consider 5, 6, or 7 as quite acceptable guesses for exercise 3A and anything from 10 to 13 as a good guess for exercise 3C. Have the children do the rest of the page on their own and, when they have finished, allow time for discussion and checking papers.

Solving Story Problems *New Brunswick and the Yukon*



	New Brunswick	Yukon
Area	72 586 km^2	530 114 km^2
Population (1971 census)	634 557	18 338
Average temperature	4°C (Fredericton)	-5°C (Whitehorse)
Highest point	Mt. Carleton 807 metres	Mt. Logan 5955 metres

1. How many more square kilometres are in the Yukon than in New Brunswick ? 457 528 km²

2. A How many more people lived in New Brunswick than in the Yukon in 1971 ? 616 219
 B How many lived in the two areas together ? 652 895

3. Mt. Logan is the highest point in Canada. Mt. McKinley is the highest point in North America. Mt. McKinley is 6096 metres high. How much higher is Mt. McKinley than Mt. Logan ? 141 m

4. Ontario is approximately two times the size of the Yukon. What is the approximate area of Ontario ? About 1 060 000 km²

Follow-up

You might encourage children to do research regarding their own province in comparison with New Brunswick or the Yukon. They might also do research regarding the area in square kilometres of their city or county to help them get some idea of the size of a square kilometre.

Using the Exercises

Study with the children the maps and the table at the top of page 239.

Most children will be familiar with the concept of negative temperature. Simply treat this as a symbol for so many degrees "below zero", and do not enter into a detailed discussion at this time. You might have the children do some research in an encyclopedia or almanac about facts such as dates when various provinces joined Confederation, geographic features, and histories of these and other areas of Canada. Encourage them to make up problems of their own with these facts. The children may benefit from group work both for the problems on this page and for making up their own problems. Whichever way you treat this page, as group activity or as independent work, allow time for discussion when all have finished.

PAGES 240–241

Objective

Given division exercises in which the divisor is a multiple of 10, the child will be able to find the quotient and remainder.

Preparation

Conduct a short oral practice to review products with multiples of 10 as factors. For example, ask the children to give products such as 5×70, 6×80, 4×90, 3×70, and 4×60. Follow this with products such as 10×10, 20×10, 40×20, 40×30, 50×60, 70×20, and 90×30. You might also have them give the missing factor in equations such as $n \times 80 = 240$, $n \times 30 = 210$, $n \times 70 = 6300$, $n \times 50 = 3000$, and $n \times 40 = 2400$.



How can we divide by multiples of 10?

Discussing the Ideas

1. A What is the number for n? 5 $\quad n \times 30 = 150$

 B How many thirties in 150? 5 $\quad 30\overline{)150}$

2. How can you find how many thirties are in 1500? $\quad 30\overline{)1500}$ $\quad 50$

 Think: Since $150 \div 30 = 5$, $1500 \div 30 = 50$.

 Check your answer by using multiplication.

3. Study the example below.

Think:		Write:
$\begin{array}{r} 4 \\ 70\overline{)305} \\ -280 \\ \hline 25 \end{array}$ ← 4×70	Since $4 \times 70 < 305$, $5 \times 70 > 305$, we can subtract 4 seventies from 305. The quotient is 4. The remainder is 25.	$\begin{array}{r} 4 \\ 70\overline{)305} \\ 280 \\ \hline 25 \end{array}$ (4)

 Use the method above to find the quotient and remainder in this problem. Check your answer with your teacher.

 $\begin{array}{r} 7 \\ 60\overline{)463} \\ 420 \\ \hline 43 \end{array}$ (7)

4. Study the example below.

Think:		Write:
$\begin{array}{r} 89 \\ 70\overline{)6230} \\ 5600 \\ \hline 630 \\ 630 \\ \hline 0 \end{array}$ ← 80×70 ← 9×70	Since $80 \times 70 < 6230$, $90 \times 70 > 6230$, we can subtract **80** seventies from 6230. Then we can subtract **9** more seventies. The quotient is **89**.	$\begin{array}{r} 89 \\ 70\overline{)6230} \\ 5600 \\ \hline 630 \\ 630 \\ \hline 0 \end{array}$ (80) (9)

 Use the method above to find the quotient and remainder in this problem.

 $\begin{array}{r} 74 \\ 80\overline{)5920} \\ 5600 \\ \hline 320 \\ 320 \\ \hline 0 \end{array}$ (70) (4)

Discussion

Although page 240 is entitled "Discussing the Ideas," it includes exercises of the investigative type. Have the children try the questions before you check the answers. Children may approach these problems differently. In exercises 1 and 2 some may begin to subtract 30's from each of the numbers; some may think of missing factors and use multiplication. You might choose to have children work together in groups of three or four to answer other questions that you put on the chalkboard, such as:

Can you find how many 40's in 240? how many 20's in 400? how many 40's in 1760? how many 70's in 3640? how many 50's in 2500?

Have the children study the example in exercise 3, and discuss it with them before they try the suggested problem. Since this example deals with quotients that are less than 10, you may want to assign problems 1 and 2 on page 241 before discussing exercise 4 on page 240. After you have discussed exercise 4, which deals with 2-digit quotients, the remainder of the exercises on page 241 might then be assigned.

Using the Ideas

1. For each exercise, when you find the largest whole number that makes the sentence true, you will have found the quotient. Write the quotient and remainder for each division problem.

 A 6 R12 ▮ × 30 < 192
 $30\overline{)192}$

 B 5 R17 ▮ × 60 < 317
 $60\overline{)317}$

2. Find the quotients and remainders.

 A $20\overline{)190}$ 9 R10 B $30\overline{)180}$ 6 C $40\overline{)297}$ 7 R17 D $70\overline{)220}$ 3 R10

 E $80\overline{)230}$ 2 R70 F $50\overline{)415}$ 8 R15 G $70\overline{)579}$ 8 R19 H $40\overline{)317}$ 7 R37

3. The largest number in {10, 20, 30, 40, . . .} that makes the sentence true is the quotient. Find the quotient and remainder for each division problem.

 A 20 R27 ▮ × 30 < 627
 $30\overline{)627}$

 B 70 R6 ▮ × 20 < 1406
 $20\overline{)1406}$

4. Find the quotients and remainders.

 A $30\overline{)1200}$ 40 B $20\overline{)816}$ 40 R16

 C $40\overline{)2036}$ 50 R36 D $50\overline{)2536}$ 50 R36

 E $20\overline{)1808}$ 90 R8 F $40\overline{)2835}$ 70 R35

 G $90\overline{)5472}$ 60 R72 H $60\overline{)4231}$ 70 R31

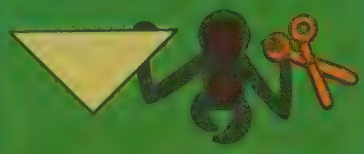
 I $30\overline{)690}$ 23 J $70\overline{)2240}$ 32

 K $50\overline{)3200}$ 64 L $40\overline{)2640}$ 66

 M $80\overline{)5200}$ 65 N $70\overline{)5670}$ 81

think

Trace and cut out 4 of these.



How many different shapes can you make by placing "like" edges together? Here are a few.



Draw your findings on graph paper.

See Solution, T.E. page 241.

Follow-up

For further practice in finding the largest multiple of a number for a missing quotient, write exercises like the following on the chalkboard, or duplicate a worksheet of problems similar to these.

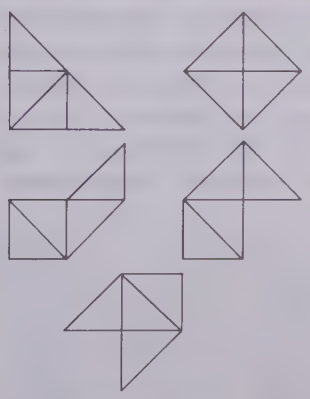
DIVIDEND equals DIVISOR times QUOTIENT plus REMAINDER.
Find the missing numbers.
234 = (40 × ___) + ___
492 = (70 × ___) + ___
375 = (50 × ___) + ___
189 = (20 × ___) + ___
758 = (80 × ___) + ___
891 = (90 × ___) + ___
562 = (80 × ___) + ___
637 = (___ × 9) + ___
804 = (90 × ___) + ___
285 = (70 × ___) + ___
491 = (___ × 8) + ___
386 = (80 × ___) + ___

Duplicator Masters, page 47
Workbook, page 77

Using the Exercises

Work through as many exercises as you consider necessary. Point out that the remainders now may be larger than in previous exercises because the divisors are larger. Also note with the children that the subtraction procedure applies in this division as it did previously. Also point out the role of the inequalities in Exercises 1 and 3: finding the largest multiple of the divisor that is less than the dividend is the way to find the correct quotient.

Solution, Think, page 241



Sample Figures

Assignments (page 241)*_____
Minimum: 1–4F. Average: 1–4.
Maximum: 1–4.

PAGES 242–243

Objective

Given division exercises with 2-digit divisors which have quotients less than 100, the child will be able to find the quotients and remainders.

Preparation

Since familiarity with products such as 20×60, 80×30, 40×50, and 90×70 is needed for successful work with larger divisors, give the children plenty of oral practice in finding these products. Also, have them find missing factors in equations such as $n \times 80 = 2400$, $50 \times n = 3500$, $90 \times n = 6300$, and $n \times 60 = 4800$.

Investigation

Explain to the children that in this investigation they will have a chance to try division with 2-digit divisors. Observe with the children that the problems in the investigation all have the same dividend. Also have them note that the divisors they will use are near 30, a kind of divisor with which they worked in the previous lesson. Remind them that division is basically a subtraction process. Then have the children work in groups of three or four, discussing how they might find the quotients.

If children understand division well enough as a subtractive process, they should be able to find these quotients even if they do not relate the investigation problems to the quotient, $1423 \div 30$. However, some may realize that, if 40 thirties can be subtracted from 1423, then at least 40 twenty-sevens can be subtracted also. This kind of thinking will help children with their first quotient estimate.

● *Let's explore 2-digit divisors.*

Investigating the Ideas

You know how to find this quotient.

```
 47 R 13
30)1423
1200   <- 40 x 30  (40)
 223
 210   <- 7 x 30   (7)
  13
```

Can you find these quotients? 44 R 15 → 32)1423 and 52 R 19 → 27)1423

See Investigation and Discussion.

Discussing the Ideas

1. Explain why one of the quotients in the Investigation is more than 47 and the other is less. See Discussion.

2. How would you improve Dick's method of finding how many forty-ones in 258?
 Sample answer: Subtract more than 2 forty-ones at a time.

3. Explain how the examples below can help you with your first quotient estimate.

Dick

```
   6
41)258
 82   (2)
176
 82   (2)
 94
 82   (2)
 12
```

A

Think: $n \times 20 < 624$

$30n \times 21 < 624$

```
 29 R 15
21)624
```

B

Think: $n \times 60 < 1357$

$20n \times 59 < 1357$

```
 23
59)1357
```



Discussion

Have children explain how they found the quotients in the investigation. Use exercise 1 to help them relate these quotients to the quotient, $1423 \div 30$. Use the following solutions as a basis for discussion.

```
   44, R15                52, R19
32)1423                27)1423
1280   (40)            1080   (40)
 143                    343
 128   (4)              270   (10)
  15                     73
                         54   (2)
                         19
```

Help the children see that the method of division used with 2-digit divisors is basically the same as that used with multiples of 10. Also stress, as exercise 3 suggests, that thinking of a 2-digit divisor in relation to a multiple of 10 makes the first quotient estimate easier.

As you discuss the solution for $1423 \div 27$ given here, and Dick's problem in exercise 2, point out that the longer methods are correct but can be shortened. Help the children realize that they should strive to make their first quotient estimates as large as possible, but do not expect them to become adept rapidly in this skill. This lesson does not seek to develop speed skills. Many of the

Using the Ideas

Find the quotients and remainders.

1. $21\overline{)126}$ (6)	2. $41\overline{)175}$ (4 R11)	3. $52\overline{)156}$ (3)	4. $59\overline{)192}$ (3 R15)
5. $69\overline{)560}$ (8 R8)	6. $21\overline{)630}$ (30)	7. $59\overline{)1357}$ (23)	8. $42\overline{)1050}$ (25)
9. $38\overline{)2014}$ (53)	10. $51\overline{)3417}$ (67)	11. $31\overline{)372}$ (12)	12. $42\overline{)966}$ (23)
13. $54\overline{)1674}$ (31)	14. $87\overline{)3262}$ (37 R43)	15. $26\overline{)832}$ (32)	16. $91\overline{)1911}$ (21)
17. $78\overline{)1560}$ (20)	18. $69\overline{)2415}$ (35)	19. $72\overline{)3456}$ (48)	20. $88\overline{)4084}$ (46 R36)

Short Picture Problems

1. IF 1 TENNIS 3 THEN 30 TENNIS ? 90
2. IF 1 EGGS 12 THEN ? EGGS 276
3. IF 3 PEARS 87 THEN 1 PEARS ? 29
4. IF 1 ONE CASE 24 THEN ? ONE CASE 408
5. IF 1 TABLET 50 THEN 25 TABLET ? 1250

More practice, page A-24, Set 44



children will be fairly slow and awkward at finding these quotients. It is more important that they do correct work even if their methods are longer.

Using the Exercises

You may consider it necessary to present a few of the exercises on page 243 as demonstrations. Then assign only selected exercises so that the children will have time to finish. The short picture problems give children an opportunity to use their division and multiplication skills in a different context. When they are finished, have some children explain their work to the whole class.

Assignments (page 243)*

Minimum: 1–10; picture problems, oral. Average: Even-numbered problems; picture problems 1–4. Maximum: All.

Follow-up

Now that the children have a certain facility with all four basic operations, suggest that they make up short picture problems involving any of the operations. They may write these on assignment cards or on paper to exchange with classmates.

Resources for Active Learning

Mathematics in Modules, WN21, Addison-Wesley.

Duplicator Masters, page 48
Workbook, pages 78–80
Skill Masters, page 48

PAGES 244–245

Objective

Given word problems which require division, the child will be able to solve the problems by finding the quotients.

Preparation

To prepare for this lesson, give the children practice in making first quotient estimates. For example, you might write problems such as

$7\overline{)324}$ $80\overline{)5321}$ $47\overline{)135}$ $54\overline{)9142}$

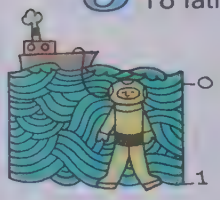
and ask children to give a first quotient estimate for each. Have them give reasons for their choices. For instance, for $47\overline{)135}$, they might say, "2, because 2 times 50 is less than 135," or for $80\overline{)5321}$, they might say, "60, because 60 times 80 is less than 5321."

Short Stories

1 60 minutes → 1 hour.
180 minutes. How many hours? 3

2 260 minutes. How many full hours? 4 How many extra minutes? 20

3 1 fathom → about 2 metres.
18 fathoms. How many metres? 36



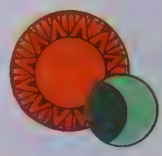
4 200 days.
How many weeks? 28
How many extra days? 4



5 1 coffee urn → 12 cups of coffee.
72 cups needed. How many urns? 6

6 Relay team → 4 runners.
112 runners.
How many teams? 28

7 12 flowers → 1 dozen.
One bush: 72 flowers.
How many dozen? 6



8 24 hours → 1 day.
198 hours. How many full days? 8
How many extra hours? 6

9 365 days → 1 year.
How many weeks? 52
How many extra days? 1



10 Run 1500 metres. How many centimetres? 150 000

11 52 weeks → 1 year.
332 weeks.
How many full years? 6
How many extra weeks? 20

12 60 seconds → 1 minute.
2058 seconds. How many full minutes? How many extra seconds? 34 min 18 sec

13 34 children → 1 classroom.
272 children. How many classrooms? 8



★14 780 maples → 1 square kilometre.
Park: 4680 maples. How many square kilometres? 6

244

Discussion

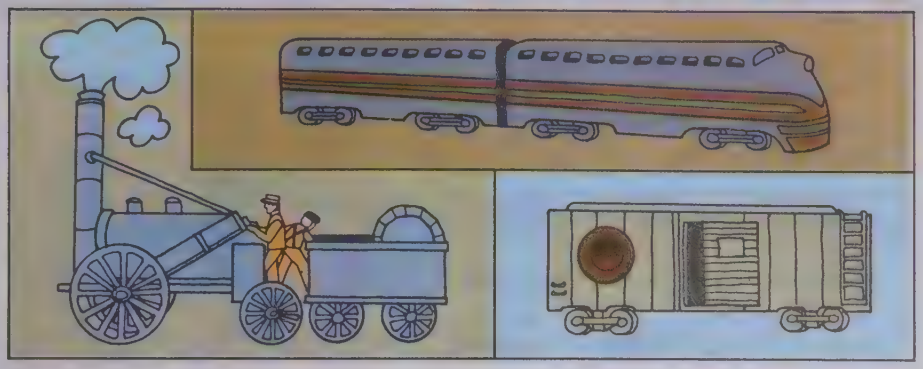
You may want to work two or three of the problems on page 244 with the class. Then have the children do the remaining problems independently.

Note that for starred exercise 14, the children must divide 4680 by 780. Those children proficient with the division algorithm will see how to do this with a series of approximations. For example, one child may subtract 2 × 780 until he gets a remainder that is less than 780. An exercise such as this will test the children's understanding of division. Do not attempt to give rules for solving this exercise. However, some of the children may recognize that they can simplify their work by considering 468 ÷ 78 instead of 4680 ÷ 780.

Assignments (page 244)

Minimum: Odd-numbered problems.
Average: 1–13. Maximum: 1–14.

Solving Story Problems TRAINS



1. A locomotive similar to the one shown in the picture above was invented in 1829. How many years ago was this?
 Answer depends on date solved.
2. The sleeping section of a modern passenger train is about 147 metres long. If each sleeping car is 21 metres long, how many cars are there? 7
3. A passenger coach holds 70 people. If 253 people are in passenger coaches on the train, how many full passenger coaches could there be? How many extra people would there be for a partly filled passenger coach? 3 full; 43 extra people
4. A boxcar is 12 metres long. A train has a boxcar section that is about 276 metres long. How many boxcars are in the section? 23
5. There are 8 tractors on each flatcar except the last. There are 543 tractors in all.
 A How many full flatcars are there? 67
 B How many extra tractors are left for the last flatcar? 7
6. Rails about 12 metres long are put end to end to make one side of a long railroad track. How many rails are needed to make 1 kilometre of one side of a track? 84
7. The average speed of a fast passenger train is 130 kilometres per hour. How many hours does it take to go 3250 kilometres? 25

More practice, page A-24, Set 45

Using the Exercises

The exercises on page 245 are also intended to be worked independently. Be careful in exercise 6 that the children understand why the answer is 84. Actually, $83\frac{1}{3}$ rails are needed but the rails cannot be broken. Follow their completion with discussion. If the discussion generates an interest in trains, their history, or in transportation of various forms, encourage children to pursue their ideas with research. Word problem sets such as these are intended not only to provide practice in mathematics but also to serve as motivating factors for the children's general learning.

Assignments (page 245)

Minimum: 1–7, oral. Average: 1–7. Maximum: 1–7.

Follow-up

Encourage children to make up short story problems of their own. Also, suggest that they write problems in relation to any new facts they find in their research.

Resources for Active Learning

[These resources offer a variety of measurement activities.]

Applied Mathematics Cards, Group 1/4, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Franklin Series: *Learning About Measurement*, pp. 60–73; 81–84, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, C23, 28; D25, Macmillan.

Nuffield Project: *Computation and Structure* 2, pp. 6–41, Wiley.

Duplicator Masters, page 49
Workbook, page 81

PAGES 246–247

The child will demonstrate his ability to work with the concepts presented in this chapter.

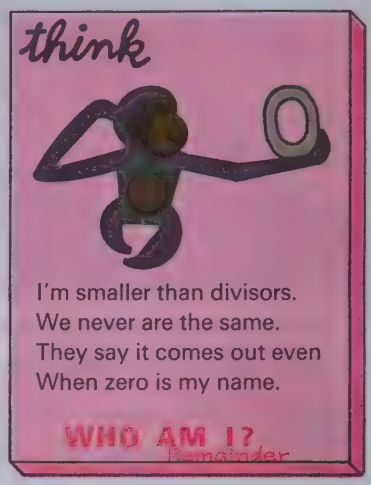
Preparation

Use the preparation time to review any concept with which the children had particular difficulty. For example, you might conduct an oral session to review estimating multiples of 10, using problems such as $n \times 80 = 5600$, $n \times 38 < 2000$, $n \times 60 = 3600$, and $n \times 77 < 3200$. Or you might review the concept of finding the average or performing the division algorithm. In any case, remember that the preparation should be kept short.

Reviewing the Ideas

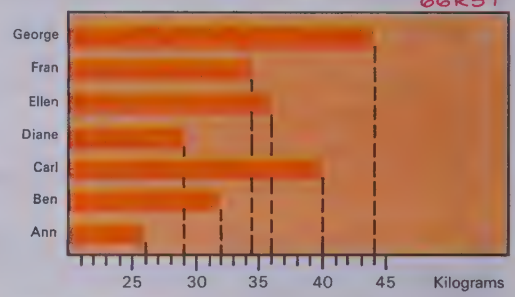
1. Copy each exercise and find the quotients and remainders. Check your work.

A $3\overline{)60}$ → 20	B $4\overline{)72}$ → 18	C $5\overline{)68}$ → 13 R3
D $6\overline{)258}$ → 43	E $7\overline{)430}$ → 61 R3	F $8\overline{)664}$ → 83
G $9\overline{)2106}$ → 234	H $7\overline{)3759}$ → 537	I $6\overline{)5239}$ → 873 R1
J $32\overline{)224}$ → 7	K $68\overline{)476}$ → 7	L $91\overline{)549}$ → 6 R3
★ M $35\overline{)1505}$ → 43	★ N $53\overline{)2491}$ → 47	★ O $76\overline{)6764}$ → 89



2. A 35 nines can be subtracted from 315. Give the quotient for 315 ÷ 9. 35
 B Start with 612. Subtract 50 × 9. Then subtract 10 × 9. Finally subtract 8 × 9. Give the quotient for 612 ÷ 9. 68
 C Start with 5799. Subtract 40 × 87. Subtract 20 × 87. Subtract 6 × 87. Give the quotient for 5799 ÷ 87. Give the remainder. 66 R57

3. The bar graph shows the weights of 7 fourth-grade children. Give the average weight of these children. 34½ kg



4. The U.S.S. Skate was the first atomic submarine to cross the Atlantic Ocean both ways submerged. The Skate went about 5058 kilometres in 9 days. About how many kilometres was this each day ? 562

Discussion

If you plan to use page 246 as an evaluation page, make sure children can read the bar graph in exercise 3. Exercise 3 may cause problems for some children. Fran's weight is $34\frac{1}{2}$ kg. The sum of the 7 weights is $241\frac{1}{2}$ kg. In dividing $241\frac{1}{2}$ by 7, confusion may result when the remainder appears as $3\frac{1}{2}$. Some children will recognize that $3\frac{1}{2}$ is one half of 7 but do not require the answer $34\frac{1}{2}$ kg. If the children approach this or if they round off Fran's weight to 35 kg and the average to 35 kg, accept this.

You may choose to assign only parts of exercise 1 to some children, to give them sufficient time to work out the long division.

The *Think* problem uses the term "come out even," which may be unfamiliar to the children. You might explain that when the remainder of a division problem is zero, the division is said to "come out even." The children should be able to solve the *Think* from the main clue in the first line. You may help the children with this *Think* by pointing out that the answer is not a number.

Solving Story Problems

ASTRONAUTS AND SPACE FLIGHT

1. The ages, weights, and heights of the first 7 United States astronauts (when they joined the space program) are given in the table. Give these averages:
 - A average age of the first 4 astronauts in the list 34
 - B average weight of the 5 youngest astronauts About 71
 - C average height in centimetres of the 2 lightest astronauts About 171

Name	Age	Height	Weight
Carpenter	34	176 cm	73
Cooper	32	173 cm	68
Glenn	37	176 cm	82
Grissom	33	168 cm	70
Schirra	36	175 cm	84
Shepard	35	178 cm	73
Slayton	35	176 cm	73

2. A satellite in orbit travels about 28 000 kilometres per hour. How far does it travel in 8 hours ? 224 000 km



3. A satellite in orbit travels about 8 kilometres per second. The airline distance from Vancouver to St. John's is about 7760 kilometres.
 - A About how many seconds would it take the satellite to go over Canada ? 970
 - B About how many minutes ? 16

4. In one of the manned satellite flights it took about 88 minutes for the satellite to make one orbit of the earth.
 - A If 1936 minutes had passed since launch time, about how many orbits would have been made? 22
 - ★ B If the astronaut slept for 8 hours while in orbit, about how many complete orbits did he make while asleep? 5



247

Using the Exercises

Have the children do the exercises on page 247. When they have finished, encourage discussion not only to point out the mathematical understandings but also to stimulate interest in space flights, their history, and related data. The diverse topics and facts covered in these exercises tend to heighten the children's interest and enjoyment, which are so necessary to successful learning.

Follow-up

While the children's interest is at a high level, you may wish to continue to provide information about space and allow them to make up more problems which will require analysis and many operations to solve.

For example, use the following data to stimulate ideas for word problems.

Planet	*Diameter (in kilometres)*	*Planet*	*Diameter (in kilometres)*
Mercury	4 849	Uranus	49 749
Pluto	5 957	Neptune	53 130
Mars	6 794	Saturn	120 911
Venus	12 210	Jupiter	139 748
Earth	12 762		

Resources for Active Learning

[These resources provide graphing activities.]

Applied Mathematics Cards, Group 1/ 23–26, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Developmental Math Cards, G²15, Addison-Wesley.

Math Activity Cards, C6, Macmillan.

Workbook, page 82

PAGES 248–249

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Unless you recognize a specific concept that you feel you should review with the children before they start page 248, use the preparation time to introduce page 249. For example, ask the children what time it will be five hours from now, ten hours from now, 12 hours from now, 24 hours from now, one day and six hours from now, etc.

Keeping in Touch with

Addition
Multiplication
Division
Measurement
Inequalities

1. Write the sign (<, >, =) that should go in each ◍.

A $25 + 24$ ◍ $23 + 26$ =
B 99×7 ◍ 700 <
C 51×4 ◍ 200 >
D $840 \div 6$ ◍ $840 \div 7$ >
E 24×36 ◍ 36×24 =
F $387 + 49$ ◍ $48 + 388$ =

2. Pretend that a unit has been chosen and the lengths, widths, and areas of the rectangles are as given. Find the missing numbers.

A 169 — 13 by 13, area: ▥

B 1548 — length: 36, width: 43, area: ▥

C 770 — length: 22, width: 35, area: ▥

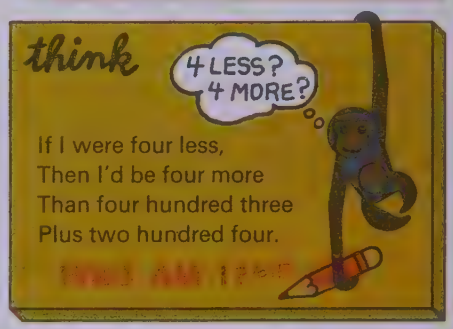
D 192 — length: 24, width: 8, area: ▥

E — length: ▥ 63, width: 9, area: 567

3. Find the sums.

A	B	C
23	613	8
40	492	407
56	876	8932
35	37	63
49	4023	4006
203	6041	13416

D $13 + 258 + 87 + 5976 + 503$ = 6837





248

Discussion

Assign page 248 as independent work and follow up with discussion.

If any children had difficulty finding the areas in exercise 2, you might draw a gridded rectangle with dimensions of four and five. By being able to see the grid and count the squares, the children can observe how multiplication is used to find the area of a rectangle.

When discussing the *Think* problem, ask children for reasons for their answers. Help them observe that since the number has to be four less in order to be four more than the given sum, it must be eight more than the given sum.



Solving Story Problems **Clock Problems**

For exercises 1-4, it is 2:30 in the afternoon.



1. In how many hours will it be
 - A 5:30 in the afternoon? 3
 - B 7:30 in the evening? 5
 - C midnight? $9\frac{1}{2}$

2. In how many minutes will it be
 - A 3:30 in the afternoon? 60
 - B 6:00 in the evening? 210
 - C 4:47 in the afternoon? 137
 - D 7:10 in the evening? 280
 - E 11:20 at night? 530
 - F 2:30 the next afternoon? 1440

3. Will the minute hand overtake the hour hand
 - A in the next 30 minutes? No
 - B in the next 40 minutes? No
 - C in the next 45 minutes? No
 - D in the next 50 minutes? Yes

4. How many times will the minute hand pass the hour hand in the next
 - A 2 hours? 2
 - B 3 hours? 3
 - C 4 hours? 3
 - D 12 hours? 11

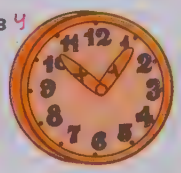
5. It is 5:37 in the afternoon.
 - A How many minutes has it been since 2:45 in the afternoon? 172
 - B How many minutes will it be before a television program at 7:45 in the evening? 128
 - C How long has it been since breakfast at 7:30 in the morning? 607 min or 10hr 7min

★ 6. If the hands of a clock point in the directions shown, which hand is the minute hand?
Answer x or y.
See Solution, T.E. page 249.

A y



B y



Using the Exercises

You may want to discuss page 249 following the preparation. Most children will need to count on the clock in order to answer these exercises. (As a matter of fact, this is a method by which many persons figure time lengths.) After the children have worked through the problems on their own or in groups of two or three, discuss their answers, asking for explanations of how they found them. You might point out techniques such as that of dropping the 30 minutes in exercise 1 and thinking: 2 to 5 is 3 hours and 2 to 7 is 5 hours and 2 to midnight is 10 hours, so 2:30 to midnight is $9\frac{1}{2}$ hours. Encourage any similar reasonable approaches which the children may have used. This entire page is intended to be treated with a light touch.

Solution, exercise 6, page 249

Since the *y*-hand of clock A points directly at the three, the time could not be a quarter to three; therefore, the time must be a quarter after nine since the *x*-hand is slightly past the nine. Similarly, in clock B, the *y*-hand must record six or seven minutes after 10, because the *x*-hand is slightly past the ten. Thus for both clocks, *y* is the minute hand.

Follow-up

The clock exercises on page 249 might stimulate the children to make up word problems in relation to telling time. You might suggest that they use actual incidents from their own experience as a basis for the problems. The following sample problems might inspire a few ideas.

1. School bell rings at 8:45.
 Morning recess — 10:00-10:15
 Lunch — 11:45-12:45
 Afternoon recess — 1:45-2:00
 Dismissal — 3:00
 How many minutes are spent in class time?
2. Jack looks at TV shows which are on at the following times.
 7:30 A.M.–7:45 A.M.
 5:00 P.M.–5:30 P.M.
 7:30 P.M.–8:30 P.M.
 How many minutes does Jack spend watching TV?

Resources for Active Learning

[These resources present materials pertaining to time concepts.]

Applied Mathematics Cards, Group 1/16–19, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Franklin Series: *Learning About Measurement* (Roman Numerals), pp. 74–80, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Franklin Series: *Patterns and Puzzles*, p. 47–48, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Freedom to Learn, pp. 134–139, Addison-Wesley.

Math Activity Cards, C31, 32, Macmillan.

Measure and Find Out, Book 1, Activities 14–17, Scott Foresman. (Available from Gage Educational)

Nuffield Project: *Computation and Structure* 2, pp. 82–90, Wiley.

Nuffield Project: *Problems*—Green Set, No. 14, Wiley.

Duplicator Masters, page 50
Skill Masters, page 50

CHAPTER 11 | Number Theory

Pages 250–261

General Objectives

To provide experiences with odd and even numbers

To introduce some properties of number endings

To explore sums and products involving odd and even numbers

To provide experiences with multiples and factors

To provide experiences with prime numbers and composite numbers

The first lesson of the chapter is devoted to work with odd and even numbers. The next two lessons emphasize concepts of factors and multiples and then concepts of common factors and greatest common factors. Following a methodical introduction to prime numbers, material is provided for general work with some prime-number concepts, including prime factorization.

Mathematics

Following the definitions of even, odd, and prime numbers, we give examples to illustrate each set of numbers defined.

The *even numbers* are the set of all numbers x such that $x = 2 \times n$ where n is a whole number.

Note that:

$n = 0 \rightarrow x = 0$
$n = 1 \rightarrow x = 2$
$n = 2 \rightarrow x = 4$
$n = 3 \rightarrow x = 6$
$\vdots \quad \vdots$

The *odd numbers* are the set of all whole numbers x such that

$x = (2 \times n) + 1$

where n is a whole number.

Note that:

$n = 0 \rightarrow x = 1$
$n = 1 \rightarrow x = 3$
$n = 2 \rightarrow x = 5$
$n = 3 \rightarrow x = 7$
$\vdots \quad \vdots$

The set of *prime numbers* is the set of all whole numbers x such that x has exactly two factors.

Thus:

2 is a prime number (factors: 2, 1).
3 is a prime number (factors: 3, 1).
5 is a prime number (factors: 5, 1).
7 is a prime number (factors: 7, 1).
11 is a prime number (factors: 11, 1).
⋮

Note that:

0 is not prime (factors: any whole number).
1 is not prime (factor: 1).
6 is not prime (factors: 1, 2, 3, 6).
25 is not prime (factors: 1, 5, 25).
91 is not prime (factors: 1, 7, 13, 91).

The whole numbers which are greater than 1 and have more than two factors are called *composite numbers*. Thus, 6, 25, and 91 are examples of composite numbers. If a number is a composite number, then it may be expressed as a product of prime numbers in exactly one way (though the order of the factors may vary).

Thus: $30 = 2 \times 3 \times 5$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
$100 = 2 \times 2 \times 5 \times 5$

These examples illustrate a basic property of the whole numbers which is sometimes called the *Fundamental Theorem of Arithmetic* or the *Unique Factorization Theorem*.

Teaching the Chapter

Materials

Overhead projector (if available) and transparencies

Vocabulary

common factor	multiple
composite number	odd number
even number	perfect number
factor	prime number
greatest common factor	

Multiple, *factor*, and *prime* are important concepts in the study of arithmetic and will come up again and again in Books 5 and 6 of this series. An important use of these words arises in the development of fractions and fractional numbers. The concept of greatest common factor is developed in preparation for reducing fractions to lowest terms. In Book 5, we will be concerned with the least common multiple, which leads toward the concept of lowest common denominator for addition of fractional numbers.

Lesson Schedule

This chapter is designed to be covered in about a week. However, you may wish to adjust this schedule to allow for the interest and motivation of your children. If the children are highly stimulated by the ideas presented here, it will be desirable either to spend more time each day on the lessons or to allot an extra few days to the chapter.

Evaluation of Progress

Although you should attempt to evaluate both understanding and mastery of certain fundamental concepts in this chapter, keep in mind that an overall purpose of the chapter is to arouse interest and stimulate the children in their study of arithmetic. When evaluating children's achievement, you should expect a certain amount of mastery in such areas as knowing and recognizing odd, even, and prime numbers; understanding the concepts of factor and multiple; and being able to find the greatest common factor of two numbers. You can best judge whether or not children understand the ideas presented by observing their remarks and their participation in discussions on a given topic on a day-to-day basis.

You will probably find that those children who are adept in mathematics will become quite excited as they work with the ideas developed in this chapter, but it is not an easy chapter. Thorough class discussions can do much to keep weak "mathematicians" from becoming discouraged by the more difficult concepts. You should try not to exhibit such concern with their progress that they feel that the entire chapter is a lost cause for them; certainly, there is much in the chapter from which they will benefit. We also suggest that you keep in mind that we do not expect that all children will master all the concepts presented in the chapter. The children will vary widely in their ability to handle this material.

Resources for Active Learning

GENERAL ACTIVITIES

Nuffield Project: *Computation and Structure* 2, "The '100' Square," pp. 51–57, Wiley

Nuffield Project: *Computation and Structure* 3, "Factors and Primes," pp. 67–69, Wiley

COMMERCIAL GAMES

Domino Number Games, Factors and Multiples (Heath)

PATTERN GAMES

Haar Hoolim Perception Games (Selective Educational Equipment)

MEM (Math Media; Selective Educational Equipment) Switch (Kohner Bros.)

PAGES 250–251

Objective

Given a number written in base-ten notation, the child will identify it as even or odd, and will use this skill to develop rules for sums and products of even and odd numbers.

Preparation

Materials

colored strips

To prepare for this lesson, you might refer to the strips and ask children to recall some of the uses they have made of them, such as for measurement and for constructing triangles and quadrilaterals.

If you prefer, you might omit any preparation and begin immediately with the investigation.

Investigation

In this investigation children concretely explore the distinctive properties of odd and even numbers. The even-number strips (2, 4, 6, 8, 10) can be matched with two strips of equal length (the 1-, 2-, 3-, 4-, and 5-strips respectively.) There are no two strips of equal length which can be matched with any of the odd-number strips. This investigation should help children realize that even-number strips may be matched with 2 equal-length strips, and odd-number strips can never be matched with just 2 equal-length strips. Every pair of equal-length strips must add on a single-unit strip in order to match an odd-number strip.

You might want the children to work as partners for this investigation. Make sure they understand what they are being asked to find, but allow them considerable freedom in working through the investigation themselves. Have children record the results of the investigation individually. For example, they might record the strip being considered and those that they matched with it.

$3 \rightarrow$ two 1-strips + 1
$(2 \times 1) + 1 = 3$

$4 \rightarrow$ two 2-strips
$(2 \times 2) = 4$

$5 \rightarrow$ two 2-strips + 1
$(2 \times 2) + 1 = 5$

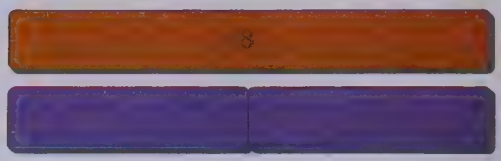
and so on.

11 Number theory

● *How are even and odd numbers different?*

Investigating the Ideas

The 8-strip matches two strips that are the same length.



The 9-strip will not match two strips that are the same length.



? What other strips can be matched with two strips that are the same length?

2-, 4-, 6-, and 10-strips

See Investigation.

Discussing the Ideas

▶ A number that is $2 \times$ (a whole number) is called an **EVEN NUMBER.**

▶ A number that is 1 + (an even number) is called an **ODD NUMBER.**

1. If you had strips for numbers up to 30, which strips would be like the 8-strip? like the 9-strip? From the definition above, which numbers are odd? even?

Odd: 1,3,5,7,9,11,13,15,17,19,21,23,25,27,29

Even: 2,4,6,8,10,12,14,16,18,20,22,24,26,28,30

2. Study this sequence of **even numbers.**

0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, . . .

tens twenties

What is true about the last digit of an **even number?**

It is 0, 2, 4, 6, or 8.

3. What is true about the last digit of an **odd number?**

It is 1, 3, 5, 7, or 9.

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Discussion

Give children an opportunity to share their results for the investigation. Relate each matching with the definitions given in the text, as in the equations shown. Then, when you discuss exercise 1, lead the children to think in terms of matching strips of equal length. For example, 14 is like the 8-strip; that is, 14 is even because two 7-strips would match a 14-strip; 25 is odd because two 12-strips plus one 1-strip would match a 25-strip. Also point out that 0 is even because $2 \times 0 = 0$. The emphasis here is on the definition of even and odd numbers.

In exercises 2 and 3 the skill of quick recognition of even and odd numbers by means of the last digit is developed. Note that the development toward *understanding* the definition of even and odd numbers precedes development of the skill in recognizing even and odd numbers.

Using the Ideas

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

1. A How many sums are there in the blue addition table? 100
B How many of the sums are **even numbers**? 50
C How many of the sums are **odd numbers**? 50

2. A How many products are there in the yellow multiplication table? 100
B How many of the products are **even numbers**? 75
C How many of the products are **odd numbers**? 25

3. A In the multiplication table, the 2 row contains only even numbers. Give the other rows that contain only even numbers. 0, 4, 6, 8
B Are there any rows that contain only odd numbers? No
C Give a row that helps show that an even number times any number gives a product that is an even number. 2 row, 4 row, 6 row, 8 row
D Does an odd number times any number give a product that is an odd number? No; the product of an odd number and an even number is even.

★ **4.** Write **O** (odd) or **E** (even) for each space in the tables at the right.

+	E	O
E (even)	E A	O B
O (odd)	O C	E D

×	E	O
E	E E	E F
O	E G	O H



Using the Exercises

You might treat page 251 as an activity page and have children work together in small groups. However, it is important to allow time for discussion of the children's conclusions. Pay special attention to the tables for exercise 4. Help the children realize that the tables, when correctly completed, show rules for all even (E) and odd (O) numbers.

Assignments (page 251)
Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Resources for Active Learning

Inquiry in Mathematics via the Geoboard, "Even-Odd Numbers" Geo-Cards 7/1,2; 8/1,2, Walker. (Available from Fitzhenry & Whiteside)

Math Activity Cards, "Odd and Even," B42, Macmillan.

Nuffield Project: *Computation and Structure* 2, "Closure and Checks of Accuracy," pp. 78–81, Wiley.

Nuffield Project: *Problems*—Green Set, No. 31, Wiley.

PAGES 252–253

Objective

Given whole numbers less than 100, the child will be able to find the factors of the numbers.

Preparation

You might want to review the terms *factor* and *product* before having children begin the investigation. However, the terms should be referred to only in regard to a multiplication

F F P

equation, such as $5 \times 6 = 30$, in order not to preempt the child's exploration in the investigation. If you think the children satisfactorily recall the meaning of the terms, begin immediately with the investigation.

Investigation

Unless the children work extremely well together as partners, have them do this investigation individually. After you have read the opening paragraph with them, explain that they should work through problems A and B and then show all the possible answers for C. For those who finish quickly, write on the chalkboard equations similar to the following.

Supply missing factors:

$\square \times \square = 12$

$\square \times \square = 20$

Through this investigation children are introduced to the idea that some numbers have many factors. The possible answers for problem C follow.

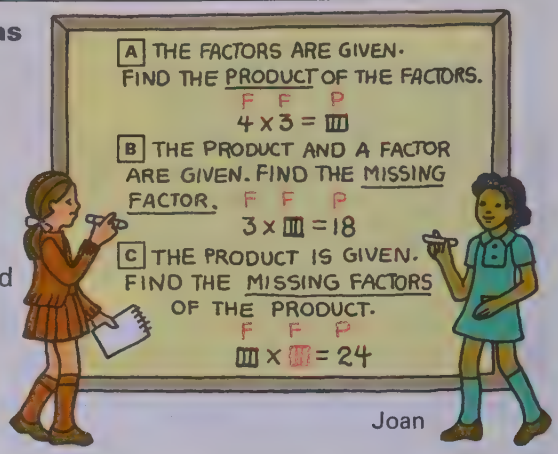
$1 \times 24 = 24$	$24 \times 1 = 24$
$2 \times 12 = 24$	$12 \times 2 = 24$
$3 \times 8 = 24$	$8 \times 3 = 24$
$4 \times 6 = 24$	$6 \times 4 = 24$

(If a child gives only four equations but uses all factors of 24, accept his answer as correct.)

Let's explore factors and products.

Investigating the Ideas

Joan gave Sara these problems to solve. Sara solved problems A and B easily. When she tried problem C, she stopped and looked very puzzled. Do you know why?



Can you give all the possible answers to problem C?
See Investigation.

Discussing the Ideas

1. Study the equations in A and B and then give the numbers for each ▥.

A Since $n \times 7 = 28$ has the solution 4, we know that 4 and 7 are factors of ▥.

B Since $n \times 6 = 28$ has no whole-number solution, we know that ▥ **is not** a factor of 28.

2. A How do you know that 9 is a factor of 36? 4x9=36
 B How do you know that 8 is not a factor of 36?
 n x 8 = 36 has no whole-number solution.
3. A How many different multiplication equations that have 18 as the product can you find? 1x18=18; 2x9=18; 3x6=18
 B Can you list all the factors of 18? 1,2,3,6,9,18

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Discussion

In discussion exercise 1, statements A and B explain how to determine whether or not a number is a factor of another number. The children may use the second statement when they are trying to decide whether or not a given number is a factor of some other number. If they cannot find a whole number that, when multiplied by their number, will give the desired number, they should conclude that the given number is not a factor of the other number. During this discussion of exercises 1 and 2, encourage the children to use the terms correctly. If the equation is $4 \times 3 = 12$, the children should say, "Four and three are factors of twelve," rather than "Four and three are factors." They should say, "Twelve is the product of four and three," rather than simply saying "Twelve is the product." The point to be made—not to the children necessarily, except through emphasis on correct usage—is that, since the concepts of factor and product are relations, it is not quite correct to say that a number is a factor. We must say that the number is a *factor of* another number. An analogous instance, in which the improper usage is more obvious, would be the statement that "The number eight is equal."

Using the Ideas

1. Write as many different multiplication equations that have these numbers as **products** as you can. Write equations using just two factors at a time. See Answers, T.E. page 253.

A 6 (Example: $1 \times 6 = 6$, $2 \times 3 = 6$) B 18 C 20 D 28

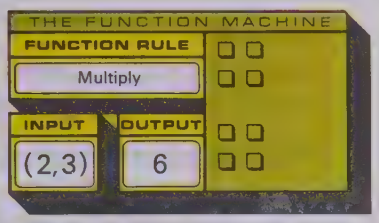
E 32 F 30 G 36 H 48 I 100

2. Solve the equation and give two factors of the product.

A n (3) $\times 5 = 15$ (Answer: 3, 5)
B $9n \times 7 = 63$ 9,7
C $12n \times 5 = 60$ 12,5
D $4n \times 12 = 48$ 4,12
E $5n \times 15 = 75$ 5,15
F $3n \times 13 = 39$ 3,13
G $8n \times 9 = 72$ 8,9
H $4n \times 18 = 72$ 4,18
I $2n \times 36 = 72$ 2,36
J $32n \times 1 = 32$ 32,1
K $16n \times 2 = 32$ 16,2
L $8n \times 4 = 32$ 8,4
M $3n \times 16 = 48$ 3,16
N $24n \times 2 = 48$ 24,2

3. A Is 4 a factor of 11? No
B Is 8 a factor of 63? No
C Is 13 a factor of 42? No
D Is 17 a factor of 51? Yes

4. Using the rule shown, you can put a **pair** of numbers into the function machine and get a single output number. Some output numbers are given below. Give as many pairs as you can that would produce each output number.



Example: 4 Answers: (1, 4) and (2, 2)

A 12 C 5 E 18 G 30 I 50 K 15
B 6 D 8 F 11 H 36 J 13 L 21

See Answers, T.E. page 253.

5. List all the factors of each number.

Example: 12 (Answers: 1, 2, 3, 4, 6, 12)

A 6 1,2,3,6
B 5 1,5
C 8 1,2,4,8
D 18 1,2,3,6,9,18
E 11 1,11
F 30 1,2,3,5,6,10,15,30
G 36 1,2,3,4,6,9,12,18,36
H 50 1,2,5,10,25,50
I 45 1,3,5,9,15,45
J 1 1



Using the Exercises

Although sample answers for the exercises on page 253 are shown in the text, it would be helpful to demonstrate for the children how to write pairs of factors of a number in parentheses. Also explain how, in exercise 4, pairs of input numbers and one function rule produce output numbers. Assign the exercises and, when the children are finished, allow time for checking papers and further discussion.

Answers, exercises 1 and 4, page 253

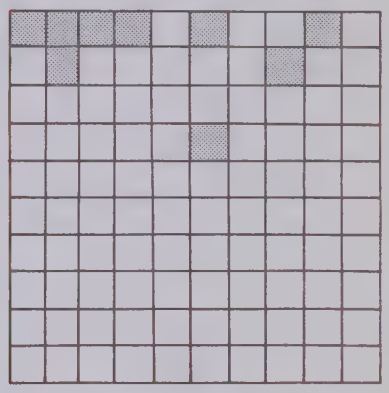
1B. $1\times18=18$, $2\times9=18$, $3\times6=18$
C. $1\times20=20$, $2\times10=20$, $4\times5=20$
D. $1\times28=28$, $2\times14=28$, $4\times7=28$
E. $1\times32=32$, $2\times16=32$, $4\times8=32$
F. $1\times30=30$, $2\times15=30$, $3\times10=30$, $5\times6=30$
G. $1\times36=36$, $2\times18=36$, $3\times12=36$, $4\times9=36$, $6\times6=36$
H. $1\times48=48$, $2\times24=48$, $3\times16=48$, $4\times12=48$, $6\times8=48$
I. $1\times100=100$, $2\times50=100$, $4\times25=100$, $5\times20=100$, $10\times10=100$
4A. (1,12), (2,6), (4,3) **B.** (1,6), (2,3) **C.** (1,5) **D.** (1,8), (2,4) **E.** (1,18), (2,9), (3,6) **F.** (1,11) **G.** (1,30), (2,15), (3,10), (5,6) **H.** (1,36), (2,18), (3,12), (4,9), (6,6) **I.** (1,50), (2,25), (5,10) **J.** (1,13) **K.** (1,15), (3,5) **L.** (1,21), (3,7)

Assignments (page 253)
Minimum: 1–3. Average: 1–5. Maximum: 1–5.

Follow-up

An activity which would help children recognize factors of a number is the construction of factor cards. These would also be useful in the following lesson. Write the following directions on the chalkboard or run off duplicated copies:

> Use two or three sheets of 1-cm graph paper. Cut out several squares that are 10 by 10. Make a master card by numbering one of the cards consecutively from 1 to 100. Choose a product number from the basic multiplication facts. Find all the factors of your number. Beginning with 1 at the top left-hand corner of another 10-by-10 square, count by ones, cutting out the small square of any number which is a factor of your chosen number.
>
> Do this with a different number for each of your cards. For example, the card for product number 36 would look like the following, where the shaded areas represent cut out sections.



Factor Card for 36

(You may need to demonstrate for the children that interior squares can most easily be cut out by folding the paper through the middle of the square and then cutting through the fold, along the grid lines.)

When the factor card is placed upon the master card, the factors for the number show through the holes that have been cut out.

These cards should prove helpful in the next lesson for finding common factors. For instance, when the card for 36 is superimposed on the card for 24 and both are placed on the master card, the common factors will appear in the holes in the cards.

Workbook, page 83

PAGES 254–255

Objective

Given two whole numbers, the child will be able to find the set of factors for each number and give the greatest common factor of the pair.

Preparation

As a warm-up activity, give the children practice in finding factors of a number. You might say something such as the following:

"I'm thinking of the numbers 1, 2, 3, 4, 5, 6; which of these is not a factor of 12?" or "I'm thinking of 1, 2, 3, 6, 8, 9; which of these is not a factor of 18?" Also include questions such as: "What are 2 factors of 30?" "Name 4 factors of 36." "Name all the factors of 12." However, since this is an oral activity, the questions should be kept as simple as possible.

Investigation

For this investigation children may work solely from the text or they may use the factor cards described in the follow-up section of the preceding lesson. If children work from the text alone, you may want them to make yellow cards for the factors of 18 and blue cards for the factors of 24, but it is not necessary. They may simply list the factors of 18 (1, 2, 3, 6, 9, 18) and of 24 (1, 2, 3, 4, 6, 8, 12, 24) and then choose the numbers which appear in both lists (common factors: 1, 2, 3, 6) and the largest of these (greatest common factor: 6). For those who finish quickly, you might suggest another set of factors, for example, factors of 32 (1, 2, 4, 8, 16, 32) with the question, "What is the largest number that is a factor of both 32 and 24?"

(If the children made the factor cards suggested in the follow-up section of the previous lesson, they might use them for this investigation. To find common factors of 18 and 24, they should take the card for each and place them one over the other to find the numbers which are cut out on both cards and to find the largest of these numbers. Depending on which factor cards the children have made, you might suggest other numbers for which they should find the common factors.)

How do you find the greatest common factor?

Investigating the Ideas

Suppose you had sets of cards like these.



? What is the largest number that is on both a yellow and a blue card? 6 See Investigation.

Discussing the Ideas

1. The numbers that are factors of **both** 18 **and** 24 are called common factors of 18 and 24. Which numbers are common factors of 18 and 24? 1,2,3,6

2. The **largest number** that is a factor of both 18 and 24 is called the greatest common factor of 18 and 24. What number is the greatest common factor of 18 and 24? 6

3. A Can you give the numbers missing from the diagram?

The factors of 8 : { 1, 2, 4, 8 }

The factors of 12 : { 1, 2, 3, 4, 6, 12 }

1, 2, and 4 are common factors of 8 and 12.

4 is the greatest common factor of 8 and 12.

B On the chalkboard, show a diagram like this for the factors of 6 and 9. See Discussion.

Discussion

Use the answers from the investigation in your discussion of exercises 1 and 2. As you discuss exercise 3, emphasize that the numbers 1, 2, and 4 are factors of a *pair* of numbers: they are factors of both 8 and 12. Hence, we use the phrase *common factors* with respect to 1, 2, and 4 and the pair of numbers 8 and 12. We say that 1, 2, and 4 are *common factors* of 8 and 12. Then have the children observe that, since 4 is the largest of these factors, it is called *the greatest common factor* of 8 and 12. The diagram called for by exercise 3B would be similar to the one shown at the right.

The factors of 6: 1, 2, 3, 6

The factors of 9: 1, 3, 9

1 and 3 are common factors of 6 and 9.

3 is the greatest common factor of 6 and 9.

You might find it helpful to work through other examples before going on to page 255.

Using the Ideas

1. A List the factors of 9. 1,3,9 B List the factors of 12. 1,2,3,4,6,12
 C List the common factors of 9 and 12. 1,3
 D What is the greatest common factor of 9 and 12? 3

2. A List the factors of 6. 1,2,3,6 B List the factors of 8. 1,2,4,8
 C List the common factors of 6 and 8. 1, 2
 D What is the greatest common factor of 6 and 8? 2

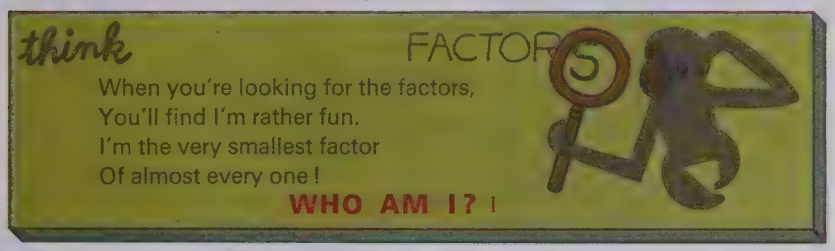
3. A List the factors of 12. 1,2,3,4,6,12 B List the factors of 16. 1,2,4,8,16
 C List the common factors of 12 and 16. 1,2,4
 D What is the greatest common factor of 12 and 16? 4

4. A List the factors of 8. 1,2,4,8 B List the factors of 9. 1,3,9
 C List the common factors of 8 and 9. 1
 D What is the greatest common factor of 8 and 9? 1

5. A List the factors of 12. 1,2,3,4,6,12 B List the factors of 18. 1,2,3,6,9,18
 C List the common factors of 12 and 18. 1,2,3,6
 D What is the greatest common factor of 12 and 18? 6

6. A List the factors of 24. 1,2,3,4,6,8,12,24 B List the factors of 32. 1,2,4,8,16,32
 C List the common factors of 24 and 32. 1,2,4,8
 D What is the greatest common factor of 24 and 32? 8

★ 7. Give an odd number and an even number between 10 and 20 that have 3 as the greatest common factor. 15,12 or 15,18



More practice, page A-25, Set 46

Using the Exercises

You might choose to work through one or two of the exercises on page 255 before you assign them as independent work. If some children wish to try the starred exercise but find it difficult, you might suggest that, for exercise 7, they begin by listing all the numbers between 10 and 20 that have 3 as a factor and then look for one that is odd and one that is even.

Encourage all the children to try to find the answer to the *Think* problem. Help them see that although one is a factor of zero ($1 \times 0 = 0$), it is not the smallest factor of 0 (any number $\times 0 = 0$); thus one is the smallest factor of "almost every one."

Assignments (page 255)*
Minimum: 1–6. Average: 1–6.
Maximum: 1–7.

Mathematics

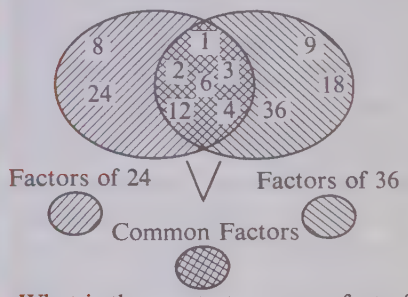
The concept of the intersection of two sets may be more evident in this lesson than in any other portion of Book 4. We give below a formal definition for the intersection of two sets.

> If A and B are sets, then the intersection of A and B, written $A \cap B$, is the set of all elements that are in both A and B (that is, those elements which A and B have in common).

In this lesson, the word *common* implies the idea of the intersection of two sets, and it is not essential to formally define the concept of the intersection of two sets or to introduce the symbol for intersection to the children. Most of them will understand that the word *common* is applied to those elements which are in both of two sets.

Follow-up

Many children might enjoy working with Venn diagrams to show the intersection of sets. As an example, an approach similar to the following one is usually successful.



What is the greatest common factor?

1. A. List the factors of 27.
 B. List the factors of 36.
 C. Draw a Venn diagram to show the *common* factors of 27 and 36.
 D. What is the greatest common factor of 27 and 36?
2. A. List the factors of 36.
 B. List the factors of 48.
 C. Draw a Venn diagram to show the common factors of 36 and 48.
 D. What is the greatest common factor of 36 and 48?

Duplicator Masters, page 51
Workbook, page 84
Skill Masters, page 51

PAGES 256–257

Objective

Given a whole number less than 100, the child will be able to classify it as a prime or a composite number.

Preparation

To prepare for this lesson, give children practice in naming factors of numbers related to the basic facts. You might ask one child to name multiplication equations for the number 36 and ask another child to list on the chalkboard the factors that are mentioned. For example, from the equations at the left, the list of factors at the right may be formed.

$9 \times 4 = 36$ {1, 2, 3, 4,
$2 \times 18 = 36$ 6, 12, 18, 36}
$3 \times 12 = 36$
$6 \times 6 = 36$
$1 \times 36 = 36$

Investigation

Read the directions with the children, pointing out how the numbers are found to form the rows and columns. It may take some children a considerable length of time to find the 10 required numbers. Point out that the number 1 cannot be among these 10 numbers since the numbers must be *between* 1 and 38. Remind the children to record their answers carefully. Some may find all 12 such numbers.

What are prime numbers?

Investigating the Ideas

Here is a multiplication table with the zero row and column and the 1 row and column left out.

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45			
4	8	12	16	20	24	28	32	36	40	44	48	52	56				
5	10	15	20	25	30	35	40	45	50	55	60	65					
6	12	18	24	30	36	42	48	54	60	66	72						
7	14	21	28	35	42	49	56	63	70	77							
8	16	24	32	40	48	56	64	72	80								
9	18	27	36	45	54	63	72	81									
10	20	30	40	50	60	70	80										
11	22	33	44	55	66	77											
12	24	36	48	60	72												
13	26	39	52	65													
14	28	42	56														
15	30	45															
16	32																
17	34																
18																	

Think of this table as going on without end.

? Can you list ten numbers between 1 and 34 that **do not** appear in the blue part of the table?
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

Discussing the Ideas

1. 7 is not one of the products in the table. Can you explain why?
 It has no factors other than 1 and 7.
2. 13 is not one of the products. Why? Only two factors, 1 and 13.
3. 29 is not one of the products. Why? Only two factors, 1 and 29
4. Name some other numbers that are not among the products.
 43, 47, 53, 59, 61, etc.
5. Write all the whole numbers from 2 to 34.
 2, 3, 4, 5, 6, . . . 32, 33, 34.
 Now mark out the numbers that are products in the table above. What you have left is the set of all prime numbers less than 34.
 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~
 ~~21~~, ~~22~~, 23, ~~24~~, ~~25~~, ~~26~~, ~~27~~, ~~28~~, 29, ~~30~~, 31, ~~32~~, ~~33~~, ~~34~~

Discussion

Encourage the children to keep in mind their results from the investigation as you work through the discussion questions. As you discuss exercises 1, 2, and 3, point out that 7, 13, and 29 have no factors other than 1 and themselves. Develop the idea that a prime number is a number which has only two factors, namely, 1 and itself. Also discuss examples of numbers which are not prime and ask children to express them as products of factors. Help them realize that all the numbers on the chart have other factors besides 1 and the number itself. Be sure they realize that the 12 numbers between 1 and 38 which they found in the investigation (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37) are all prime numbers.

Using the Ideas

1. Complete each sentence with an equation.
 I know that 20 is **not prime** because $4 \times 5 = 20$.
 - A I know that 12 is **not prime** because ▮ × ▮ = 12. 2×6 or 4×3
 - B I know that 14 is **not prime** because ▮ × ▮ = 14. 2×7
 - C I know that 24 is **not prime** because ▮ × ▮ = 24. 2×12 or 3×8 or 4×6

2. Tell whether each number is prime or not prime.
 A 15 No B 17 Yes C 19 Yes D 21 No E 23 Yes

3. Copy each equation. Give the missing numbers.
 A ▮(1) × ▮(2) = 2 B ▮(1) × ▮(3) = 3 C ▮(1) × ▮(5) = 5

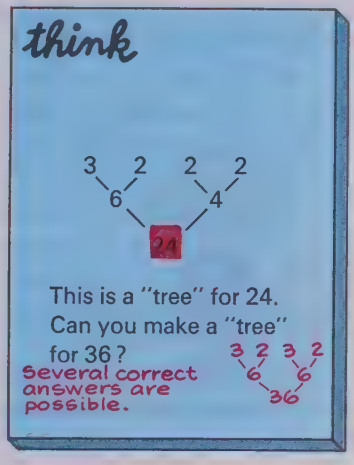
4. Could you find a different pair of factors for any part of exercise 3? No

5. A 2 is a prime number. What are the factors of 2? 1, 2
 B 3 is a prime number. What are the factors of 3? 1, 3
 C 5 is a prime number. What are the factors of 5? 1, 5
 D How many different factors does a prime number have? 2

6. List all the factors of each of the following numbers.
 A 7 1, 7 B 1 1 C 13 1, 13 D 17 1, 17 E 19 1, 19

★7. List all prime numbers less than 50. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

★8. Give the correct number or word.
 - A The number ▮ is a factor of every number. 1
 - B Each prime number has exactly ▮ factors. 2
 - C The factors of a prime number are the number itself and ▮. 1
 - D The number ▮ has exactly one factor. 1
 - E If a number has more than two factors, it is not a __?__ number. prime





Using the Exercises

In the exercises on page 257, the children have an opportunity to apply the definition of prime numbers. Exercise 2 might need an explanation before children begin to work on it. Help them realize that as soon as a third factor of a number is discovered they will know that it is not a prime number. You might have them show exercise 2 by either listing the factors for each number or writing multiplication equations which demonstrate a third and fourth factor. For example, for exercise 2A, a child might write: 15 is not prime because $3 \times 5 = 15$. Starred exercise 7 may be more time consuming for some children than it is for others, but encourage them all to try it. Although exercise 8 is also starred, you might use these statements as a basis for discussion.

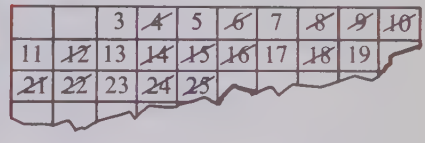
Assignments (page 257)*
Minimum: 1–6. Average 1–6.
Maximum: 1–8.

Follow-up

A Greek mathematician named Eratosthenes (3rd century B.C.) invented a relatively simple method of finding prime numbers. To find all the primes less than 100, for instance, he wrote down the numerals for the *odd numbers* beginning with 3, struck out the multiples of 3 greater than 3, then the multiples of 5 greater than 5, and the multiples of 7 greater than 7. The remaining numerals, along with 2, represent the prime numbers less than 100. This method has been called the Sieve of Eratosthenes.

Adapt the sieve technique for your class by giving each child a duplicated sheet ruled into 10 rows and 10 columns to make 100 squares. Direct the children to write the numerals from 1 through 99 in the spaces. Remind them that since a prime number has exactly two factors, they should circle 2 as the first prime. Again remind them that since all the multiples of 2, the even numbers, have at least 1, 2, and the number itself as factors, they should cross out all the even numbers except 2. Next, have them begin at 3 and cross out every third number, and continue similarly with 5 and 7.

One child may wish to take a piece of parchment paper, write the numerals 3 through 99 in the manner shown below, and punch out the numerals that are not primes. A veritable sieve will result.



Resources for Active Learning

Discovery, Section I, Activities 17–24, pp. 20–25, Encyclopaedia Britannica Educational Corp.

Teaching Aids for Elementary Mathematics, pp. 32–35, Holt, Rinehart and Winston.

Math Activity Cards, "Factors and Primes," B43, Macmillan.

Mathex: Numeration No. 7, "Prime Numbers," pp. 6–9; "Sieve of Eratosthenes," pp. 9–12 (pupil pages 5–8), Encyclopaedia Britannica Publications Ltd.

Workbook, page 85

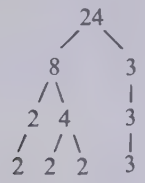
PAGES 258–259

Objective

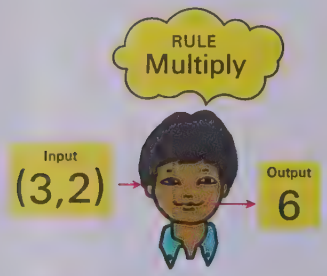
Given composite numbers, the child will be able to find their prime factors.

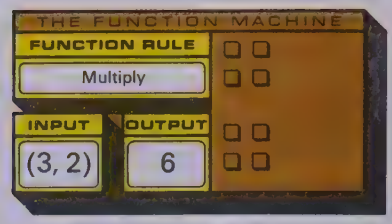
Preparation

To prepare for this lesson, give children practice in breaking down composite numbers into prime factors. For example, write 24 on the chalkboard and have the children break it down step by step.



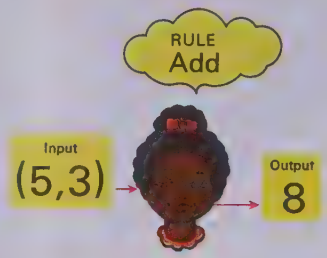
The Function Machine

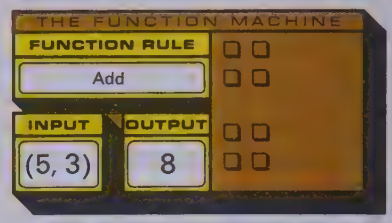




1. Give a pair of **prime numbers** that will produce each of these output numbers. A 4 [Answer: (2, 2)]

B 10 2,5 C 9 3,3 D 14 2,7 E 15 3,5 F 21 3,7 G 22 2,11 H 25 5,5 I 26 2,13 J 33 3,11

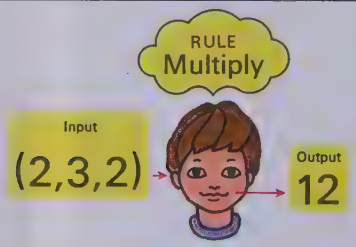


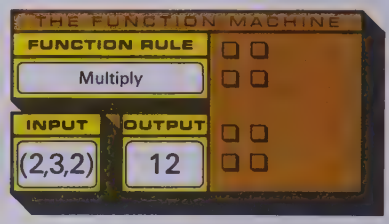


2. Give a pair of **prime numbers** that will produce each output number. See Answers, T.E. page 258. A 9 [Answer: (7, 2)]

B 4 C 12 D 6 E 14 F 10 G 15 H 16 I 18 J 20

K 21 L 22 M 24 N 25 O 26 P 28 Q 30 R 32 S 33





3. Give the output number for each number triple.

A (1, 4, 2) 8 B (2, 4, 2) 16 C (5, 3, 0) 0 D (1, 1, 1) 1 E (3, 4, 5) 60

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Discussion

Discuss the rule for each function machine. Notice that the set of prime numbers is unique for each output number.

The exercises here are actually based on the *Fundamental Theorem of Arithmetic*, which states that every composite number can be factored into a unique set of prime numbers except for the order in which the factors are written. (See the mathematics section at the beginning of this chapter.)

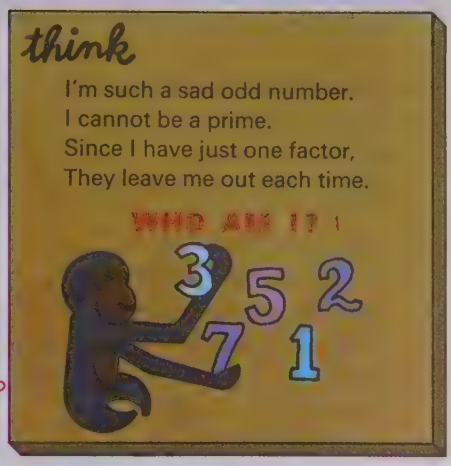
Answers, exercise 2, page 258

2B. 2,2 **C.** 5,7 **D.** 3,3 **E.** 3,11 or 7,7 **F.** 3,7 or 5,5 **G.** 2,13 **H.** 5,11 or 3,13 **I.** 5,13 or 7,11 **J.** 3,17 or 7,13 **K.** 2,19 **L.** 3,19 or 5,17 or 11,11 **M.** 7,17 or 5,19 or 11,13 **N.** 2,23 **O.** 3,23 or 7,19 or 13,13 **P.** 5,23 or 11,17 **Q.** 7,23 or 11,19 or 13,17 **R.** 3,29 or 13,19 **S.** 2,31

Prime Number Problems

1. The whole numbers greater than 1 that are not prime numbers are called **composite** numbers.

- A List the composite numbers less than 40. See Answers, T.E. page 259.
- B Give a composite number between 90 and 100. 91, 92, 93, 94, 95, 96, 97, 98, or 99



2.

- A List all the factors of 12. 1, 2, 3, 4, 6, 12
- B List the factors of 12 which are prime. 2, 3
- C List the factors of 30. 1, 2, 3, 5, 6, 10, 15, 30
- D List the prime factors of 30. 2, 3, 5

3. The equation $30 = 5 \times 2 \times 3$ shows that 30 is a product of prime numbers. Write equations to show that 35 and 36 are products of prime numbers. $35 = 5 \times 7$; $36 = 2 \times 2 \times 3 \times 3$

4. Give the word or number for each blank.

- A Every prime number is odd except __?__. 2
- B The number __?__ is a factor of every number. 1
- C Each prime number has exactly __?__ factors. 2
- D The only factor of 1 is __?__. 1
- E Since the only factors of 73 are 1 and 73, 73 is a __?__ number. prime
- F Since 69 has four factors, 69 is a __?__ number. composite
- G 57 is not a prime number because $3 \times n = 57$. 19
- H __?__ is the only prime number between 61 and 71. 67
- ★ I __?__ is the only prime number between 79 and 89. 83
- ★ J __?__ is the only prime number between 103 and 109. 107
- ★ K __?__ is the only prime number between 113 and 131. 127

★ **5.** The numbers 3 and 5 are called **twin primes** because their difference is 2. Another pair of twin primes is 41 and 43. Give four more pairs of twin primes. 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; 59, 61; etc.

More practice, page A-25, Set 47

Follow-up

Encourage the children to make factor trees similar to the one shown in the preparation section. An activity of this kind should impress upon the children the fact that the set of prime numbers for a composite number is unique; no matter how they factor a composite number they will get just one set of prime factors. Suggest that the children make factor trees for 36, 32, 40, and 56 and so on.

Resources for Active Learning

Nuffield Project: *Problems*—Green Set, No. 25, Wiley.

Using the Exercises

On page 259, explain the term *composite* as presented in exercise 1. In exercise 2, make sure the children understand that *prime factors* are prime numbers which are factors of a particular number.

Note that exercises 4I, J, and K and exercise 5 are challenging. Most children will be able to do these exercises, if given enough time. These starred exercises are not conceptually difficult; they simply involve searching for larger prime numbers or pairs of prime numbers. All children will benefit from a discussion of the exercises once the correct answers are given.

The number of children who solve the *Think* problem will give you an indication of how many of them understand the definition of a prime number. Most of the class should solve it successfully if they completed exercise 4 correctly.

Answers, exercise 1A, page 259

1A. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39

Assignments (page 259)*_____

Minimum: 1–3. Average: 1–4H. Maximum: 1–5.

PAGES 260–261

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

As preparation for this lesson, have the children identify several numbers as even or odd. Then have them identify the prime numbers between 0 and 20. You might also have them make factor trees for some composite numbers, such as 48, 64, 72, 81.

Reviewing the Ideas

1. Answer **E** if the number is even and **O** if it is odd.

 A 68 E C 2001 O E 7642 E
 B 83 O D 3958 E F 7050 E

2. The first even number is 0. The second is 2. The third is 4. The fourth is 6. The fifth is 8.
 - A What is the sixth even number? 10
 - B What is the ninth even number? 16
 - C What is the seventeenth even number? 32

think

I'm slightly less than 30,
But more than 22.
They say that I am perfect,
And here is why they do.
Just add up all my factors,
And then divide by two.
You'll find that I'm the answer.
I'll say no more to you.

WHO AM I? 28

3. The first odd number is 1. The second is 3.
 - A What is the third odd number? 5
 - B What is the eighth odd number? 15
 - C What is the fourteenth odd number? 27

4. List the composite numbers between 20 and 30. 21,22,24,25,26,27,28

5. A List the factors of 18. 1,2,3,6,9,18 B List the factors of 24. 1,2,3,4,6,8,12,24
 - C List the common factors of 18 and 24. 1,2,3,6
 - D What is the greatest common factor of 18 and 24? 6

6. A List the factors of 30. 1,2,3,5,6,10,15,30 B List the factors of 42. 1,2,3,6,7,14,21,42
 - C List the common factors of 30 and 42. 1,2,3,6
 - D List the common **prime** factors of 30 and 42. 2,3
 - E What is the greatest common factor of 30 and 42? 6

7. A Write an equation to show that 38 is the product of two prime numbers. 38 = 2 × 19
 - B Write an equation to show that 39 is the product of two prime numbers. 39 = 3 × 13

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Discussion

Have the children work the exercises on page 260. Even though you may be using this exercise set as a test and thus choose not to have the children check their own papers, you should discuss the answers with the children immediately after they have completed the page.

The *Think* problem presents an interesting mathematical idea—that of a perfect number. The first three perfect numbers are 6, 28, and 496; this problem concerns the perfect number 28. You will notice that the factors of 28 are 1, 2, 4, 7, 14, and 28; and the sum of these factors is twice 28. One definition of perfect numbers may be stated as follows.

> A perfect number is a number such that the sum of all its factors that are less than the number is the same as the number.

That is, the sum of the factors 1, 2, 4, 7, and 14 is 28. The way the *Think* problem is stated, however, we add *all* the factors and then divide by two and get 28.

Keeping in Touch with Addition Subtraction Multiplication Division

1. Solve the equations.

A $10 \times 10 = n$ 100
B $100 \times 10 = n$ 1000
C $10 \times 1000 = n$ 10000
D $100 \times 100 = n$ 10000
E $100 \div 10 = n$ 10
F $1000 \div 10 = n$ 100
G $10\,000 \div 10 = n$ 1000
H $10\,000 \div 100 = n$ 100
I $10\,000 \div 1000 = n$ 10

2. Give the missing numbers.

A Since $5 \times 7 = 35$, we call [redacted] a multiple of 7. 35
B 48 is a multiple of 6 because $n \times 6 = 48$. 8
C Since $5 \times 9 = 45$, 45 is a multiple of both 5 and [redacted]. 9
D Since $63 + 78 = 141$, we know that $141 - 78 = n$. 63
E Since $156 - 79 = 77$, we know that $77 + 79 = n$. 156
F Since $7 \times 38 = 266$, we know that $266 \div 7 = n$. 38
G Since $147 \div 3 = 49$, we know that $49 \times 3 = n$. 147
H Since $39 + 39 = 78$, we know that $78 - (39 + 39) = n$. 0

3. Find the sums and differences.

A	B	C	D	E
35	132	5076	8206	30 042
+89	−59	+3938	−39	−5 978
124	73	9014	8167	24 064

4. Find the products.

A	B	C	D	E	F
9	38	434	32	627	347
×6	×7	×9	×25	×34	×562
54	266	3906	800	21 318	195 014

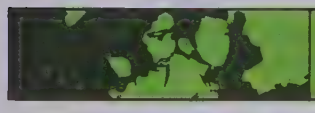
5. Find the quotients.

A $7\overline{)294}$ → 42
B $8\overline{)3448}$ → 431
C $23\overline{)2093}$ → 91

think

Multiply me by myself,
Or find my sum with two.
Your answer is the same
No matter which you do.

WHO AM I? 2



You are invited to explore

ACTIVITY CARD 13 Page 353

Using the Exercises

You may assign the exercises on page 261 as independent work or use them for a review discussion. If children understand the relationships treated in exercise 2, they should be able to give the answers without performing the computation. When they have finished, allow time for checking papers and discussion.

Follow-up

Encourage children to work with prime numbers less than 100 to find patterns. They may be motivated to explore patterns using other numbers also. For example, present them with the following exploratory questions. They should see that, if 2 is excluded, definite patterns emerge.

1. Find the sum of two prime numbers. What kind of number do you get? Is it odd, even? Is it prime, composite? Is there a pattern?
2. Subtract one prime number from another prime number. What kind of number do you get? Is it odd, even? Is it prime, composite? Is there a pattern?
3. Multiply 2 prime numbers. What kind of a number do you get? Is it odd, even?
4. Compare any patterns you found with those for odd and even numbers.
5. Make a square card and number the small squares from 0 through 99. Can you find any interesting patterns? (A sample is shown below.)

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Resources for Active Learning

Developmental Math Cards, G³5, Addison-Wesley.

Franklin Series: *Mirror Magic*, "Symmetry," pp. 58–72, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Inquiry in Mathematics via the Geoboard, "Congruent Triangles," Geo-Card 22, Walker. (Available from Fitzhenry & Whiteside)

Math Activity Cards, "Halving," B17, Macmillan.

Mathex: Geometry No. 4, "Plane Geometric Figures," pp. 31–32; "Geometric Solids," p. 33 (pupil pages 50–52), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Shape and Size* 2, "From 3-dimensional to Two," pp. 36–61, Wiley.

Workbook, page 86 (Use with text page 260.)

General Objectives

To provide a formal introduction to the concept of fractions

To introduce the concept of equivalent fractions

To provide experience in working with sets of equivalent fractions

To provide background material for a smooth transition to fractional numbers

The opening pages of the chapter establish a relationship between the concept of a number pair and that of a fraction. Formally, we take the point of view that a fraction is a written symbol for a number pair. However, we agree that often it is convenient to use the word fraction in place of number pair, and you will notice from time to time that we will occasionally say fraction when we mean number pair. It should, however, be clear at all times from the context of the material whether we are speaking of the number pair or the symbol. In the Chapter 14, fractions will be treated as symbols for fractional numbers.

The subsequent lessons cover the concepts of fractions and sets, fractions and parts of an object, and the concept of equivalent fractions. Most of the remainder of the chapter builds readiness for understanding fractional-number concepts by developing understanding of equivalent fractions and providing work with sets of equivalent fractions.

The words *numerator*, *denominator*, and *improper fraction* are introduced. A check to determine whether or not two fractions are equivalent is provided, and then, toward the end of the chapter, a lesson develops the idea that for each "complete" set of equivalent fractions, there is a unique fraction in the set that is called the *lowest-terms fraction* for that set.

You will note the absence of any attempt to teach particular skills for reducing fractions to lowest terms or for finding the lowest-terms fraction for another fraction. Our intent is to introduce the concept so that the children understand it completely before they are given work in Book 5 for developing particular skills in these operations.

Mathematics

This chapter is concerned primarily with developing concepts of fractions in preparation for the introduction of fractional-number concepts in Chapter 14. The study of fractions is conceptually different from the study of the numbers represented by fractions (fractional numbers). You will observe that, except for the brief references to pairs of numbers early in the chapter, essentially nothing is said about numbers throughout Chapter 12. Fractions are considered to be symbols for number pairs in the study of this chapter, and in Chapter 14 they will be considered to be symbols for fractional numbers.

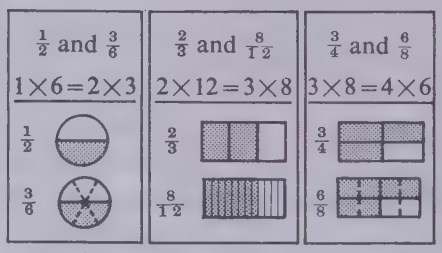
One of the clearest explanations we can give for the concepts presented in this chapter appears at the top of page 264 of the children's text. On this page we introduce the idea of thinking about number pairs and writing fractions to represent them. We may use the word *fraction* instead of *number pair*, but it should always be clear from the context whether we are referring to the symbol or to the pair of numbers involved.

Work with equivalent fractions is the most important part of Chapter 12 in preparing for the development of fractional-number concepts. We do not give the children a formal definition of equivalent fractions because such a definition would be too abstract for them. We intend that the children recognize two fractions as equivalent by seeing them in terms of sets or parts of a region.

A formal definition of equivalent fractions, for the teacher, follows:

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent to each other, if and only if $a \times d = b \times c$.

Some examples of pairs of equivalent fractions are shown below. The diagrams illustrate the equivalence relation with regard to part of an object.



Having provided the children with experiences in working with equivalent fractions, we lead them to build sets of equivalent fractions, starting with lowest-terms fractions such as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, etc. The following sets are built from such fractions:

Set A $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$,
Set B $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$,
Set C $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$.

Clearly, if we continue the established pattern indefinitely, each set contains an unlimited number of fractions. It is important to note that any fraction that is equivalent to one half is in set A (assuming an unlimited continuation of the obvious pattern) and that a fraction that is not equivalent to one half is not in set A. Similar statements are true for sets B and C, as well as for any other similarly constructed set of fractions.

We list below facts concerning equivalent fractions and sets of equivalent fractions; these facts are obvious, but are essential to the general concepts of Chapter 12.

a) The definition of equivalent fractions divides the fractions into separate and disjoint classes.

b) Any two fractions in one class are equivalent.

c) A fraction from one class and a fraction from a different class are not equivalent.

d) In every class, there is an unlimited number of fractions.

e) Every fraction is in exactly one class.

Since this chapter is concerned only with fractions and not with fractional numbers, we do not want to write equalities such as $\frac{1}{2} = \frac{2}{4}$. Eventually, we will give meaning to statements of this type, but only when one half and two fourths refer to fractional numbers. Clearly, one half and two fourths are different fractions, but they are equivalent. We could, if we thought it expedient, invent a symbol for equivalence of fractions and write a mathematical sentence about the equivalence between one half and two fourths. However, this is not necessary for our study. We do want to make it clear to children that pairs of fractions, such as one half and two fourths, are in a sense the same; we use the word *equivalent* to denote this.

Teaching the Chapter

Materials

Balance scales (if possible, 1 set for every 5 children)
Colored strips; Counters
Crayons; Geoboard or dot paper
2-cm graph paper or paper on which 2-cm squares have been duplicated
Mail-order catalogues, advertisements, recipes, etc., to show uses of fractions
Rubber bands for geoboard
Scissors
Sets of weights, including 1g, 5g, 10g, 20g, 50g, and 100g, 200g, 500g, 1kg sets
Short pieces of string (approximately 12 for every three children)
Slips of paper approximately 5 by 7 cm
Strips of paper approximately 20 or 25 cm long suitable for folding

Vocabulary

denominator	lowest terms
equivalent fraction	number pair
fraction	numerator
improper fraction	

Although most of the children's early experiences with fractions have been in terms of working with parts of sets or parts of regions, you should find it helpful to provide additional experiences of this type. Be alert to guide your children toward more abstract thinking as their experiences with fractions intensify in this chapter. In the early part of the chapter, use the concrete experiences as needed; and then, as the children are able to think more and more abstractly about fractions, reduce the use of materials. If most of your children are very advanced, limit the use of set ideas to the drawings on the chalkboard and those in the text itself. However, all should benefit from the concrete investigations suggested in the text.

Many vocabulary words in this chapter are familiar to the children. However, such words as *numerator*, *denominator*, and *improper fraction* are almost sure to be new. We attempt to play down the term *improper fraction* in this development because of the misleading connotation of the word *improper* with respect to certain types of fractions. Avoid having the children think that there is something wrong with a fraction such as $\frac{8}{5}$. The expression *improper fraction* has historical significance much like that of *negative* number. The connotations of such words show that those who first thought of, used, and named these numbers had little faith in them. Expose the children to the term *improper fraction*, but play it down, and use it infrequently. Whenever possible, just use the word *fraction* for the improper fraction.

Lesson Schedule

Plan to cover the material in this chapter in about three-and-a-half or four weeks. Since this chapter is quite long, you must adjust this recommended schedule to the needs and abilities of your children. Certainly, if your children have had considerable background material in fractions, you will be able to move through the chapter much more rapidly than if they have had very limited experience with fractions.

Evaluation of Progress

The most important evaluation criteria in this chapter are general understanding of fraction concepts and the children's ability to construct a set of equivalent fractions with reasonable knowledge of the ideas involved. The heavy emphasis on equivalent fractions in this chapter is necessary as background for understanding fractional numbers, presented in Chapter 14. Evaluation based primarily on understanding of concepts is difficult, and therefore, your evaluation should be based primarily on daily observations of the children rather than on the use of cumulative test materials.

The role this chapter plays in the overall development of fractional-number concepts is an important one. The children need to get some feeling for a quantitative relationship, of quantitative sameness, between pairs of fractions from the set $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\}$. If you can successfully build their comprehension of the quantitative sameness between pairs of equivalent fractions, much will be gained in terms of readiness for the more abstract concepts presented in Chapter 14.

Resources for Active Learning

General Activities

Activities for developing fractional number concepts: *Mathex:* Numeration No. 2, "Fractions," pp. 48–52, (pupil pages 47–60), Encyclopaedia Britannica Publications Ltd.
Developmental Math Cards, $H^2$2, $H^1$18, Addison-Wesley
Nuffield Project: *Computation and Structure* 3, "Fractions," pp. 70–79, Wiley
Math Activity Cards, B21, C34, D35, Macmillan. [Three recipes]
Notes on Mathematics in Primary Schools, "Fractions," pp. 190–195, Cambridge University Press. [Using geoboards] (Available from Macmillan of Canada)

Manipulative Devices

Geoboards (Addison-Wesley)
Lowenfeld Poleidoblocs Set G (Responsive Environments Corp.; Selective Educational Equipment)
Modulated Cut-outs (Encyclopaedia Britannica Publications Ltd.)
Unifix Fractions Kit (Educational Teaching Aids)

Commercial Games

Action Fractions (Math Media)
Competitive Fractions (Selective Educational Equipment)
Fraction Dominoes (Responsive Environments Corp.; Selective Educational Equipment)
Fractions Are as Easy as Pie (Milton Bradley; school supplier)

PAGES 262–263

Objective

Given pairs of strips, the child will match each pair with a number pair and express that number pair as a fraction.

Preparation

Materials

colored strips

You might review previous uses of the strips, such as in exploring measurement concepts, addition and subtraction, odd and even numbers, etc. Then explain to the children that once again these strips will help them understand some important mathematical ideas.

Investigation

In this investigation, children use the strips to relate one whole number to another. They might take any pair of strips from their set and write the fraction related to it. If a child chooses two strips of equal length, for example two 8-strips, he still can write a fraction such as $\frac{8}{8}$. Or he may think of the 9-strip in relation to the 3-strip and write $\frac{9}{3}$. However, if he follows the suggestions in the text, he will think of the 3-strip covering 3 out of 9 parts of the 9-strip. Note that the number pair refers not to the two separate strips but to how many parts of the strip are covered in relation to how many parts that strip has. Encourage the children to try showing at least 8 or 10 fractions. If a child has difficulty, ask him to show a specific fraction such as $\frac{4}{7}$, saying something like, "Can you show a situation in which 4 parts out of 7 are covered?"

12 Fractions

Let's explore number pairs and fractions.

Investigating the Ideas

A



parts out of are covered.

("two ") of the purple strip is covered.

B



3 parts out of 5 are covered.

$\frac{3}{5}$ ("three fifths") of the yellow strip is covered.

"$\frac{2}{4}$" and "$\frac{3}{5}$" are called **fractions.**

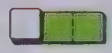
? Can you show "cover-ups" like these with other strips and write the fractions?

Answers will vary. See Investigation.

Discussing the Ideas

1. A What **pair of numbers** do we think about in A? 2,4 in B? 3,5
 B We wrote the **fraction** $\frac{2}{4}$ in A. What fraction did we write in B? $\frac{3}{5}$

2. A What number pair do you think about in this picture? 1,3



 B What fraction of the light green strip is covered? $\frac{1}{3}$

3. A What number pair do you think about in this picture? 2,3



 B What fraction of the light green strip is covered? $\frac{2}{3}$

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Discussion

As you work through the discussion exercises, have the children observe the correct manner of reading fractions. However, correct usage will develop naturally and easily more from your example than from lengthy explanation. One of the main points of this lesson is to express a number pair as a fraction. Thus, in exercise 2, we think about the number pair 1,3 (or 1 part covered, 3 parts in all), so we write $\frac{1}{3}$ to express how much of the green strip is covered.

Extend the discussion with other examples. For instance, have children describe a pair of strips such as "the yellow covering the blue" and ask others to name and write a fraction related to each pair.

Using the Ideas

1. Give the **number pair** and the **fraction** for each picture.

A



? out of ? is covered.
? of the yellow strip
is covered. 1/5

B



? out of ? are covered.
? of the brown strip
is covered. 3/8

C



? out of ? is covered.
? of the red strip
is covered. 1/2

D



? out of ? are covered.
? of the orange strip
is covered. 9/10

E



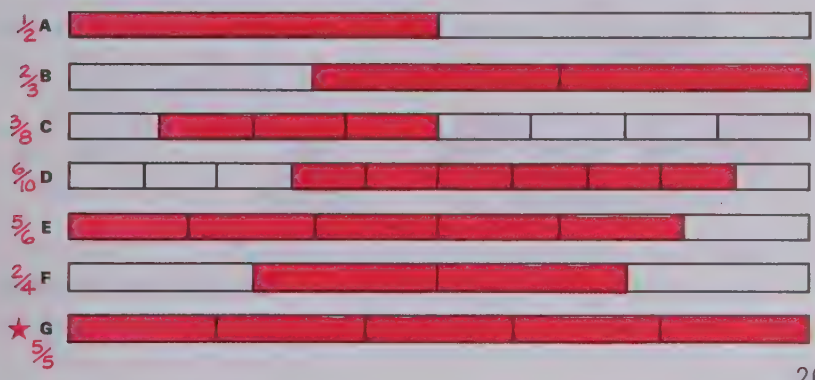
? out of ? are covered.
? of the purple strip
is covered. 3/4

F



? out of ? are covered.
? of the brown strip
is covered. 5/8

2. What fraction of each long strip has been shaded red?





Using the Exercises

Have the children do the exercises on page 263 independently. When they have finished, allow time for checking papers and for further discussion. Point out to the children that each fraction expresses a relation between how many parts of a strip are covered and how many parts that strip has in all. To help children develop an understanding of this relation, you might read the text question for each exercise in a manner similar to this example for exercise 1A: "What fraction of the yellow strip is covered?" Continue to stress how a number pair may be symbolized by a fraction.

Assignments (page 263)

Minimum: 1–2B. Average: 1–2F. Maximum: 1–2.

Mathematics

Although we establish in this lesson that we consider fractions to be symbols for number pairs, we will sometimes say *fraction* when in context it is clear that we are speaking of the number pair involved rather than of the symbol. However, just as with the use of the words *number* and *numeral* when speaking of whole numbers, it will always be clear from the context whether we are referring to the number itself or to the symbol for the number. When fractional numbers are introduced in Chapter 14, it will be seen that fractions are symbols for fractional numbers. Here, the word fraction will be used occasionally for fractional numbers rather than for a specific symbol, but the ideas involved should be clear from the context.

Again, we make the recommendation that we made for the use of number-numeral terminology in the study of whole numbers: use the language which will most clearly illustrate the ideas involved, even if such language does not precisely accord to definitions. For example, just as you might tell a child to write a *number* (when, precisely, you mean *numeral*), you might also ask, "What *fraction* of the objects is blue?" Although such a question can be answered by writing a fraction, it clearly implies that you do not really mean "What *symbol* of the objects is blue?" but, rather, "What *part* of the objects is blue?"

Follow-up

Suggest that children roll four dice and see how many pairs of numbers shown by combinations of the dice they can write as fractions. They may then try to match each fraction with a pair of strips.



$\frac{2}{5} \rightarrow \frac{\text{red strip}}{\text{yellow strip}}$ $\frac{2}{4} \rightarrow \frac{\text{red strip}}{\text{purple strip}}$

$\frac{2}{6} \rightarrow \frac{\text{red strip}}{\text{dark green strip}}$ $\frac{5}{6} \rightarrow \frac{\text{yellow strip}}{\text{dark green strip}}$

Workbook, page 87

PAGES 264–265

Objective

Given a region or set which is pictured so that a fractional part of the region or set is indicated, the child will be able to give a number pair or a fraction for the indicated part of the region or set.

Preparation

Since there is no investigation for this lesson, you might use your preparation time for a review of the previous lesson and as an introduction to the discussion exercises.

For example, name a number pair and ask the children to cover one strip with another to match the number pair. Thus, if you said, "2,5," they would place the red strip on top of the yellow strip. Continue with several number pairs such as (1,4), (2,3), (5,8), (7,10), (8,9), (6,6), (4,8), (2,7), and so on. Have a child write on the chalkboard a fraction for each number pair.



Can you write and read fractions?

Discussing the Ideas

1.

We see	We think	We write	We say
	3 red parts 4 parts in all	$\frac{3}{4}$ of the region is red.	three fourths of the region is red.
	2 parts covered 3 parts in all	▭ of the light green strip covered. $\frac{2}{3}$	▭ of the light green strip is covered. two thirds
	4 blue parts 6 parts in all	▭ of the region is blue. $\frac{4}{6}$	▭ of the region is blue. four sixths
	1 green dot 5 dots in all	▭ of the dots are green. $\frac{1}{5}$	▭ of the dots are green. one fifth

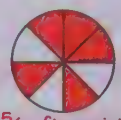
In this column, we think about a number pair.

In this column, we write a fraction for the number pair.

Can you write and read the fraction for each part?

2. Write and read a fraction for the shaded part of each figure.

A  $\frac{3}{8}$, three eighths

B  $\frac{5}{8}$, five eighths

C  $\frac{9}{10}$, nine tenths

D  $\frac{6}{11}$, six elevenths

3. What fraction is suggested by each of these?

A 2 out of 5 $\frac{2}{5}$
B 7 out of 10 $\frac{7}{10}$
C 1 out of 2 $\frac{1}{2}$
D 2 out of 3 $\frac{2}{3}$
E 12 out of a hundred $\frac{12}{100}$
F 9 times out of 10 $\frac{9}{10}$

Discussion

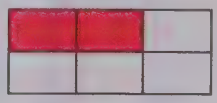
In this lesson, children learn to associate fractions not only with the parts of strips but also with various regions and sets. Discuss each illustration and the given number pair and have the children write and read the related fraction. It would be helpful for you to write the answers to the last two columns on the chalkboard. For example, for the first part of exercise 1, you would write $\frac{3}{4}$, and say "three fourths." Work through exercises 2 and 3 orally, having children take turns in writing the fractions on the chalkboard.

Depending on the ability and background of the children, provide set demonstrations patterned on the illustrations of exercise 1. Care must be taken to see that the children give the correct number pairs. For example, if you exhibit a set of five objects, of which two are blue and three are black, the children might err by saying, "Two blue, three black; therefore two thirds are blue." You must see that the children say, "Two blue, five in all; therefore two fifths are blue."

If necessary, provide children with more practice in reading fractions. It is important that children learn to read fractions effectively; otherwise, they will have difficulty communicating in future lessons.

Using the Ideas

Give the missing numbers. Then give a fraction to answer each question.

1. 

 A ▨ parts are red. 2
 B ▨ parts in all. 6
 C What part of the region is red? 2/6

2. 

 A ▨ dots are black. 6
 B ▨ dots in all. 20
 C What part of the dots are black? 6/20

3. ▲ ▲ ▲ ▲ ▲
 △ △ △ △ △

 A ▨ triangles are red. 5
 B ▨ triangles in all. 10
 C What part of the triangles are red? 5/10

4. 

 A ▨ sections are red. 2
 B ▨ sections in all. 4
 C What part of the region is red? 2/4



5. Write a fraction for this number-pair story. Tom said, "5 of the 8 birds in our yard are cardinals." What fraction of the birds are cardinals? 5/8



6. Write a number-pair story about each fraction.

 A 1/4 B 2/3 C 5/6 D 1/2 E 7/8 F 3/4

 Answers will vary.

265

Using the Exercises

Have the children work through the exercises on page 265 independently. When you check their work and discuss the exercises, accept any fraction equivalent to the indicated answer. For example, in answer to exercise 3, some children may very well say "five tenths," while others may say "one half," of the triangles are red. However, the problem is designed to encourage the response "five tenths," and the children, particularly those who answer "one half," should realize this.

Assignments (page 265)
Minimum: 1–4. Average: 1–5.
Maximum: 1–6.

Follow-up

To help children who need more experience in associating fractions with given parts of sets and regions, give them a worksheet similar to the following one.

Color the number of objects as directed.
Write a fraction that shows what part is colored.

Color 4 ★'s. ____ of the set is colored.
★ ★ ★ ★ ★ ★ ★ ★
★ ★ ★ ★ ★ ★ ★ ★

Color 6 △'s. ____ of the set is colored.
△ △ △ △ △ △ △ △ △ △ △ △

Color 3 ⌓'s. ____ of the set is colored.
⊕ ⊕ ⊕

Color 8 ○'s. ____ of the set is colored.
○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○

Color 7 ♡'s. ____ of the set is colored.
♡ ♡ ♡ ♡ ♡ ♡ ♡
♡ ♡ ♡ ♡ ♡ ♡ ♡

Color 4 □'s. ____ of the set is colored.
□ □ □ □ □ □ □
□ □ □ □ □ □ □

For others, you might write several fractions on the chalkboard and have them write a number-pair story for each or draw a picture to illustrate each.

Resources for Active Learning
Developmental Math Cards, G120, Addison-Wesley.

Workbook, page 88

PAGES 266–267

Objective

Given parts of regions or parts of an object, the child will show his understanding that fractional parts of an object must be the same size. He will show this by identifying fractional parts.

Preparation

Materials

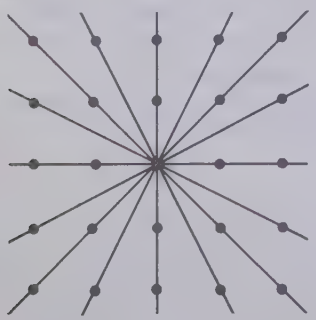
geoboard for each child or dot paper (*Duplicator Masters*, page 64)

It would be appropriate to begin immediately with the investigation and omit any particular preparation. However, if children are not very familiar with a geoboard or dot paper, spend a few minutes showing a few shapes that can be formed on the board or paper. Be sure to use shapes that are unrelated to the investigation activity so that you do not preempt the children's opportunity for discovery.

Investigation

In this investigation, children explore the concept that in order for a region to be divided into halves it must not only be separated into two parts but must be separated into parts that are the same size.

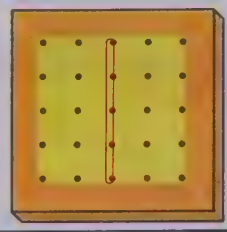
If children use geoboards, remind them to keep a count of each division they find. The number of divisions possible depends on the dimensions of the region used. For example, if the geoboard or dot paper is 5 by 5, the divisions that can be made by a straight line are as shown below.



Let's explore fractions and parts of an object.

Investigating the Ideas

Use a nailboard and rubber bands or dot paper and lines.



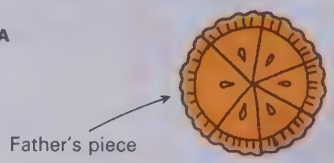
? How many different ways can you find to use one straight rubber band (line) and divide the yellow part into halves?
See Investigation.

Discussing the Ideas

1. How can you tell when you have divided the yellow part into halves? Sample answer: There will be the same number of nails in each half. Each half will have the same area.

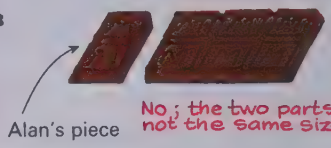
2. Answer the questions and explain your answers.

A



Is Father's piece $\frac{1}{6}$ of the pie? No; the pieces are not all the same size.

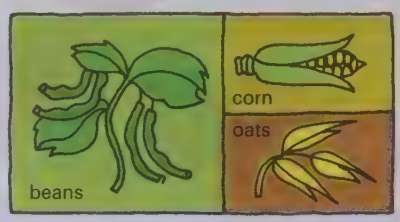
B



No; the two parts are not the same size.

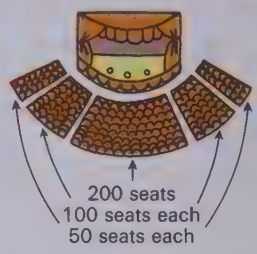
Is Alan's piece $\frac{1}{2}$ of the candy bar?

C



Did Mr. White plant $\frac{1}{3}$ of his field with oats?
No; the three areas are not the same size.

D



Does the centre section have $\frac{1}{5}$ of the seats?
No; the sections are not the same size.



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Discussion

Usually children intuitively grasp the idea that in order for 2 parts of a region to be halves they must be equal in size. Thus, the criterion used in exercise 1 is that both regions called halves are the same size. As you discuss exercise 2, continue to emphasize that when fractional portions of *one particular object* are being considered, the portions must be of the *same size*. You might stress this by asking children to estimate what fractional parts Father's piece of pie and Alan's piece of candy are. Estimation of what fractional part of the field is planted with oats might be quite difficult for some. The important point is for children to recognize that the field is not divided into sections that are the same size.

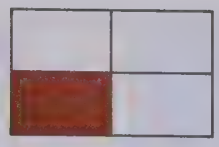
As you discuss the concepts above, it would be helpful to point out to the children that, when they are considering fractional parts of a *set*, the parts can have objects of *different sizes* as long as each part has the same number of objects. For instance, for a set consisting of a horse, a cow, a pig, and a chicken, we could say that one fourth of the animals are in the bird family even though the chicken does not represent one fourth of the total bulk of the four animals. However, when we speak of

Using the Ideas

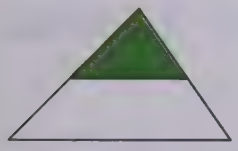
Answer **true** or **false** for each exercise.

1. 

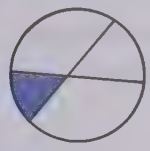
$\frac{1}{2}$ of the region is red. False

2. 

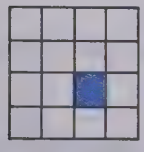
$\frac{1}{4}$ of the region is brown. True

3. 

$\frac{1}{2}$ of the region is green. False

4. 

$\frac{1}{4}$ of the region is blue. False

5. 

$\frac{1}{16}$ of the region is blue. True

6. 

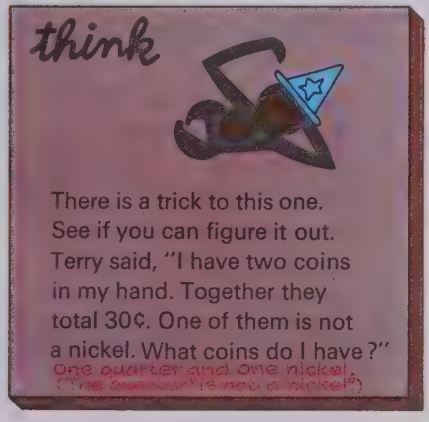
$\frac{1}{7}$ of the region is yellow. False

7. 

$\frac{3}{4}$ of the region is orange. True

8. 

$\frac{3}{4}$ of the region is red. False



More practice, page A-26, Set 48

a fractional part of a region, each of the pieces must be the same size.

Using the Exercises

Have the children work the exercises on page 267 independently. When they have finished, allow time for discussion and checking papers.

Encourage all to try the *Think* problem and discuss it before you comment on which answer is correct. Although the trick is easy to see when pointed out, many will not find it. The statement, "One of them is not a nickel," is often misinterpreted as, "Neither of them is a nickel."

Assignments (page 267)

Minimum: 1–5, oral. Average: 1–6. Maximum: 1–8.

Follow-up

Suggest to the children that they think of and draw examples similar to those on page 267. When they have made up four or five true or false exercises, have them exchange papers and work each others' problems. Stress the fact that any examples they intend to have considered true must be very carefully drawn so that their picture is as accurate as possible.

Duplicator Masters, page 52
Workbook, page 89
Skill Masters, page 52

PAGES 268–269

Objective

Given a comparison of parts of a region to the whole region, the child will be able to write more than one fraction to express the comparison.

Preparation

Materials

strips of paper approximately 20 or 25 cm long, suitable for folding

You might begin immediately with the investigation. However, if you prefer, spend a few minutes to provide the children with practice in reading and writing fractions.

Investigation

In this investigation, children explore, through physical experiences, the idea that the same fractional amount of a whole may be named by more than one fraction. As children work through this investigation, make sure it is one half of the strip that they color. They may need to be reminded to fold the strip in half before they begin to color. By folding one strip of paper first in half, then into fourths, and finally into eighths, the child names the same amount with the fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$.

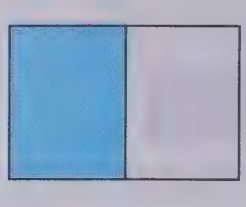
You may wish to extend the investigation by having the children color one half of another strip which is the same size as the first, and then challenge them to fold it in such a way as to show that $\frac{3}{6}$ of the strip has been colored.

When the child folds the same size strip in half and then into thirds to show six equal partitions, it is important that he realizes that his $\frac{3}{6}$ section is the same amount as the $\frac{4}{8}$ section of his first strip. Only by concretely relating the $\frac{4}{8}$ and $\frac{3}{6}$ to the same amount will he get the idea that the fractions $\frac{4}{8}$ and $\frac{3}{6}$ represent the same idea.

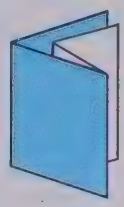


Can two fractions show the same amount?

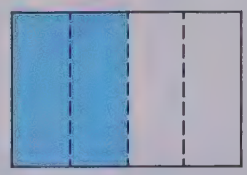
Investigating the Ideas



$\frac{1}{2}$ of the paper is colored.



Fold the paper twice.

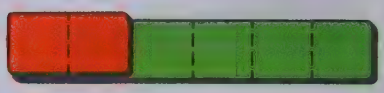


$\frac{2}{4}$ of the paper is colored.

Can you fold and color a piece of paper to show $\frac{4}{8}$ of it colored? See Investigation.

Discussing the Ideas

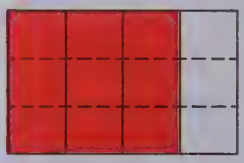
1. What part of the dark green strip is covered by the red strip? Can you give more than one fraction to show the part that is covered?



$\frac{2}{6}$

Yes; $\frac{2}{6}$ and $\frac{1}{3}$, for example.

2. A Explain what you are thinking if you say, "$\frac{9}{12}$ of the region is red." Sample answer: nine of the twelve squares are red.



B Explain what you are thinking if you say, "$\frac{3}{4}$ of the region is red." Sample answer: Three of the four columns are red.

3. Explain two different ways you might think about the part of the small square region that is shaded.



Sample answer: One of the 2 larger triangles is shaded. Two of 4 smaller triangles are shaded.

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Discussion

Spend sufficient time discussing the idea that the fractions $\frac{4}{8}$ and $\frac{3}{6}$ name the same amount. If possible, use a transparency on an overhead projector to show how the fractions $\frac{4}{8}$ and $\frac{3}{6}$ represent the same amount even though they represent different partitions of that amount.

Discussion exercise 1 stresses the fact that the fractions $\frac{2}{6}$ and $\frac{1}{3}$ name the same amount. Children may need help in identifying the thirds on the strip. Ask them how they are thinking when they say the red strip is covering $\frac{1}{3}$ of the strip and when they say the red strip is covering $\frac{2}{6}$ of the strip.

As you discuss exercises 2 and 3, continue to stress how the same amount can be named by more than one fraction. It would also be helpful to mention that the denominator (bottom numeral) indicates into how many parts the whole has been divided, and that the numerator (top numeral) indicates how many of those parts are being considered.

Using the Ideas

1. First give the missing number. Then give the fraction for the number pair.

A 3 of the ▨ parts are red. 6
▨ of the region is red. $\frac{3}{6}$



B 1 of the ▨ parts is red. 2
▨ of the region is red. $\frac{1}{2}$



C 4 of the ▨ parts are red. 8
▨ of the region is red. $\frac{4}{8}$



2. Give at least two fractions to tell what part of each region is shaded.

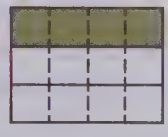
A $\frac{2}{6}, \frac{1}{3}$



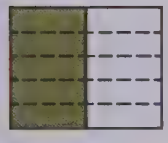
B $\frac{4}{8}, \frac{1}{2}$



C $\frac{4}{12}, \frac{1}{3}$



D $\frac{5}{10}, \frac{1}{2}$



E $\frac{2}{8}, \frac{1}{4}$



F $\frac{4}{6}, \frac{2}{3}$



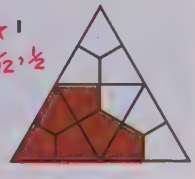
G $\frac{6}{12}, \frac{1}{2}$



★ H $\frac{3}{9}, \frac{1}{3}$



★ I $\frac{6}{12}, \frac{1}{2}$



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Follow-up

Children will enjoy paper-folding activities which help them work with fractions that name the same amount. For example, suggest that they fold and color pieces of paper to show that each of the following pairs of fractions names the same amount.

$\frac{1}{4}, \frac{2}{8}$ $\quad$ $\frac{4}{6}, \frac{2}{3}$

$\frac{3}{4}, \frac{6}{8}$ $\quad$ $\frac{6}{12}, \frac{1}{2}$

$\frac{8}{16}, \frac{1}{2}$ $\quad$ $\frac{8}{12}, \frac{2}{3}$

$\frac{4}{16}, \frac{2}{8}$ $\quad$ $\frac{4}{16}, \frac{1}{4}$

Using the Exercises

Encourage the children to try to do the exercises on page 269 independently. When they finish, ask volunteers to explain the exercises. You may have to give help on some problems.

In talking about exercise 2, have the children discuss how, for each figure, they can think of more than just one fraction to describe the part of the figure that is shaded. The line segments that divide each region should help the children think about different fractions for a given shaded portion.

Assignments (page 269)

Minimum: 1. Average: 1–2G.
Maximum: 1–2.

PAGES 270–271

Objective

Given a set of objects, the child will be able to name a fractional part of the set by using more than one fraction.

Preparation

Materials

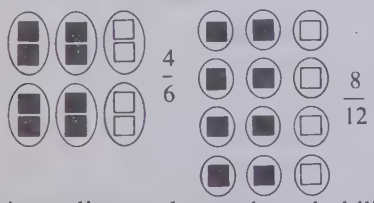
2-cm graph paper or duplicated worksheets ruled into squares (*Duplicator Masters*, page 65); crayons; pieces of string (Counters may be used in place of the squares and crayons.)

It would be appropriate to begin immediately with the investigation. However, if you prefer, prepare for this lesson by reviewing how 2 fractions can name the same amount. For example, display geometric shapes or strips of paper showing regions which may be named by more than one fraction, and ask the children to name them.

Investigation

In order for the children to circle the sets to show $\frac{4}{6}$, they must group the squares in twos. If they are using the 12 squares, 8 of which are blue, then 4 of these pairs will be colored and 2 not colored. Note that they are actually comparing part of a set to the total in the set. If the part is described as $\frac{8}{12}$ or as $\frac{4}{6}$, it is the same part of the set which is being compared to the total.

Two of the possible fractional names which show this comparison of the colored part of the set to the total set are as follows:



$\frac{4}{6}$ $\frac{8}{12}$

According to the needs and ability of the class, supply the children with added materials in order to find fractions which would show a comparison of 8 colored squares out of a total of 24 squares. They should circle the squares in groups of 8, 4, 2, and 1 to name $\frac{1}{3}$, $\frac{2}{6}$, $\frac{4}{12}$, and $\frac{8}{24}$. Alternatively, you might have children work in groups of two or three using counters and string. For example, if the children show 12 counters they could circle 8 of them with string to show $\frac{8}{12}$.

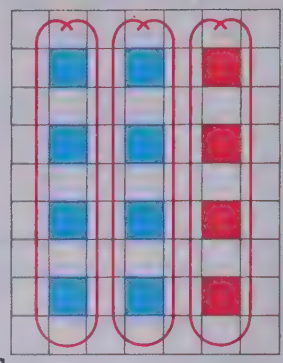


Let's explore fractions and sets.

Investigating the Ideas

2 sets out of 3 sets are blue.
$\frac{2}{3}$ of the squares are blue.

On your graph paper, color 12 squares like this, with 8 of them blue.

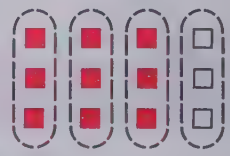


? Can you circle sets of squares to show that $\frac{4}{6}$ of the squares are blue?

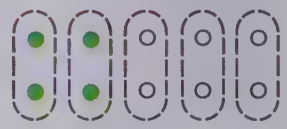
See Investigation.

Discussing the Ideas

1. A Explain what you are thinking if you say, "$\frac{9}{12}$ of the squares are red." "9 out of 12 squares are red."
 B Explain what you are thinking if you say, "$\frac{3}{4}$ of the squares are red." "3 of the 4 ringed groups are colored."



2. A Explain what you are thinking if you say, "$\frac{4}{10}$ of the circles are green." "4 out of 10 circles are green."
 B Explain what you are thinking if you say, "$\frac{2}{5}$ of the circles are green." "2 of the 5 ringed groups of circles are green."



3. Explain two different ways you might think about what part of the set of triangles is red.



See Discussion.

Discussion

One of the important points of this lesson is for children to realize that, for a given part of a set, they can often think of more than one fraction simply by considering different groupings. As you carefully discuss exercises 1 through 3, point out for the children how the dotted rings around the parts of a set help them think of the set in different ways. In exercise 3, for example, they might think, "1 ring of dots out of 2 rings of dots is colored" or "2 triangles out of 4 triangles are colored." Stress the idea that they are considering a part of a total set compared to the total set. Later they will use fractions to compare two separate sets.

Continue with the discussion, using as examples the groupings the children explored in the investigation. Ask the children to explain their thinking for each fraction they gave in the investigation.

Using the Ideas

1. First give the missing number. Then give the fraction for the number pair.

 A 8 of the ▨ squares are red. 12
 ▨ of the squares are red. $\frac{8}{12}$



 B ▨ of the 6 sets have red squares. 4
 ▨ of the squares are red. $\frac{4}{6}$

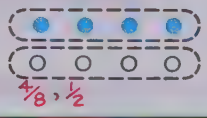


 C ▨ of the 3 sets have red squares. 2
 ▨ of the squares are red. $\frac{2}{3}$

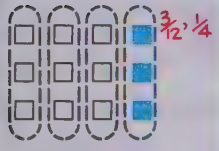


2. For each set, give at least two different fractions to tell what part of the set is colored.

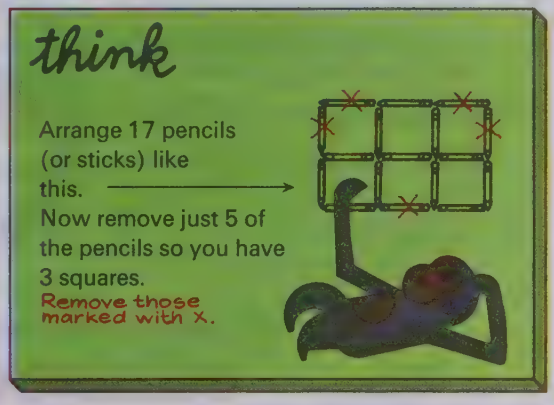
A  $\frac{3}{6}, \frac{1}{3}$

B  $\frac{4}{8}, \frac{1}{2}$

C  $\frac{4}{12}, \frac{1}{3}$

D  $\frac{3}{12}, \frac{1}{4}$

E  $\frac{6}{8}, \frac{3}{4}$

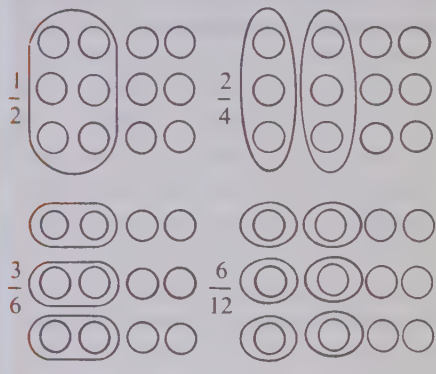


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Follow-up

Some children may benefit from use of the counters with pieces of string, whether or not they used these materials in the investigation for this lesson. Encourage children to work together in small groups so that they can demonstrate for each other different fractions with the counters and string. For example, write the fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{6}{12}$ on the chalkboard. Suggest that children use 12 or 24 counters and circle the appropriate number of counters to show each of the given fractions.



Resources for Active Learning

Mathex: Numeration No. 2, "Spin and Say Game," pp. 51–52, Encyclopaedia Britannica Publications Ltd.

Using the Exercises

Before you assign the exercises on page 271, make sure children realize that for the second statement in exercises 1A, 1B, and 1C they must give a *fraction*. Also point out how the dotted rings help them think of the sets in a slightly different way, even though the same number of objects is shaded for all three parts.

When the children have finished the exercises and you are checking exercise 2, note that there are more than 2 correct fractions for each set.

For exercise 2C, the children might give $\frac{1}{3}$ or $\frac{4}{12}$. With some imagination, one might suggest $\frac{2}{6}$. Although the fraction $\frac{3}{9}$ would not be incorrect, this set picture does not suggest the fraction $\frac{3}{9}$ as telling what part of the set is colored.

For the *Think* problem, you may wish to give the children 17 pencils or rods to manipulate to help them discover a solution. The problem may be solved in several ways, one of which is indicated on the annotated student page above.

Assignments (page 271)

Minimum: 1, oral. Average: 1–2. Maximum: 1–2.

PAGES 272–273

Objective

Given equivalent fractions, the child will be able to recognize and use the phrases "equivalent fractions" and "is equivalent to."

Preparation

To review previous lessons, exhibit a set of 6 objects, 3 of which are red and 3 another color. Ask the children to write two fractions for this set. They will probably write "$\frac{1}{2}$" and "$\frac{3}{6}$". Use other examples from pages 268 through 271 and ask the children to give you a pair of fractions for each figure.

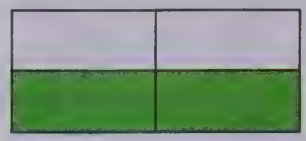


What are equivalent fractions?

Discussing the Ideas

For exercises 1-3, see Discussion.

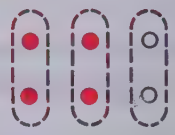
1. A Explain what you are thinking if you say, "$\frac{1}{2}$ of the region is shaded."
 B Explain what you are thinking if you say, "$\frac{2}{4}$ of the region is shaded."



- Such pairs of fractions are called equivalent fractions

$\frac{1}{2}$ is equivalent to $\frac{2}{4}$.

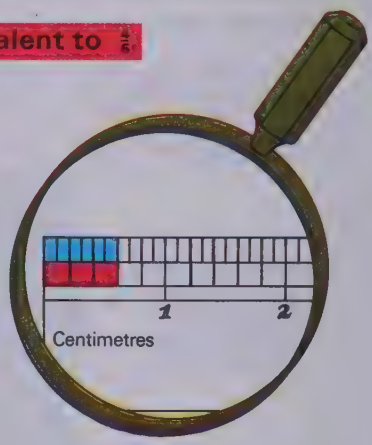
2. A Explain what you are thinking if you say "$\frac{2}{3}$ of the dots are red."
 B Explain what you are thinking if you say "$\frac{4}{6}$ of the dots are red."



- Such pairs of fractions are called equivalent fractions

$\frac{2}{3}$ is equivalent to $\frac{4}{6}$.

3. A Explain what you are thinking if you say, "$\frac{\blacksquare}{5}$ of the way to the 1-cm mark is shaded red."
 B Explain what you are thinking if you say, "$\frac{\blacksquare}{10}$ of the way to the 1-cm mark is shaded blue."



- Such pairs of fractions are called equivalent fractions

$\frac{1}{5}$ is equivalent to $\frac{2}{10}$.

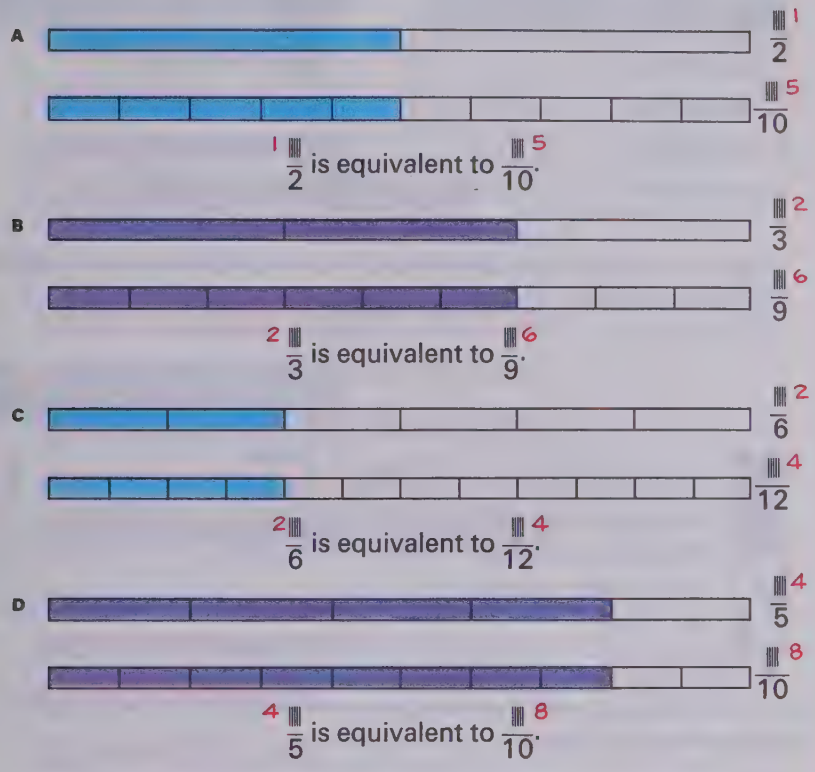
Discussion

In this lesson, children are introduced to the vocabulary for the concepts which were developed in the previous lessons. Have several children give their explanations for each exercise. Be sure that for each exercise children see the relationship between the two fractions and the picture. For example, in exercise 2, if we think, "Two of the three sets of dots are red," or if we think, "Four of the six dots are red," we are thinking of the same part of the whole and the two fractions, $\frac{2}{3}$ and $\frac{4}{6}$, are equivalent.

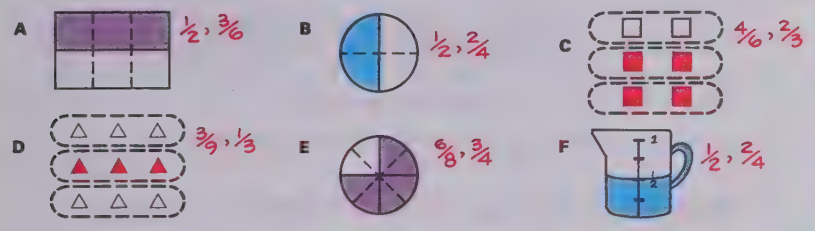
Note that children are not expected at this time to write equations such as $\frac{2}{3} = \frac{4}{6}$. (Recall the explanation in the mathematics section of the introduction to the chapter: since this chapter does not treat fractional numbers per se, equalities such as $\frac{2}{3} = \frac{4}{6}$ should not be written here.)

Using the Ideas

1. Complete the fraction suggested by the shading of each bar. Then copy the sentence and complete the fractions.



2. Write the pair of equivalent fractions suggested by each picture.



Using the Exercises

You might use exercise 1A on page 273 as a basis for discussion. For example, the children should observe that the fractions for exercise 1A are equivalent because the same amount of the bar is shaded for each one; but for the second, they think of $\frac{5}{10}$ because of the divider marks, and for the first bar, they think of $\frac{1}{2}$. Be sure that the children are given ample opportunity to explain these ideas.

Following this discussion, instruct the children to do the remaining exercises. When they have finished, ask them to explain how they arrived at the two fractions for each exercise.

Assignments (page 273):
Minimum: 1, oral. Average: 1–2C. Maximum: 1–2.

Mathematics

In this lesson, rather than provide a formal definition for the equivalence of two fractions, we continue to develop the idea in terms of sets and parts of a region in such a way that children perceive intuitively the general idea of equivalent fractions.

The standard definition for equivalent fractions follows:

The fraction $\frac{a}{b}$ is equivalent to $\frac{c}{d}$ if and only if $a \times d = b \times c$.

The fact that this definition expresses the same general idea as does the intuitive definition given in the student text requires further consideration. Certainly, using either the intuitive definition or the standard definition, we could easily observe that a fraction is equivalent to itself. Let us examine the fraction *two thirds*. Two thirds is equivalent to two thirds because $2 \times 3 = 3 \times 2$. Now, let us examine the two fractions *two thirds* and *four sixths*. Of course, if we write the numerator and denominator of four sixths in factored form, we are saying that two thirds is equivalent to the fraction

$$\frac{2 \times 2}{2 \times 3}.$$

That is, four sixths differ from two thirds only in having a factor of 2 in both the numerator and the denominator. Hence, $2 \times (2 \times 3) = 3 \times (2 \times 2)$. Note that the difference between this equality and $2 \times 3 = 3 \times 2$ is simply a factor of 2 on each side.

The importance of the idea of equivalent fractions lies in the fact that it allows us to separate our fractions into large classes from which we shall abstract the concept of fractional number. For example, we want to associate just one number with the two fractions, two thirds and four sixths, and we will also want to observe that there are other fractions for this same number, such as $\frac{6}{9}$, $\frac{8}{12}$, $\frac{10}{15}$.

Resources for Active Learning

Mathex: Numeration No. 7, "Equivalent Fractions," pp. 41–42 (pupil pages 32–41), Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 53
Workbook, page 90
Skill Masters, page 53

PAGES 274–275

Objective

Given charts or bars showing sets of equivalent fractions, the child will show his understanding of sets of equivalent fractions by listing the next three or four fractions of the set.

Preparation

To prepare for this lesson, review some familiar pairs of equivalent fractions. For example, give the children $\frac{1}{2}$ and ask for an equivalent fraction. Continue with examples such as $\frac{4}{6}$, $\frac{5}{10}$, $\frac{8}{16}$, $\frac{9}{12}$, $\frac{2}{3}$, $\frac{3}{6}$, $\frac{2}{8}$. If children have difficulty thinking of equivalent fractions, explain that they will soon learn a way to find them more easily.

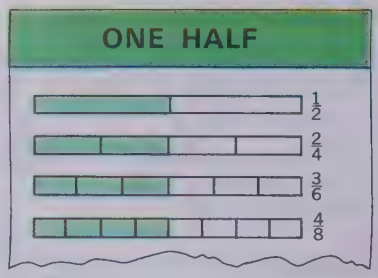
Investigation

Before the children begin the investigation, ask them to study the two charts on the page. Be sure they understand how the divider marks on the various bars help them think of different fractions for the same shaded portion. Have them observe that the same length on each bar in a set is shaded. Note with them that the fractions listed below the chart are all names for the same shaded bar length. Explain that they are to study the third set and try to give at least the fractions for the bars in the chart, and then try to give other fractions that are in the same set.

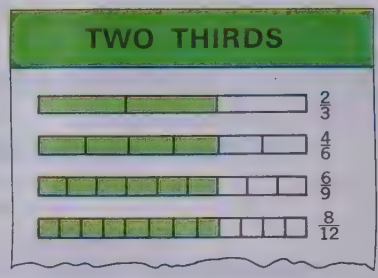
If some children finish quickly, encourage them to find other fractions in the sets shown in the first two charts.

Let's think about sets of equivalent fractions.

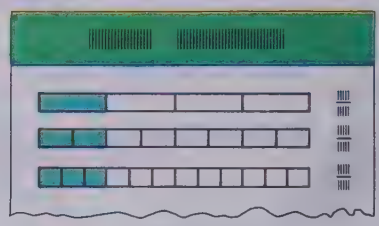
Investigating the Ideas



This chart shows some fractions that are equivalent to $\frac{1}{2}$:
$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$.



This chart shows some fractions that are equivalent to $\frac{2}{3}$:
$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$.



This chart shows some fractions that are equivalent to ▥: $\frac{1}{4}$
{▥, ▥, ▥, . . .}. $\frac{1}{4}$, $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, . . .

Can you find the missing fractions for this chart and give some more fractions for the set?

See Investigation and Discussion.



Discussing the Ideas

1. Can you describe what the next bar in the "One Half" chart would look like? Five of ten parts would be shaded.
2. What would the next bar in the "Two Thirds" chart be like? Ten of fifteen parts would be shaded.
3. Is the same amount of the bar shaded for $\frac{2}{4}$ as for $\frac{4}{8}$? Yes
4. What name would you give to the last chart? "One Fourth"

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Discussion

In discussing the results of the investigation, be sure that children observed initially that, as in the first two charts, the same length on each bar in the third chart is shaded. Have volunteers explain how they decided what fraction names the situation illustrated by each bar in the chart. Encourage such explanations as, "One of four parts is shaded, so one fourth of the bar is shaded."

To extend the set beyond the fractions illustrated, we would like the children to think of $\frac{4}{16}$ as the next fraction in the set of equivalent fractions for the *one-fourth* chart. There are a number of ways the children might look at this. They might think of the sequence of the set of numerators, {1,2,3,4, . . .}, and the sequence of the set of denominators, {4,8,12,16, . . .}. Or, they might observe that each succeeding bar has four more parts than the bar before it; hence, if there are 16 parts, 4 of them would be required to make up one fourth.

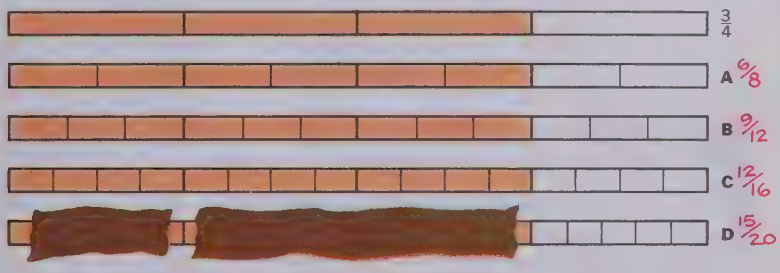
It is not important for the children to give precise descriptions of how they decide on $\frac{4}{16}$; rather, they should learn that there are various ways of arriving at $\frac{4}{16}$ as the next fraction in the set. The pattern of the previously listed fractions intuitively implies that $\frac{4}{16}$ is the next fraction in this set, and this is what we want the children to see. However, notice that the next fraction could just as well be $\frac{5}{20}$ or even $\frac{25}{100}$.

Using the Ideas

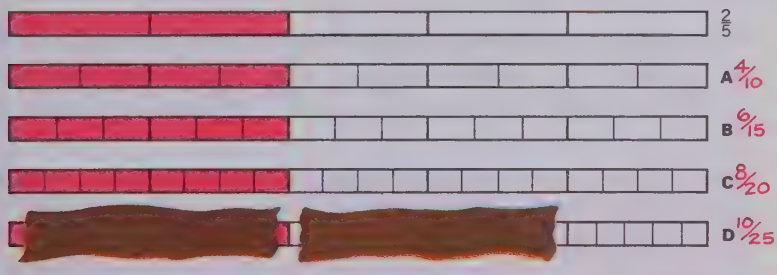
1. Study the bars and give the missing fractions.



2. Study the bars and give the missing fractions.



3. Study the bars and give the missing fractions.



★ 4. Give the next three fractions for each set of equivalent fractions.

A $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$ $\frac{5}{20}, \frac{6}{24}, \frac{7}{28}$

B $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\}$ $\frac{15}{25}, \frac{18}{30}, \frac{21}{35}$

C $\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \ldots\}$ $\frac{4}{32}, \frac{5}{40}, \frac{6}{48}$

D $\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \ldots\}$ $\frac{28}{40}, \frac{35}{50}, \frac{42}{60}$

More practice, page A-26, Set 49



Using the Exercises

Notice with the children that in each exercise on page 275 part of the last bar is covered, so they cannot count the divisions; hence, they will have to figure out another way to tell what the next fraction is. In each case, the first fraction is given (in a later lesson, the children will learn to call this fraction the *lowest-terms fraction* for the set). During the discussion of exercises 1 through 3, ask the children to tell the various methods they used to arrive at the answer to part D for each exercise.

Exercise 4 is starred because no particular method has been suggested for finding the next three fractions in these sets; however, some children will be able to do this quite readily on the basis of an intuitive grasp of the concepts presented thus far.

Assignments (page 275)

Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

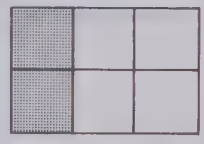
Follow-up

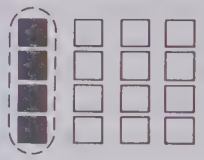
To provide children with more practice with equivalent fractions, you might prepare a worksheet similar to the following.

Give a pair of equivalent fractions to match each picture.

1.  $(\frac{6}{12}, \frac{2}{4})$

2.  $(\frac{3}{4}, \frac{9}{12})$

3.  $(\frac{1}{3}, \frac{2}{6})$

4.  $(\frac{1}{4}, \frac{4}{16})$

5. Draw a picture that would match each pair of equivalent fractions: $(\frac{2}{3}, \frac{6}{9})$; $(\frac{7}{14}, \frac{1}{2})$; $(\frac{2}{10}, \frac{1}{5})$; $(\frac{8}{12}, \frac{2}{3})$.

You might want to refer to the "Resources for Active Learning" section in the introduction to this chapter for ideas for suitable fraction games. Fraction Dominoes, for example, would be an appropriate follow-up activity for this and subsequent lessons.

Workbook, page 91

PAGES 276–277

Objective

Given the first three or four members of a set of equivalent fractions, the child will be able to list other members of the set.

Preparation

It would be appropriate to begin this lesson immediately with the investigation. However, if you prefer, you might display charts similar to the bar illustrations in the previous lesson and ask children to give equivalent fractions to correspond to the bar lengths.

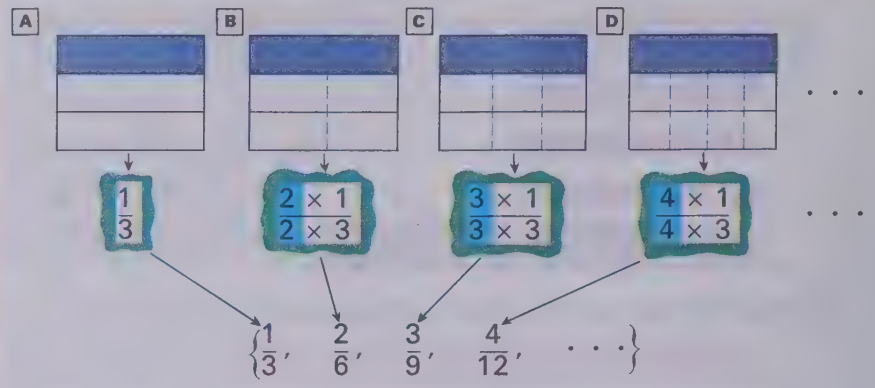
Investigation

This investigation could effectively be handled as a group activity. Have children work in groups of three or four. Direct them to interpret and discuss among themselves what is shown by illustrations A, B, C, and D. Then they should try to build a set of equivalent fractions as indicated in the investigation question. As you move around the room, make sure that all children have correctly identified the first member of the set as $\frac{1}{4}$. Do not expect the pictures which the children draw to be precise, but see that the partitions or folds they make follow the proper pattern, that is, first in eighths, then twelfths, sixteenths, twentieths, and so on. Some may tend to copy the fourths and then fold eighths and then sixteenths, forgetting about twelfths. If so, you might suggest that they draw a second picture of $\frac{1}{4}$ and divide it by 2 vertical folds. In any case, help them associate the picture to the pattern shown in the text for building the set for $\frac{1}{3}$.



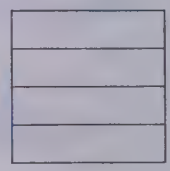
How can you build sets of equivalent fractions?

Investigating the Ideas



A set of **equivalent fractions**

? Can you cut out and color four papers, like this, and fold them to show a set of equivalent fractions? See Investigation.



Discussing the Ideas

1. A How many parts are shaded in A? How many parts in all? 1, 3
 B How many parts are shaded in B? How many parts in all? 2, 6
 C How many times as many parts are shaded in C as in A? 3

2. How would you find the next three fractions in the set above?
 Multiply by 5/5, 6/6, and 7/7.

3. How would you continue this set?

$$\left\{\frac{1\times3}{1\times4}, \frac{2\times3}{2\times4}, \frac{3\times3}{3\times4}, \frac{4\times3}{4\times4}, \frac{5\times3}{5\times4}, \frac{6\times3}{6\times4}, \frac{7\times3}{7\times4}, \frac{8\times3}{8\times4}, \ldots\right\}$$

4. Give the set of fractions for exercise 3. $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \ldots\right\}$

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Discussion

One of the important points to stress in this lesson is the relation of each fraction of higher terms to the first fraction in the set. Note that exercise 1 develops this idea in part C. Continue with other examples to focus on this progression: the second figure (figure B) shows two times as many parts shaded and unshaded as in A; the third figure (figure C) shows three times as many parts shaded and unshaded as in A; the fourth figure, four times as many; the fifth figure, five times as many; and so on. Exercises 2 and 3 develop the skill of building sets of equivalent fractions based on the understanding discussed in exercise 1. It may be helpful to exhibit on the chalkboard or overhead projector examples similar to the following:

$$\frac{2}{3} \quad \frac{4}{6} \quad \frac{6}{9} \quad \frac{8}{12} \quad \frac{10}{15}$$

$$\frac{1\times2}{1\times3} \quad \frac{2\times2}{2\times3} \quad \frac{3\times2}{3\times3} \quad \frac{4\times2}{4\times3} \quad \frac{5\times2}{5\times3}$$

Using the Ideas

1. Find the missing fractions.

$\frac{1 \times 1}{1 \times 6}$, ↓ $\frac{1}{6}$	$\frac{2 \times 1}{2 \times 6}$, ↓ $\frac{2}{12}$	$\frac{3 \times 1}{3 \times 6}$, ↓ A $\frac{3}{18}$	$\frac{4 \times 1}{4 \times 6}$, ↓ B $\frac{4}{24}$	., ↓ C $\frac{5}{30}$	., ↓ D $\frac{6}{36}$
$\frac{1 \times 3}{1 \times 8}$, ↓ $\frac{3}{8}$	$\frac{2 \times 3}{2 \times 8}$, ↓ $\frac{6}{16}$	$\frac{3 \times 3}{3 \times 8}$, ↓ E $\frac{9}{24}$	$\frac{4 \times 3}{4 \times 8}$, ↓ F $\frac{12}{32}$	., ↓ G $\frac{15}{40}$	., ↓ H $\frac{18}{48}$
$\frac{1 \times 4}{1 \times 5}$, ↓ $\frac{4}{10}$	$\frac{2 \times 4}{2 \times 5}$, ↓ $\frac{8}{10}$	$\frac{3 \times 4}{3 \times 5}$, ↓ I $\frac{12}{15}$	$\frac{4 \times 4}{4 \times 5}$, ↓ J $\frac{16}{20}$	., ↓ K $\frac{20}{25}$	., ↓ L $\frac{24}{30}$
$\frac{1 \times 1}{1 \times 4}$, ↓ $\frac{1}{4}$	$\frac{2 \times 1}{2 \times 4}$, ↓ M $\frac{2}{8}$	$\frac{3 \times 1}{3 \times 4}$, ↓ N $\frac{3}{12}$	., ↓ O $\frac{4}{16}$	., ↓ P $\frac{5}{20}$	., ↓ Q $\frac{6}{24}$

2. Find the next three fractions for each set of equivalent fractions.

A $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$ $\frac{5}{15}, \frac{6}{18}, \frac{7}{21}$

B $\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\}$ $\frac{5}{40}, \frac{6}{48}, \frac{7}{56}$

C $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \ldots\}$ $\frac{20}{24}, \frac{25}{30}, \frac{30}{36}$

D $\{\frac{2}{7}, \frac{4}{14}, \frac{6}{21}, \ldots\}$ $\frac{8}{28}, \frac{10}{35}, \frac{12}{42}$

think

Find the missing numbers. $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots, \frac{25}{50}, \ldots, \frac{50}{100}\}$

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots, \frac{30}{90}, \ldots, \frac{100}{300}\}$ $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots, \frac{30}{40}, \ldots, \frac{75}{100}, \ldots, \frac{200}{400}\}$

More practice, page A-27, Set 50



Using the Exercises

Before assigning the exercises on page 277, make sure that children understand the method by which the missing fractions can be found from those that are given. For example, in exercise 2C we would like the children to observe that the fraction $\frac{10}{12}$ is found by considering

$$\frac{2 \times 5}{2 \times 6}.$$

So the fraction after $\frac{15}{18}$ would be

$$\frac{4 \times 5}{4 \times 6} \text{ or } \frac{20}{24}$$

You may choose to have all the children attempt the *Think* problem, but with the caution that these sets may be more difficult than the others. Many of the children will benefit from a discussion of the problem after the correct answers have been given.

Assignments (page 277)

Minimum: 1. Average 1–2.
Maximum: 1–2.

Duplicator Masters, page 54
Workbook, page 92
Skill Masters, page 54

PAGES 278–279

Objective

Given a fraction, the child will be able to identify the numerator and the denominator of the fraction.

Preparation

Materials

sale advertisements; recipes; mail-order catalogues

To prepare for this lesson, review building sets of equivalent fractions. For example, begin a listing of the fractions $\frac{2}{5}$, $\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{20}$ and ask children to continue to list several fractions. Use similar examples, such as

$\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots$

$\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots$

$\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots$

Investigation

Many children will be able to think of ordinary uses of fractions without referring to the catalogues or advertisements; for example, they might list hat and shoe sizes, prices in grocery stores, news announcements regarding business, gas quantities for the car, distances indicated on road signs, and so on. However, it would be worthwhile to have reference materials for the children to use after they have thought of as many fractions on their own as they can.

Children will enjoy working on this investigation together. You might have three or four children work together to compile a list. Note that some of the fractions which children find, such as sizes of shoes and other apparel, will be written in mixed-numeral form. You need not point this out; the use of the fraction is the point to be emphasized at this time.

Let's find out more about fractions.

Investigating the Ideas

The pictures show some common uses of fractions.



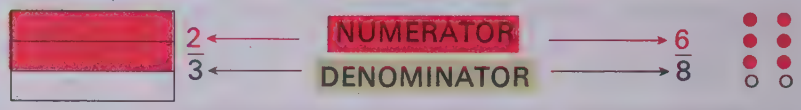
? Can you make a list of other ways fractions are used?

See Investigation.

Discussing the Ideas

1. Can you think of any difficulties we might have if there were no fractions?

See Discussion.

2. 

In these examples the **numerator** tells how many parts are red. The **denominator** tells how many parts in all. Can you read each fraction in the Investigation and give its numerator and denominator?

3. Which group of fractions do you think might be called "the eighths"? "the sixths"? "the thirds"? Why?

The sets of fractions whose denominators are 8, 6, 3, etc.

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Discussion

Lead children in a discussion of the uses of fractions, and then, as exercise 1 suggests, encourage children to suggest the difficulties we might have if there were no fractions. Some may suggest difficulties such as the following: prices would have to be given in terms of exact cents; we would need more units of measurement; we would have trouble describing the relation of parts to their whole. During this discussion, have children write on the chalkboard samples of fractions from their lists. Then refer to these as you discuss the terms *numerator* and *denominator*. Stress the role of the numerator as the term which designates how many parts of a whole are being considered and the denominator as the term which designates into how many parts the whole has been partitioned. Do not be concerned with the technicality that the numerator is the number represented by the top numeral in the fraction and the denominator is the number represented by the bottom numeral in the fraction; in order not to confuse the children, simply observe with them that the numerator is the top number and the denominator is the bottom number.

Using the Ideas

1. Give the word or numeral for each blank.

A In $\frac{4}{5}$, the numerator is __?__ 4.

B In $\frac{7}{9}$, 9 is the __?__ denominator

C $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ are all __?__ fourths

D There are __?__ 5 sixths in $\frac{5}{6}$.

E In $\frac{3}{4}$, 3 is the __?__ numerator

F In $\frac{6}{10}$, the denominator is __?__ 10

2. Write a fraction for each part of the exercise.

A Denominator: 7, Numerator: 2 — $\frac{2}{7}$

B Numerator: 4, Denominator: 10 — $\frac{4}{10}$

C Denominator: 12, Numerator: 8 — $\frac{8}{12}$

D The denominator is 10, and it is 2 times the numerator. $\frac{5}{10}$

E The numerator is 6, and the denominator is 3 more than 6. $\frac{6}{9}$

F The denominator is 100, and the numerator is half that. $\frac{50}{100}$

3. Copy each set on your paper. Write the missing numerators and denominators to form a set of equivalent fractions.

1	2	3	4	A	B	7	D	9
2	4	6	8	10	12	C 14	16	E 18

F
100

1	2	3	4	G	6	I	J	9
3	6	9	12	15	H 18	21	24	K 27

100
L 300

3	6	9	12	15	18	O	P	27
5	10	15	20	M 25	N 30	35	40	Q 45

300
R 500

3	6	9	12	S	18	U	24	27
10	20	30	40	50	T 60	70	V 80	W 90

X
1000

★ **4.** The sum of the numerator and denominator of a fraction is 10. The denominator is 4 times the numerator. Give the fraction. $\frac{2}{8}$



Follow-up

Encourage children to bring in and display advertisements which use fractions. They might also make illustrations or short reports to describe other uses of fractions.

Workbook, page 93

Using the Exercises

On page 279, you might choose to work through exercise 1 orally. Then have children work exercises 2 and 3 independently. For exercise 3, explain that the color coding shows a red screen over the word *numerator*, with a corresponding red screen over all the numerators, and a tan screen over the word *denominator*, with a tan screen over all the denominators. Simply point out to the children that this color coding is to remind them of the names *numerator* and *denominator* and their meanings.

Exercise 4 is quite challenging. Be sure that all the children who choose to do exercise 4 have time to complete it before the correct answer is revealed. When the answer is given, even those who were not able to do this exercise should be able to see that the sum of the numerator and denominator of the fraction $\frac{2}{8}$ is 10 and that the denominator is four times the numerator.

Assignments (page 279)

Minimum: 1, oral; 2. Average: 1–3. Maximum: 1–4.

PAGES 280–281

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Use one of the warm-up activities that children enjoyed most during previous chapters. For example, you might use the "What's My Rule" game to review basic operations (see page 65 for details); or you might give them mental chain games such as those which follow.

"Start with 24 . . . Divide by 8 . . . Add 7 . . . Divide by 2 . . . Multiply by 9. What's your answer?" (45)

"Start with 3 . . . Multiply by 7 . . . Subtract 3 . . . Divide by 9 . . . Multiply by 6 . . . Divide by 4. What's your answer?" (3)

"Start with 42 . . . Divide by 6 . . . Multiply by 5 . . . Add 1 . . . Divide by 6 . . . Multiply by 9. What's your answer?" (54)

"Start with 56 . . . Divide by 8 . . . Subtract 2 . . . Multiply by 5 . . . Subtract 1 . . . Divide by 6. What's your answer?" (4)

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Inequalities

1. Give the correct sign (= or ≠) for each ◍. The sign ≠ means "is **not** equal to."

A 63 427 + 1000 ◍ 64 427 =
B 100 × 1000 ◍ 1 000 000 ≠
C 872 000 ÷ 10 ◍ 8720 ≠
D 1 000 000 ◍ 999 999 + 1 =
E 10 000 − 1 ◍ 99 000 ≠
F 6285 × 100 ◍ 628 500 =
G 70 × 1000 ◍ 70 000 =
H 832 070 − 10 000 ◍ 831 070 ≠
I 10 × 10 × 10 ◍ 10 000 ≠
J 10 000 ◍ 100 × 100 =
K 10 × 10 × 10 × 10 ◍ 100 × 10 ≠
L 6322 − 302 ◍ 6020 =

2. Find the sums.

A	B	C	D	E
2	32	428	7465	82 471
7	58	106	8321	93 265
8	46	750	2405	47 721
6	71	623	3106	65 132
23	207	1907	21 297	288 589

3. Find the differences.

A 75 − 37 = 38
B 604 − 29 = 575
C 7028 − 4639 = 2389

4. Find the products.

A 27 × 6 = 162
B 32 × 12 = 384
C 58 × 26 = 1508

5. Find the quotients and remainders.

A 283 ÷ 6 = 47 R1
B 4286 ÷ 7 = 612 R2
C 316 ÷ 13 = 24 R4
D 228 ÷ 24 = 9 R12

think



The can contains only red beads and black beads. The beads are mixed up and, without looking, Jan drew out 100 beads, 90 red and 10 black. Guess how many black beads were in the can at first. 100



You are invited to explore

ACTIVITY CARD 14 Page 354

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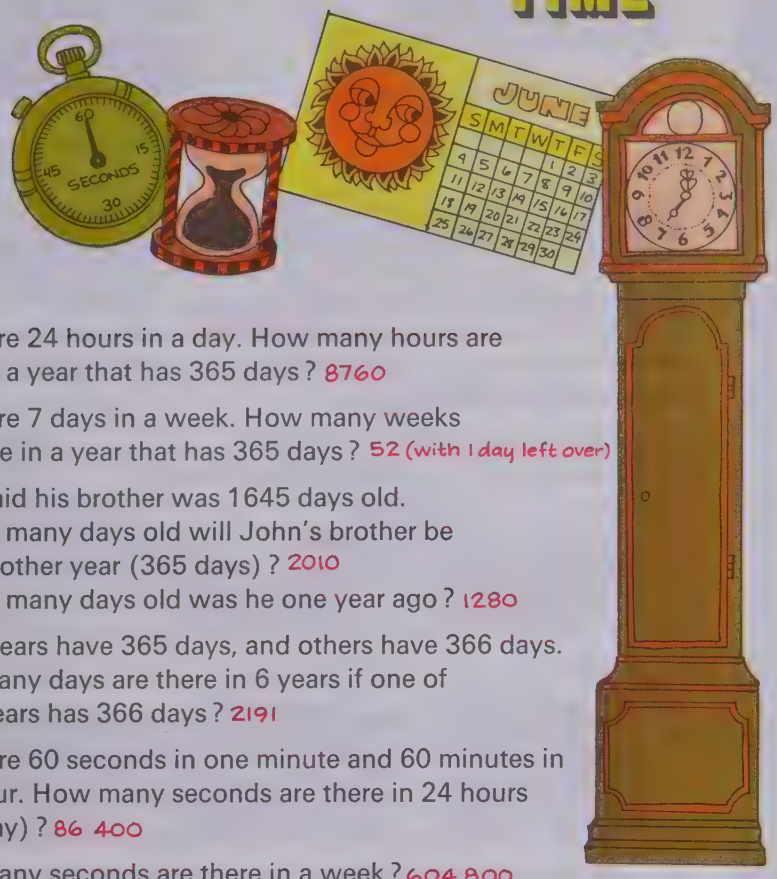
Discussion

Before you assign the exercises on page 280, you might briefly work through one or two examples of both the multiplication and division algorithm. Also, explain the symbol for "is not equal to" (≠). Exercise 1 is not intended to provide computational practice; it is intended instead to provide practice in the use of reasoning.

When the children finish the exercises, have volunteers explain how they arrived at their answers. You might have a few children exhibit some algorithmic exercises on the chalkboard.

The children may employ many approaches for solving the *Think* problem. One possible method might be to reason like this: "Out of one hundred beads, I found only ten that were black. There are ten hundreds in one thousand, so I'd expect, in all, about ten times as many black beads as I counted. Then, since 10 × 10 = 100, I'd expect to find about 100 black beads in the can."

TIME



1. There are 24 hours in a day. How many hours are there in a year that has 365 days? 8760
2. There are 7 days in a week. How many weeks are there in a year that has 365 days? 52 (with 1 day left over)
3. John said his brother was 1645 days old.
 - A How many days old will John's brother be in another year (365 days)? 2010
 - B How many days old was he one year ago? 1280
4. Some years have 365 days, and others have 366 days. How many days are there in 6 years if one of the 6 years has 366 days? 2191

★ 5. There are 60 seconds in one minute and 60 minutes in one hour. How many seconds are there in 24 hours (one day)? 86 400

★ 6. How many seconds are there in a week? 604 800

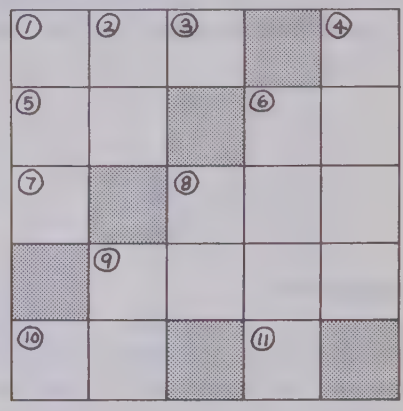
★ 7. March has 31 days.
 - A Without looking at a calendar, tell what day in April is 3 weeks after March 20. April 10
 - B What day in April is 4 weeks after March 10? April 7

Using the Exercises

Assign page 281 as independent work. Allow sufficient time for discussion when the children finish. Ask those who tried exercise 7 to explain fully how they arrived at a given answer. For example, they might explain that three weeks is 21 days, and 21 and 20 are 41; March has 31 days, so they subtract 31 from 41 to find that the date for part A should be April 10. Of course, for part B, starting with March 10 and adding 28 days would make 38; subtracting 31 from 38 would leave 7; hence the date would be April 7. The children will discover many ways to think about this exercise.

Follow-up

Most children will enjoy solving a cross-number puzzle as a way of reviewing operations. For this activity, duplicate a puzzle similar to the following one. (Or, make the puzzle on a chart and display it on a bulletin board. Cover the chart with acetate and allow the children to complete it during the checking period.) Depending on the ability of the children, you might extend the puzzle to 36 squares, and make the problems more complicated.



Across:

1. First three odd numbers
4. The even prime number
5. Three quarters total ___¢
6. Half a hundred
7. The number we cannot divide by
8. 1 less than 1000
9. The number of seconds in an hour (Hint: 60 × 60 = ?)
10. The number of days in 10 weeks
11. Half a dozen

Down:

1. 200 — 30
2. 7 × 5
3. 500 ÷ 100
4. 10 less than 2100
6. 5000 + 900 + 6
8. 100 — 4
9. (2 + 1) × 10
10. 63 ÷ 9

PAGES 282–283

Objective

Given pairs of colored strips, the child will be able to express a comparison between two strips by using a proper or improper fraction.

Preparation

Materials

colored strips

The nature of this investigation makes it appropriate to omit any specific preparation. However, if you wish, you might review equivalent fractions. For example, ask children to write three or more fractions for each set:

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\}$

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \ldots\}$

$\{\frac{3}{4}, ?, \frac{9}{12}, ?, \frac{15}{20}, \ldots\}$

$\{\frac{2}{5}, ?, \frac{6}{15}, ?, \frac{10}{25}, \ldots\}$

Investigation

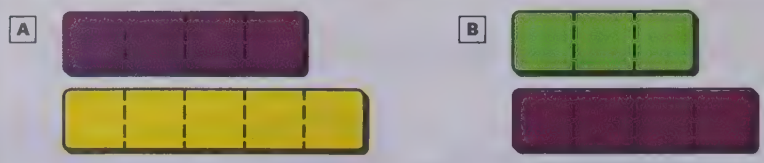
Explain to the children that they should compare the strips systematically by comparing the top strip to the bottom strip, so that in each case the numerator corresponds to the top strip and the denominator to the bottom strip. The illustrations in the text give examples for the children to study. In this investigation it is expected that children will work with improper fractions as well as proper fractions. As you move around the room, observe whether or not children are working with both proper and improper fractions; make any appropriate suggestions to those who are not. For example, suggest that they compare the blue strip as the top strip with the light green strip as the bottom.

It would be helpful to show the children a sample table of how they might record their comparisons. For instance, write on the chalkboard a suggestion similar to the following:

Pairs of Strips		*Fractions*
purple	→	$\frac{4}{5}$
yellow	→	
black	→	$\frac{7}{4}$
purple	→	
blue	→	$\frac{9}{5}$
yellow	→	
brown	→	$\frac{8}{2}$
red	→	

Can fractions be used to compare objects?

Investigating the Ideas



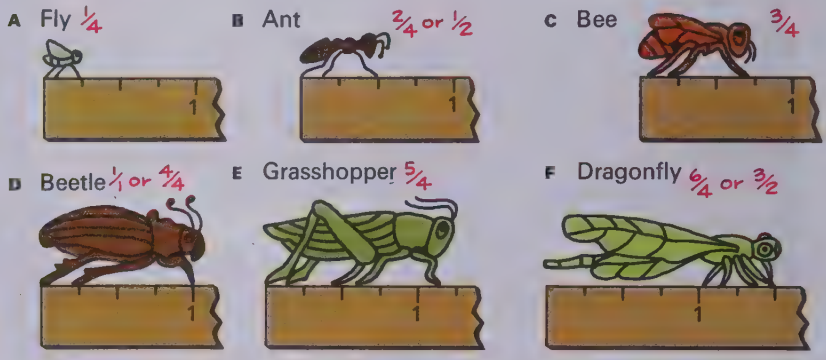
The purple strip is $\frac{4}{5}$ as long as the yellow strip.

The light green strip is $\frac{3}{4}$ as long as the purple strip.

? Can you use some other pairs of strips and give the fractions that compare them?
Answers will vary. See Investigation.

Discussing the Ideas

1. What fractions would you use to compare each insect with the unit shown?



2. A fraction with its numerator equal to or greater than its denominator is called an improper fraction
 - A Which of the fractions in exercise 1 would you call improper fractions? D, E, F
 - B Did you find any improper fractions in the Investigation? Answers may vary.

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Discussion

One of the most important points of this lesson is to help children realize that the numerator of a fraction may be greater than or equal to the denominator. Do not belabor the term *improper* fraction. Children should be familiar with the term as it applies to fractions having numerators equal to or greater than their denominators, but do not stress the word; we do not want the children to think that there is anything at all "wrong" about improper fractions.

After you discuss exercises 1 and 2, use the children's results from the investigation as a basis for discussion. Ask children to name a pair of strips which they used and the fraction which compares the two. Then ask others to identify the fraction as proper or improper, and give another fraction showing a comparison of one of these pairs of strips.

Using the Ideas

1. Give the missing numerator for each fraction.

A



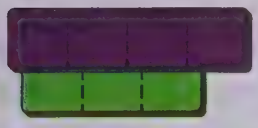
The red strip is $2\frac{\square}{3}$ as long as the light green strip.

B



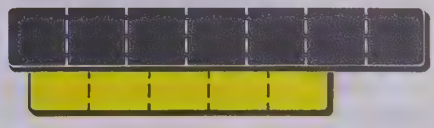
The yellow strip is $5\frac{\square}{6}$ as long as the dark green strip.

C



The purple strip is $4\frac{\square}{3}$ as long as the light green strip.

D



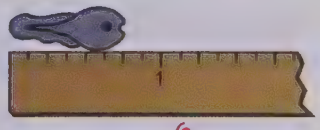
The black strip is $7\frac{\square}{5}$ as long as the yellow strip.

E



The fraction $\frac{\square}{4}$ compares the snail with the unit.

F



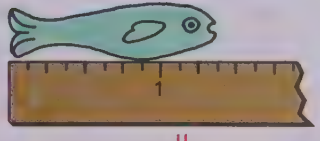
The fraction $\frac{\square}{8}$ compares the tadpole with the unit.

G



The fraction $\frac{\square}{4}$ compares the worm with the unit.

H



The fraction $\frac{\square}{8}$ compares the minnow with the unit.

2. Which of the fractions in exercise 1 are improper fractions?

3. Which is larger, a worm that is 2 units long or one that is $\frac{7}{4}$ units long?



Follow-up

Suggest to the children that they use their strips and measurement to find fractions. For instance, think of the light green strip as a unit which has three parts. Measure various objects in the classroom. Write proper or improper fractions to show the length of these objects. For example, a pencil measured with this unit might be written as $\frac{19}{3}$.

Workbook, page 94

Using the Exercises

Assign the exercises on page 283 as independent work. When the children finish, provide ample time for discussion. It would be helpful to use a clear plastic ruler on the overhead projector as you discuss parts A through F of exercise 1. Be particularly careful that the children count the fractional parts correctly and are not misled by the whole number 1. You might use these exercises to point out which fractions are other names for 1.

Assignments (page 283)

Minimum: 1. Average: 1–2.
Maximum: 1–3.

PAGES 284–285

Objective

Given sets of equivalent fractions, including those with zero numerators, the child will be able to write more fractions for each set.

Preparation

Before beginning the investigation, review equivalent fractions by exhibiting on the chalkboard a set of equivalent fractions such as $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$. Notice with the children that they can get

$\frac{6}{8}$ by thinking about $\frac{2 \times 3}{2 \times 4}$;

$\frac{9}{12}$ by thinking about $\frac{3 \times 3}{3 \times 4}$;

$\frac{12}{16}$ by thinking about $\frac{4 \times 3}{4 \times 4}$;

$\frac{15}{20}$ by thinking about $\frac{5 \times 3}{5 \times 4}$.

Give the children similar exercises in constructing sets of equivalent fractions, but do not extend such a review beyond the time customarily allotted for preparation.

Investigation

In this investigation, children study a variety of fractions, including fractions with zero numerators. It would be appropriate to have the children work either individually or in groups of two or three. Direct them to see if they can figure out the distinctive characteristic of each set and form other fractions for each. Some may relate building the set to the review that was used as preparation. For example, Type D follows the pattern

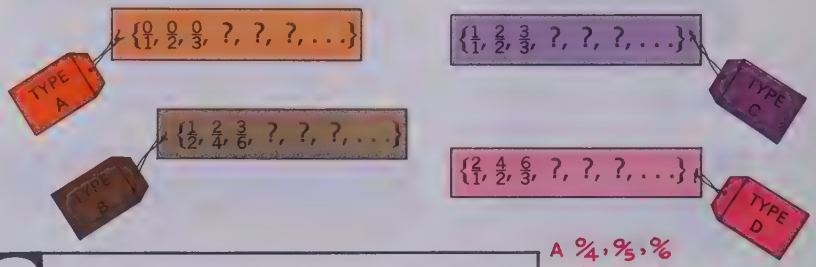
$$\frac{2 \times 2}{2 \times 1}, \frac{3 \times 2}{3 \times 1}, \frac{4 \times 2}{4 \times 1}, \ldots$$

However, many children will be able to give the next three fractions without thinking through such a definite pattern.

Let's explore different types of fractions.

Investigating the Ideas

These are special sets of equivalent fractions.



? Can you give the next three fractions for each set of equivalent fractions? See Investigation.

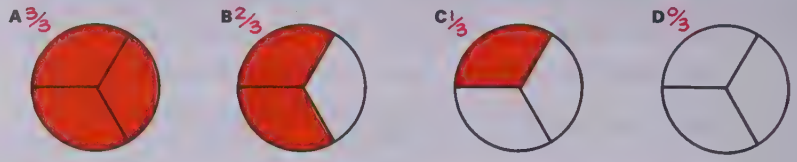
A 4/4, 5/5, 6/6
B 4/8, 5/10, 6/12
C 4/4, 5/5, 6/6
D 8/4, 10/5, 12/6

Discussing the Ideas

1. Can you explain what is special about each of the types of fractions in the Investigation? Fractions within each set are equivalent.

2. A Which type has zero numerators? Type A
 B Which type has numerators (not zero) less than denominators? Type B (proper fractions)
 C Which type has numerators greater than denominators? Type D (improper fractions)
 D Which type has numerators equal to denominators? Type C

3. Give the fraction that tells what part of each region is red.



4. Which type (A, B, C, or D) is each fraction you found in exercise 3? Why? 3/3 Type C – num = denom; 2/3 and 1/3 Type B – num < denom; 0/3 Type A – num = 0

Discussion

As you work through discussion exercises 1 and 2, have children explain how they found the next three fractions for each set in the investigation. Show how the set $\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \ldots\}$ is constructed from the first fraction by the same method as was illustrated previously; that is:

$$\left\{\frac{2 \times 0}{2 \times 1}, \frac{3 \times 0}{3 \times 1}, \frac{4 \times 0}{4 \times 1}, \frac{5 \times 0}{5 \times 1}, \ldots\right\}$$

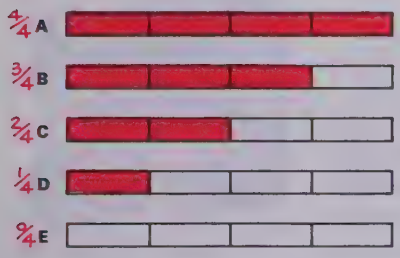
This is a particularly important point because we consider this set of fractions in our construction of fractional numbers in Chapter 14.

The sets of fractions in the investigation are labelled with the letters A, B, C, and D simply for convenience in discussion; the children should realize that these labels have no significance otherwise.

One important point is to help children identify the fractions as proper fractions when the numerator is less than the denominator; as improper fractions when the numerator is equal to or greater than the denominator; and finally as zero when the numerator is zero.

Using the Ideas

1. Give a fraction to tell what part of each bar is red.



2. Write the next three fractions.

A $\frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, ?, ?, ?, .\ \frac{8}{8}, \frac{9}{8}, \frac{10}{8}$

B $\frac{6}{8}, \frac{5}{8}, \frac{4}{8}, \frac{3}{8}, ?, ?, ?, .\ \frac{2}{8}, \frac{1}{8}, \frac{0}{8}$

C $\frac{6}{5}, \frac{5}{5}, \frac{4}{5}, \frac{3}{5}, ?, ?, ?, .\ \frac{2}{5}, \frac{1}{5}, \frac{0}{5}$

D $\frac{6}{10}, \frac{5}{10}, \frac{4}{10}, \frac{3}{10}, ?, ?, ?, .\ \frac{2}{10}, \frac{1}{10}, \frac{0}{10}$

3. Which of these fractions are improper fractions?

$\frac{3}{4}, \frac{4}{3}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}, \frac{9}{8}, \frac{5}{10}, \frac{10}{5}, \frac{1}{1}, \frac{7}{6}, \frac{12}{12}, \frac{11}{9}$ $\frac{4}{3}, \frac{8}{8}, \frac{9}{8}, \frac{10}{5}, \frac{1}{1}, \frac{7}{6}, \frac{12}{12}, \frac{11}{9}$

For exercises 4 and 5, copy the fractions on your paper. Write three more fractions for each set.

4.

A $\{\frac{3}{2}, \frac{6}{4}, \frac{9}{6}, \ldots\}$ $\frac{12}{8}, \frac{15}{10}, \frac{18}{12}$

B $\{\frac{4}{3}, \frac{8}{6}, \frac{12}{9}, \ldots\}$ $\frac{16}{12}, \frac{20}{15}, \frac{24}{18}$

C $\{\frac{5}{2}, \frac{10}{4}, \frac{15}{6}, \ldots\}$ $\frac{20}{8}, \frac{25}{10}, \frac{30}{12}$

D $\{\frac{5}{3}, \frac{10}{6}, \frac{15}{9}, \ldots\}$ $\frac{20}{12}, \frac{25}{15}, \frac{30}{18}$

E $\{\frac{0}{4}, \frac{0}{8}, \frac{0}{12}, \frac{0}{16}, \ldots\}$ $\frac{0}{20}, \frac{0}{24}, \frac{0}{28}$

F $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\}$ $\frac{15}{25}, \frac{18}{30}, \frac{21}{35}$

G $\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \ldots\}$ $\frac{12}{28}, \frac{15}{35}, \frac{18}{42}$

H $\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \ldots\}$ $\frac{12}{40}, \frac{15}{50}, \frac{18}{60}$

5.

A $\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots\}$ $\frac{4}{4}, \frac{5}{5}, \frac{6}{6}$

B $\{\frac{2}{1}, \frac{4}{2}, \frac{6}{3}, \ldots\}$ $\frac{8}{4}, \frac{10}{5}, \frac{12}{6}$

C $\{\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \ldots\}$ $\frac{12}{4}, \frac{15}{5}, \frac{18}{6}$

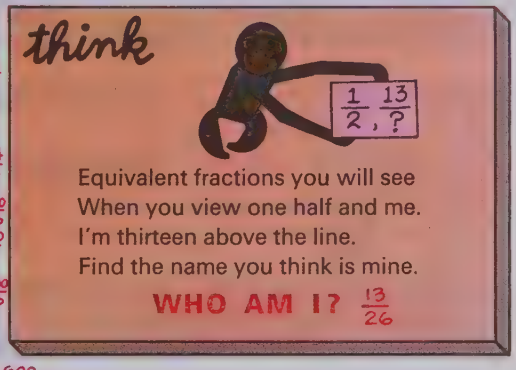
D $\{\frac{4}{1}, \frac{8}{2}, \frac{12}{3}, \ldots\}$ $\frac{16}{4}, \frac{20}{5}, \frac{24}{6}$

E $\{\frac{10}{1}, \frac{20}{2}, \frac{30}{3}, \ldots\}$ $\frac{40}{4}, \frac{50}{5}, \frac{60}{6}$

F $\{\frac{25}{1}, \frac{50}{2}, \frac{75}{3}, \ldots\}$ $\frac{100}{4}, \frac{125}{5}, \frac{150}{6}$

G $\{\frac{50}{1}, \frac{100}{2}, \frac{150}{3}, \ldots\}$ $\frac{200}{4}, \frac{250}{5}, \frac{300}{6}$

H $\{\frac{100}{1}, \frac{200}{2}, \frac{300}{3}, \ldots\}$ $\frac{400}{4}, \frac{500}{5}, \frac{600}{6}$



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Using the Exercises

Depending on your children's need and ability, you may choose to work through exercises 1, 2, and 3 on page 285 orally with the children. Assign exercises 4 and 5 as independent work. When the children have finished, have them explain how they found the next three fractions of each set.

Most children who understand equivalent fractions will have no difficulty with the *Think* problem. Ask those who solve it to tell the rest of the class how they thought through the problem.

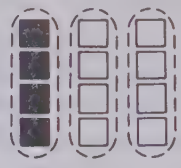
Assignments (page 285)

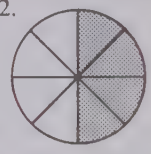
Minimum: 1–2. Average: 1–4.
Maximum: 1–5.

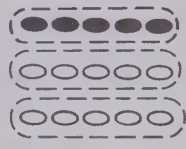
Follow-up

You might find it helpful to provide children with a worksheet similar to the following.

Give a pair of equivalent fractions to match each picture.

1. 

2. 

3. 

4. 

Write the next three fractions in each of the following sets.

5. $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \ldots\}$

6. $\{\frac{5}{7}, \frac{10}{14}, \frac{15}{21}, \ldots\}$

7. $\{\frac{1}{4}, \frac{2}{8}, \ldots\}$

8. $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \ldots\}$

Workbook, page 95

PAGES 286–287

Objective

Given two fractions, the child will be able to determine if they are equivalent by comparing their "cross products."

Preparation

You might review the topics covered in the previous lesson and ask children to give two more members of some sets of equivalent fractions such as:

$\{\frac{3}{2}, \frac{6}{4}, \frac{9}{6}, \ldots\}$

$\{\frac{2}{7}, \frac{4}{14}, \frac{6}{21}, \ldots\}$

$\{\frac{0}{1}, \frac{0}{2}, \ldots\}$

$\{\frac{4}{4}, \frac{7}{7}, \ldots\}$

$\{\frac{12}{4}, \frac{18}{6}, \frac{24}{8}, \frac{30}{10}, \ldots\}$

Investigation

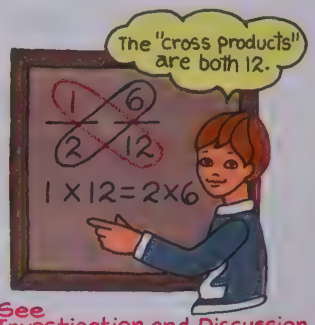
As children work on this investigation, they will soon realize that they are limited in their choice of fractions because all terms in any fraction they choose must be factors of 12. Furthermore, the terms of these fractions must be so arranged that their cross products equal 12. Besides the pair illustrated $(\frac{1}{2}, \frac{6}{12})$, these pairs meet the stated conditions:

$(\frac{12}{12}, \frac{1}{1})$ $(\frac{2}{1}, \frac{12}{6})$ $(\frac{3}{12}, \frac{1}{4})$
$(\frac{6}{6}, \frac{2}{2})$ $(\frac{3}{2}, \frac{6}{4})$ $(\frac{12}{3}, \frac{4}{1})$
$(\frac{3}{3}, \frac{4}{4})$ $(\frac{2}{3}, \frac{4}{6})$ $(\frac{12}{4}, \frac{3}{1})$
$(\frac{2}{12}, \frac{1}{6})$ $(\frac{3}{6}, \frac{2}{4})$ $(\frac{4}{12}, \frac{1}{3})$
$(\frac{12}{2}, \frac{6}{1})$ $(\frac{6}{3}, \frac{4}{2})$

Can you tell if two fractions are equivalent?

Investigating the Ideas

Sometimes the two "cross products" of a pair of fractions are equal. →



? How many other pairs of fractions can you find and record in which the "cross products" are both 12?

See Investigation and Discussion.

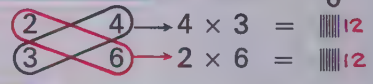
Discussing the Ideas

1. The two fractions, $\frac{1}{2}$ and $\frac{6}{12}$, in the Investigation are equivalent. What do you notice about the pairs of fractions you found? The pairs are equivalent; they have equal cross products.

2. A Does this picture show that $\frac{2}{3}$ is equivalent to $\frac{4}{6}$? Yes



B How would you complete this sentence?

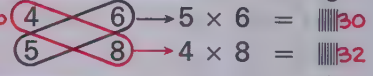


If the two "cross products" are the same, then the fractions are __?__. equivalent

3. A Does this picture show that $\frac{4}{5}$ is equivalent to $\frac{6}{8}$? No



B Are the "cross products" equal? No
Are the fractions equivalent? No



If the two "cross products" are not the same, then the fractions are not equivalent.

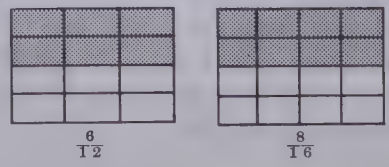


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Discussion

As you discuss exercise 1, help children study the pairs of fractions which they found in the investigation. Be sure they recognize that each pair that has equal cross products is a pair of equivalent fractions. In exercises 2 and 3, be sure children observe the shaded amounts of the bar and relate them to the fractions. Be particularly careful that they see that the amount of the bar shaded for $\frac{4}{5}$ is greater than the amount shaded for $\frac{6}{8}$.

You might want to accompany this discussion with several illustrations of parts of objects as a check to see that two fractions are equivalent. Then you could compare the illustrations with the results of the cross-product method of checking for equivalence. The overhead projector would be appropriate for this. For example, prepare a transparency of two congruent rectangles. Use an overlay on each of these and draw partitions corresponding to the two fractions you are comparing. Example:



$6 \times 16 = 12 \times 8$

Using the Ideas

1. Find the "cross products" that are ringed. Then tell whether the pair of fractions is equivalent or not.

A $\frac{2}{4}$ $\frac{5}{10}$ 20 / 20 Yes

B $\frac{3}{6}$ $\frac{5}{9}$ 30 / 27 No

C $\frac{1}{2}$ $\frac{6}{12}$ 12 / 12 Yes

D $\frac{6}{8}$ $\frac{4}{6}$ 32 / 36 No

E $\frac{2}{3}$ $\frac{6}{9}$ 18 / 18 Yes

F $\frac{5}{6}$ $\frac{12}{18}$ 72 / 90 No

G $\frac{2}{5}$ $\frac{8}{20}$ 40 / 40 Yes

H $\frac{8}{12}$ $\frac{10}{15}$ 120 / 120 Yes

2. Tell whether or not the two fractions are equivalent.

A $\frac{1}{2}$ $\frac{7}{14}$ Yes

B $\frac{6}{8}$ $\frac{7}{10}$ No

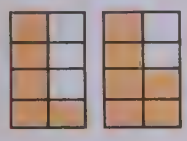
C $\frac{3}{3}$ $\frac{8}{8}$ Yes

D $\frac{3}{8}$ $\frac{9}{24}$ Yes

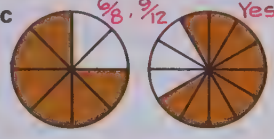
E $\frac{6}{10}$ $\frac{9}{15}$ Yes

F $\frac{0}{9}$ $\frac{0}{1}$ Yes

3. For each part of the exercise, write the two fractions suggested by the shaded parts of the two regions given. Then tell whether or not the two fractions are equivalent.

A  $\frac{5}{8}$, $\frac{6}{8}$ No

B  $\frac{3}{4}$, $\frac{6}{8}$ Yes

C  $\frac{6}{8}$, $\frac{9}{12}$ Yes

4. There are 30 children in Ann and Janet's class. 15 of the children are girls. Ann said, "$\frac{15}{30}$ of the children are girls." Janet said, "$\frac{3}{6}$ of the children are girls."

A Use the cross-product method to show that the two fractions are equivalent.

B Are both girls right? Yes

15 × 3 = 30 × 3 = 90
30 × 6 = 15 × 6 = 90

5. Mike and Tim are both in the school science club. Mike said, "$\frac{2}{3}$ of the children in our club are boys." Tim said, "$\frac{7}{10}$ of the children in our club are boys."

A Use the cross-product method to show that these two fractions are not equivalent.

B Did one of the boys make a mistake? Yes

2 × 7 = 3 × 7 = 21
3 × 10 = 2 × 10 = 20





Using the Exercises

Be sure children understand the cross-product method for checking equivalence before assigning exercises 1 and 2 as independent work. When children finish the exercises, allow sufficient time to discuss them. In particular, ask some children to explain their answers for exercises 4 and 5.

Assignments (page 287)* _____
Minimum: 1–2. Average: 1–3.
Maximum: 1–5.

Follow-up

The game described below provides practice in finding pairs of equivalent fractions. Number the faces of six wooden cubes as follows:

2 cubes numbered 1 through 6
2 cubes with even numbers 2 through 12
1 cube with 3, 6, 9, 12, 15, 18
1 cube with 4, 8, 12, 16, 20, 24

All six cubes are rolled at once. The player who rolls the cubes tries to form a pair of equivalent fractions made up from the numbers on the top faces of the cubes. Score one point for each pair found. The game can be played until one player reaches a predetermined number of points. Let the children make up their own rules for challenging incorrect pairs or overlooked pairs of equivalent fractions.

Workbook, page 96

PAGES 288–289

Objective

Given a set of equivalent fractions, the child will be able to recognize or find the lowest-terms fraction for the set.

Preparation

Materials

crayons; small slips of paper

Provide children with practice in checking equivalent fractions. For example, write pairs of fractions on the chalkboard and ask children to identify those which are equivalent fractions by the check they learned in the previous lesson. Samples:

$\frac{1}{2}, \frac{3}{4}$ ($1 \times 4 \neq 2 \times 3$, not equivalent)

$\frac{2}{3}, \frac{4}{6}$ ($2 \times 6 = 3 \times 4$, equivalent)

$\frac{4}{8}, \frac{6}{12}$ ($4 \times 12 = 6 \times 8$, equivalent)

$\frac{3}{4}, \frac{9}{12}$ ($3 \times 12 = 4 \times 9$, equivalent)

$\frac{5}{6}, \frac{18}{20}$ ($5 \times 20 \neq 6 \times 18$, not equivalent)

$\frac{2}{5}, \frac{1}{4}$ ($2 \times 4 \neq 1 \times 5$, not equivalent)

Investigation

In this investigation, children use two sets of whole numbers to form fractions, and then match those which are equivalent. They should first make and record a fraction from the numbers in the first set; they should use the second set to try to make a fraction which is equivalent to the fraction they made from the first set. It would be appropriate to have the children work in groups of three or four. Remind the children to check the pairs they think are equivalent fractions by using the method developed in the previous lesson and reviewed in the preparation period. Many pairs may be found, some of which follow:

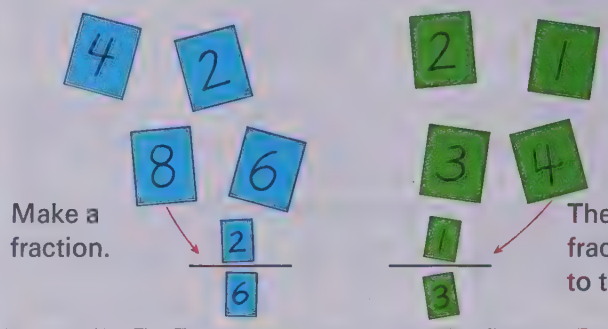
$\frac{4}{8}, \frac{1}{2}$; $\frac{4}{2}, \frac{2}{1}$; $\frac{6}{8}, \frac{3}{4}$; $\frac{4}{8}, \frac{2}{4}$;
$\frac{2}{8}, \frac{1}{4}$; $\frac{6}{4}, \frac{3}{2}$; $\frac{4}{6}, \frac{2}{3}$; $\frac{8}{4}, \frac{2}{1}$

If you choose to extend this investigation, you might include the number 12 in the first set and the number 6 in the second.

When is a fraction in lowest terms?

Investigating the Ideas

Color and mark slips of paper like these.



Make a fraction.

Then make a different fraction equivalent to the first.

? How many pairs of equivalent fractions can you find in this way? See Investigation.

Discussing the Ideas

1. Each fraction you made from the green set, except with 2 and 4, is a **lowest-terms** fraction. Can you use the green set to make a lowest-terms fraction equivalent to $\frac{2}{4}$? $\frac{1}{2}$

2. Find a fraction from the green set that is equivalent to each of these.

 A $\frac{5}{10}$ $\frac{1}{2}$ or $\frac{2}{4}$ B $\frac{3}{9}$ $\frac{1}{3}$ C $\frac{3}{12}$ $\frac{1}{4}$ D $\frac{4}{12}$ $\frac{1}{3}$ E $\frac{9}{12}$ $\frac{3}{4}$

3. The fraction $\frac{1}{2}$ is the lowest-terms fraction for each fraction in this set. $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\}$

 Find the lowest-terms fraction for each of these sets.

 A $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\right\}$ $\frac{1}{4}$ B $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}$ $\frac{2}{3}$

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Discussion

Use exercise 1 as a basis for a discussion of the investigation results. After you point out that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$, and is in lowest terms, use the fractions $\frac{4}{8}$, $\frac{2}{4}$, and $\frac{1}{2}$ to bring out the idea that $\frac{2}{4}$ is in lower terms than $\frac{4}{8}$ but that $\frac{1}{2}$ is in the *lowest terms*. Also observe with the children that the phrase *lowest terms* is used only in reference to equivalent fractions. For example, we do not say that $\frac{1}{4}$ is in lower terms than $\frac{1}{5}$. Rather, both $\frac{1}{4}$ and $\frac{1}{5}$ are in lowest terms compared to any fractions which are equivalent to either.

Use other sets of equivalent fractions to expand exercise 3. It is important for children to realize that a fraction is in lowest terms when there is no equivalent fraction in lower terms than itself. Have them note that for each set of equivalent fractions there is only one of the fractions which is in *lowest terms*.

Using the Ideas

1. Build a set of equivalent fractions from each lowest-terms fraction. Find at least six fractions for each set.

A $\frac{1}{2}$ $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}$ B $\frac{2}{3}$ $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}$ C $\frac{1}{5}$ $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \frac{7}{35}$ D $\frac{3}{10}$ $\frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \frac{15}{50}, \frac{18}{60}, \frac{21}{70}$ E $\frac{5}{6}$ $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}, \frac{35}{42}$ F $\frac{7}{4}$ $\frac{14}{8}, \frac{21}{12}, \frac{28}{16}, \frac{35}{20}, \frac{42}{24}, \frac{49}{28}$

2. In each set, find the fraction for the ■.

A $\{\frac{1}{8}, \frac{2}{16}, ■, \frac{4}{32}, \ldots\}$ $\frac{3}{24}$ $(\frac{1}{8})$*

B $\{\frac{5}{4}, \frac{10}{8}, \frac{15}{12}, ■, \ldots\}$ $\frac{20}{16}$ $(\frac{5}{4})$

C $\{\frac{1}{1}, ■, \frac{3}{3}, \frac{4}{4}, \ldots\}$ $\frac{2}{2}$ $(\frac{1}{1})$

D $\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, ■, \ldots\}$ $\frac{0}{4}$ $(\frac{0}{1})$

E $\{\frac{2}{1}, \frac{4}{2}, ■, \frac{8}{4}, \ldots\}$ $\frac{6}{3}$ $(\frac{2}{1})$

F $\{\frac{5}{8}, ■, \frac{15}{24}, \ldots\}$ $\frac{10}{16}$ $(\frac{5}{8})$

G $\{■, \frac{2}{18}, \frac{3}{27}, \ldots\}$ $\frac{1}{9}$ $(\frac{1}{9})$

H $\{■, \frac{4}{14}, \frac{6}{21}, \ldots\}$ $\frac{2}{7}$ $(\frac{2}{7})$

I $\{■, \frac{6}{4}, \frac{9}{6}, \ldots\}$ $\frac{3}{2}$ $(\frac{3}{2})$

J $\{■, \frac{12}{10}, \frac{18}{15}, \ldots\}$ $\frac{6}{5}$ $(\frac{6}{5})$

* Answers for exercise 3 are in parentheses.

3. Give the lowest-terms fraction for each part of exercise 2.

4. Find the lowest-terms fraction in each set.

A $\{\frac{8}{12}, \frac{20}{30}, \frac{14}{21}, \frac{6}{9}, \frac{2}{3}\}$ $\frac{2}{3}$

B $\{\frac{15}{20}, \frac{3}{4}, \frac{9}{12}, \frac{21}{28}, \frac{12}{16}\}$ $\frac{3}{4}$

C $\{\frac{10}{100}, \frac{2}{20}, \frac{1}{10}, \frac{5}{50}, \frac{7}{70}\}$ $\frac{1}{10}$

D $\{\frac{80}{100}, \frac{8}{10}, \frac{40}{50}, \frac{16}{20}, \frac{4}{5}\}$ $\frac{4}{5}$

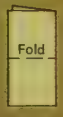
think

Cut out of a sheet of notebook paper a square about 20 cm on each side. Then:

1 Fold it once.



2 Fold it again.



3 Hold the corner with four tips and fold along the diagonal.



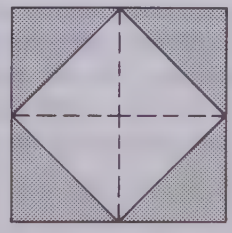
4 Now unfold it and **color it** to answer this question.
How can **exactly half** of a square window be painted so that the unpainted part remains a perfect square ?

More practice, page A-27, Set 51

Using the Exercises

On page 289, exercise 1 emphasizes the fact that, starting with the lowest-terms fraction, a set of equivalent fractions can be built in which all succeeding fractions are equivalent to this lowest-terms fraction. Exercises 3 and 4 provide practice in finding fractions in lowest terms. However, the emphasis should be, not on having children develop skills for finding fractions in lowest terms, but on having children think about constructing sets of equivalent fractions.

To do the *Think* problem properly, children must carefully note the third step of the directions, to take hold of the corner which has the four separate tips of paper. If they do this carefully, their unfolded colored square should look like this:



Assignments (page 289)*

Minimum: 1–2. Average: 1–3, Maximum: 1–4.

Duplicator Masters, page 55
Workbook, page 97
Skill Masters, page 55

Objectives

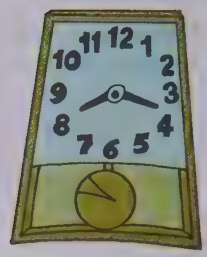
Given related units of measure, the child will be able to use fractions to express the relation of one unit to another.

The child will have an opportunity to experiment with fourths of a square.

Preparation

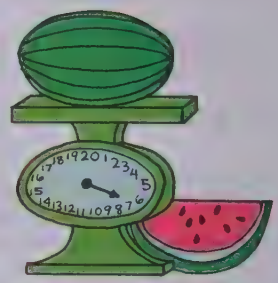
It would be appropriate to begin this lesson immediately with page 290, omitting any specific preparation.

Fraction Exercises



1. Write a fraction for each ▮.
(Example: A second is $\frac{1}{60}$ of a minute.)

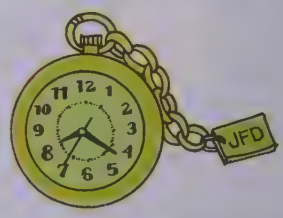
- A A minute is ▮ of an hour. $\frac{1}{60}$
- B An hour is ▮ of a day. $\frac{1}{24}$
- C A day is ▮ of a week. $\frac{1}{7}$
- D A month is ▮ of a year. $\frac{1}{12}$
- E A year is ▮ of a decade. $\frac{1}{10}$
- F A decade is ▮ of a century. $\frac{1}{10}$
- G A centimetre is ▮ of a decimetre. $\frac{1}{10}$
- H A decimetre is ▮ of a metre. $\frac{1}{10}$
- I A metre is ▮ of a kilometre. $\frac{1}{1000}$
- J A gram is ▮ of a kilogram. $\frac{1}{1000}$



★ **2.** Give the lowest-terms fraction for each ▮.

- A 4 months is ▮ of a year. $\frac{1}{3}$
- B 40 minutes is ▮ of an hour. $\frac{2}{3}$
- C 400 grams of steak weigh ▮ of a kilogram. $\frac{2}{5}$
- D 30 centimetres is ▮ of a metre. $\frac{3}{10}$
- E 5 decimetres is ▮ of a metre. $\frac{1}{2}$
- F 100 metres is ▮ of a kilometre. $\frac{1}{10}$
- G A quarter is ▮ of a dollar. $\frac{1}{4}$
- H A dime is ▮ of a dollar. $\frac{1}{10}$
- I A nickel is ▮ of a dollar. $\frac{1}{20}$
- J 50 centimetres is ▮ of a metre. $\frac{1}{2}$
- K 12 seconds is ▮ of a minute. $\frac{1}{5}$
- L A half centimetre is ▮ of a metre. $\frac{1}{200}$





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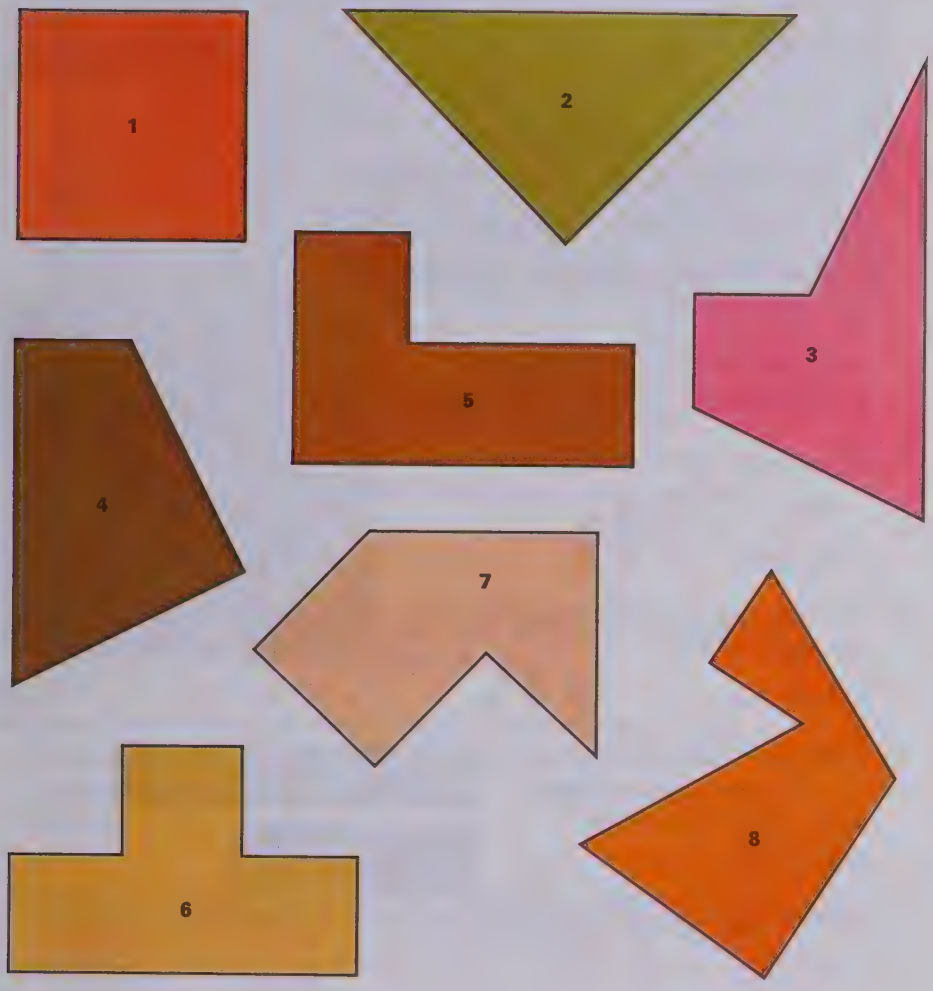
Discussion

Both of these pages may be treated either as small-group activities or as independent investigations. All of the required information is in the Tables of Measures at the back of the book (page A—38) or in previous lessons (refer children to page 46 for the meaning of *decade* and *century*).

Assignments (page 290)*_____
Minimum: 1. Average: 1–2D.
Maximum: 1–2.

FUN WITH FOURTHS

Each piece is exactly $\frac{1}{4}$ of a square. Trace and cut out four copies of each, and see if you can put them together to form a square. See Using the Exercises and Solutions, T.E. page 291.



Using the Exercises

Page 291 provides the children with an opportunity to work with fourths in relation to geometric shapes. Children might be amazed to realize in how many ways a square can be divided into fourths.

Since all of these shapes can be formed with rubber bands on a geoboard, you may prefer to have the children work with a 4-by-4 square on the geoboard.

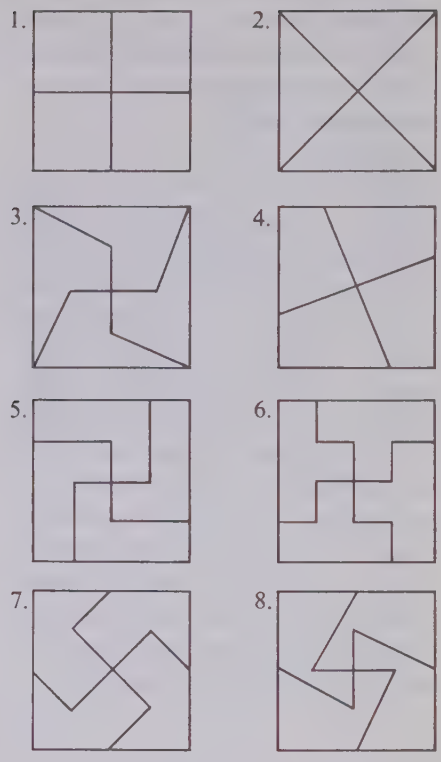
Some children may become discouraged if they find some shapes too difficult to work with. Give hints to such children (see "Solutions"), but do not weaken their opportunity for discovery. Help them to realize that some of these are very difficult and they should not expect to solve all of them in a short amount of time.

If the children cut out the fourths from paper, you might have them show their results by pasting the pieces in position.

Assignments (page 291)*

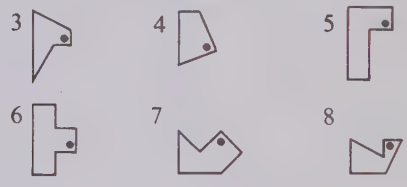
Minimum: 1–3. Average: 1–6.
Maximum: 1–8.

Solutions, page 291



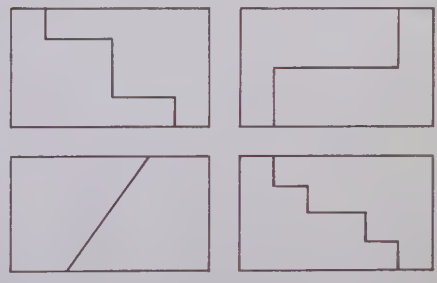
Possible hints

The tiny dot on each figure designates the corners that meet at the centre of the square.



Follow-up

You might suggest that some children try to find how many ways they can form halves of a rectangle on a geoboard. Below are a few ways that a 6-by-4 rectangle may be divided into halves.



PAGES 292–293

Objective

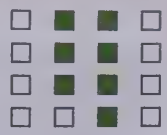
The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Spend a short time reviewing any concept presented in this chapter which was particularly troublesome for the children. For example, review the construction of sets of equivalent fractions beginning with the lowest-terms fraction (one of the most important topics of the chapter). During the construction of these sets, present several examples to illustrate the equivalence of pairs of fractions within the set. You should also have the children select two fractions from a given set of equivalent fractions and check them for equivalence by using the cross-product method.

1. Give a fraction to tell what part of each picture is shaded.

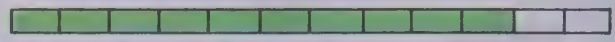
A  3/10

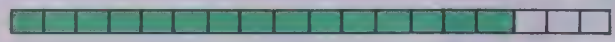
B  7/16

C  17/18

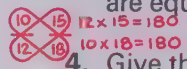
2. Give a fraction to tell what part of each bar is shaded.

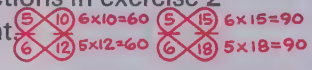
A  5/6

B  10/12

C  15/18

3. Use the cross-product method to show that your fractions in exercise 2 are equivalent.

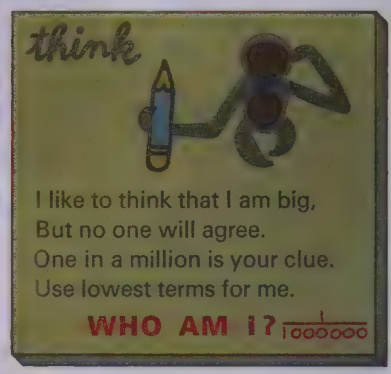




4. Give three more fractions for each set.

A $\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \ldots\}$ $\frac{12}{28}, \frac{15}{35}, \frac{18}{42}$

B $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots\}$ $\frac{16}{20}, \frac{20}{25}, \frac{24}{30}$



5. Tell whether or not the two fractions are equivalent.

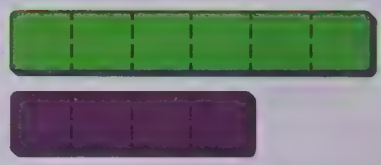
A $\frac{2}{4}$ $\frac{8}{16}$ Yes

B $\frac{8}{3}$ $\frac{25}{9}$ No

C $\frac{10}{8}$ $\frac{15}{12}$ Yes

D $\frac{7}{8}$ $\frac{21}{16}$ No

6. Give an improper fraction that tells how the dark green strip compares with the purple strip. 6/4



7. Write a set of six equivalent fractions for each lowest-terms fraction.

A $\frac{1}{2}$ $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}$

B $\frac{1}{4}$ $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}$

C $\frac{2}{3}$ $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}$

D $\frac{3}{4}$ $\frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}$

E $\frac{7}{8}$ $\frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \frac{35}{40}, \frac{42}{48}, \frac{49}{56}$

F $\frac{3}{10}$ $\frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \frac{15}{50}, \frac{18}{60}, \frac{21}{70}$

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Discussion

Assign page 292 for children to do independently. When they have finished, use the exercises as a review of the most important topics in the chapter. Children should realize that they should use the cross-product method in exercise 5. Spend ample time discussing exercise 7; it deals with one of the more important topics in the chapter.



1. Canada has ten provinces. Two of them border on James Bay. Give two fractions that tell what part of the provinces border on James Bay. $\frac{2}{10}, \frac{1}{5}$

2. Ontario touches four of the five Great Lakes. What fraction of the lakes touch Ontario ? What fraction of the lakes do not touch Ontario ? $\frac{4}{5}, \frac{1}{5}$

3. Eight of the provinces have a seacoast. Give two fractions to tell what part of the provinces this is. $\frac{8}{10}, \frac{4}{5}$

4. Seven of the provinces border on the United States. What fraction of the provinces share some border with the United States ? What fraction of the provinces do not share a border with the United States ? $\frac{7}{10}, \frac{3}{10}$

5. Five provinces have seacoasts on the Atlantic Ocean. Give two fractions to tell what part of the provinces are on the Atlantic Ocean. $\frac{5}{10}, \frac{1}{2}$

Resources for Active Learning

Franklin Series: *Patterns and Puzzles*, "Fraction Fun," pp. 64–68, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, "Map Coloring," C17, Macmillan.

Mathex: Numeration No. 7, "Puzzles" and "Games," pp. 43–45, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Green Set, No. 21, Wiley.

Workbook, page 98

Using the Exercises

As part of the chapter review, page 293 emphasizes equivalent fractions as well as the role of fractions in expressing the relationships of parts to the whole. Exercise 2 may challenge some children, but when they realize that Ontario does not touch one of the lakes, they should see that $\frac{1}{5}$ of the lakes do not touch Ontario.

Keep in mind during the discussion of these exercises that one value of such an exercise set is that it encourages children to develop interest in other topics and in the application of arithmetic to these topics. For this reason, you should encourage discussion of the geography of Canada and, perhaps, give the children an opportunity to make up some problems of their own with respect to the map on this page.

PAGES 294–295

Objectives

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Given suitable materials, the child will be able to weigh various objects, including himself, to find approximate weights.

Preparation

Materials

Sets of weights (if possible, 1 set for every 5 children) that include 100-g, 200-g, 500-g, 1-kg weights; demonstration set that includes 1-g, 5-g, 10-g, 20-g, 50-g weights; sets of balance scales

To prepare for this lesson, review any topic with which the children previously experienced difficulty. If no such topic is apparent, you might review measurement to the nearest half unit or you might review the division algorithm using 2-digit divisors.

Page 295 is a discussion page and should be treated lightly. If possible, have models set up in the classroom like the first two illustrations in the pupil text. If sets of weights and balance scales are available, divide the children into groups and let them handle the weights and scales and play with them to become familiar with them.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Measurement
Inequalities

1. Give the correct sign (+, −, ×, ÷) for each ◍.

A 48 ◍ 48 > 100 ×
B 127 ◍ 29 = 98 −
C 564 ◍ 1 > 564 +
D 329 ◍ 0 ≠ 329 ×
E 0 ◍ 28 > 0 +
F 764 ◍ 764 < 1 −
G 168 ◍ 12 < 156 ÷
H 67 ◍ 35 = 102 +
I 369 ◍ 7 > 376 ×
J 0 ◍ 1 ≠ 0 +
K 1 ◍ 1 < 1 −
L 4500 ◍ 12 < 4000 ÷
M 10 ◍ 100 = 1000 ×
N 1000 ◍ 1 = 999 −
O 12 ◍ 12 > 140 ×

2. Find the length of each segment to the nearest half centimetre.

A ——— 9½
B ——— 6
C ——— 7½
D ——— 13
E ——— 5½
F ——— 11

3. Find the sums and differences.

A 347 +685 = 1032	B 982 −643 = 339	C 803 −265 = 538	D 8654 +7283 = 15937	E 9402 −657 = 8745
F 842 −764 = 78	G 927 +849 = 1776	H 600 −475 = 125	I 8020 −6735 = 1285	J 9830 +7692 = 17522

4. Find the products and quotients.

A 83 × 5 415
B 648 ÷ 4 162
C 76 × 13 988
D 108 ÷ 12 9
E 319 × 7 2233
F 4285 × 3 12855
G 545 ÷ 5 109
H 207 ÷ 25 8 R7
I 2816 × 27 76032
J 264 ÷ 52 5 R4
K 9024 × 35 315840
L 391 ÷ 43 9 R4
M 128 ÷ 34 3 R26
N 3276 × 51 167076
O 600 ÷ 70 8 R40
P 4379 × 68 297772
Q 700 ÷ 73 9 R43
R 3269 × 48 156912
S 527 ÷ 62 8 R31
T 1346 × 37 49802
U 435 × 6 2610
V 252 ÷ 12 21
W 3412 × 12 40944
X 4444 ÷ 4 1111

Discussion

Assign the exercises on page 294 as independent work. When the children finish, carefully check their work with them, allowing ample time for discussion. It might be helpful to have some parts of exercises 3 and 4 put on the chalkboard and explained to the entire class. Also, some children may need careful explanation of the measurements in exercise 2.

The aim of page 295 is to acquaint the children with the units used in measuring mass or weight to the extent that the children are aware of the relative sizes. Go over page 295 with the children, demonstrating the different weights using the models you set up previously.



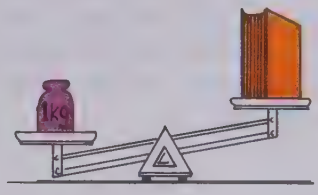
Weights

1 kilogram (kg) is 1000 grams(g).
1 tonne (t) is 1000 kilograms.

A gram has less weight than a paperclip.
A paperclip weighs about 2 grams.



A kilogram weighs a little more than this book.
The weight of your book is ⅘ that of a kilogram.



A tonne weighs less than most small cars.

This car  weighs 1¼ tonnes.

What unit would you use to weigh

A a piece of meat? Kilogram
B your pencil? gram
C a truck? tonne
D a bag of candy? Kilogram or gram
E your desk? Kilogram
F a piece of paper? gram
G yourself? Kilogram



You are invited to explore

ACTIVITY CARD 15
Page 354

Follow-up
Provide groups of objects, such as a chalkboard eraser, 2 or 3 different-sized blocks, a book, a drinking cup, a bag of marbles, and so on, for the children to weigh. Encourage them to record the weights in chart form.

General Objectives

To introduce co-ordinates to describe locations in the co-ordinate plane

To provide background experience in preparation for graphing

To develop the ideas of symmetry, similarity, translations

To provide experience in making and interpreting bar graphs

To develop the concept of graphing sets of co-ordinates and observing the pattern

To introduce the graphing of a function

To introduce negative numbers (negative integers)

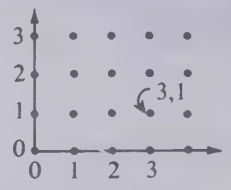
In the first few lessons, children learn to associate number pairs (co-ordinates) with points on the co-ordinate plane. From graphing a point, to graphing a figure, children work into the concepts of symmetry and similarity. They also study how points on a graph may be "slid" (translated) a given distance in a given direction. Then children have an opportunity not only to work with bar graphs but also to collect data needed to make a bar graph. The last two lessons of the chapter deal with two important concepts: graphing a function and negative numbers. The chapter concludes with a review of the graphing techniques studied in this chapter and a review of basic concepts and skills studied in previous chapters.

Mathematics

In a sense, measurement brings geometry and arithmetic together: a number is assigned to a given geometric figure by a count of the units.

This chapter combines geometry and arithmetic in a different way. Geometry is used to picture, or graph, certain arithmetic relations in a Cartesian co-ordinate system (named after mathematician and philosopher Rene Descartes).

To set up a rectangular co-ordinate system, we choose two perpendicular number lines that intersect at zero and then associate points in the plane with number pairs. In the beginning we shall use number rays, rather than lines, with the whole-number points labelled. Having chosen two rays, we can locate a point for any pair of whole numbers, provided we agree on which number in the pair refers to which ray. It is conventional to agree that the first number refers to the horizontal line, or ray, and the second to the vertical line, or ray. Usually, the two number rays look like this:



The arrow indicates the point for the number pair.

The concept of symmetry of a figure is extended in this chapter to include drawing the remaining half of a symmetric figure in the co-ordinate plane when one half of the figure is given. Also, the idea of a translation is treated as a movement of every point on a figure by a given amount in the co-ordinate plane, such as "3 over and 4 up".

It is a simple step from graphing number pairs in the co-ordinate plane to relating the pairs to the input-output numbers of the function machine and thus drawing the graph for a given function rule. Finally, negative numbers are introduced in a realistic setting and related to graphs in the co-ordinate plane. No computation with negative numbers is introduced at this level.

Teaching the Chapter

Materials

Graph paper, 1-cm grid

Grid for demonstration (approximately 1 metre square)

Overhead projector (if available) and 10-by-10 unit grid on a transparency

Ruler or straightedge

Small mirrors or reflectors

Vocabulary

axis
bar graph
coordinate axes
co-ordinate axes
co-ordinates
data
graph
negative numbers
similar figures
similar figures
symmetric figure

Most of the lessons in this chapter provide the children with practice in graphing points, "point pictures," or functions. Consequently, it is essential that you have an adequate supply of graph paper. If commercial graph paper is not available, carefully rule a duplicating master with a grid of approximately 1-cm units. For some lessons, such as the one on similar figures, you may want to use graph paper with smaller or larger unit demarcations, such as 5 mm or 2 cm. Throughout this chapter, encourage group activity. Also, plan to display numerous examples of the children's work. Demonstrations on a large grid or overhead projector should be used selectively; if children understand the graphing technique, keep your demonstrations to a minimum.

Lesson Schedule

Since most of the lessons require that children actually graph figures, the amount of time you should allocate for this chapter will vary according to your children's graphing skill. However, you should probably plan to spend at least two weeks on the chapter.

Evaluation of Progress

Most of your evaluation will come from your daily observation of the children as they work through each lesson. Do not place too high a value on the neatness and exactness of the graphing. A child who is not well co-ordinated may have a grasp of the concepts but simply be unable to do exact work. The review lesson at the end of the chapter will help you in your evaluation.

Resources for Active Learning

GENERAL ACTIVITIES

[The first five resources listed below combine art and mathematics with curve stitching.]

Experiments in Mathematics, Stage 1, pp. 30–31, Houghton Mifflin (Available from Thomas Nelson & Sons)

Franklin Series: *Patterns and Puzzles*, pp. 36–43, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Line Designs—direction booklet (Creative Publications)

Mathex: Geometry No. 4, pupil pages 45–47, Encyclopaedia Britannica Publications Ltd.

Mathex: Geometry No. 9, pp. 38–42 (pupil pages 56–59), Encyclopaedia Britannica Publications Ltd.

Freedom to Learn, pp. 81–94, Addison-Wesley [Ideas for graphing]

Mathematics Using String, Cards 7/1–5, Math Media

Notes on Mathematics in Primary Schools, "Visual Representation," pp. 89–110, Cambridge University Press (Available from Macmillan of Canada)

Nuffield Project: *Graphs Leading to Algebra 2*, "Introduction to Co-ordinates," pp. 2–11, Wiley

MANIPULATIVE DEVICES

Geoboards (Addison-Wesley)

Pegboards (school supplier)

COMMERCIAL GAMES

Battleship (buy locally)

8-Peg Game (World Wide Games)

Go (or Gomoku) (Math Media; World Wide Games)

Tower of Hanoi (or Hindu Pyramid Puzzle) (Creative Publications; World Wide Games)

Vectors (Selective Educational Equipment)

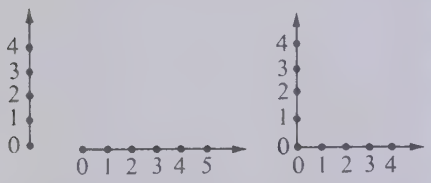
PAGES 296–297

Objective

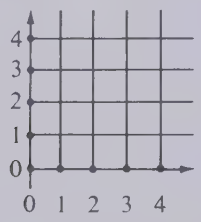
Given locations on a graph, the child will be able to identify the number pair (co-ordinates) corresponding to the locations.

Preparation

To prepare for this lesson, it would be helpful to review the number line for whole numbers. Show a horizontal number line beginning at the left with zero and continuing to the right with an arrow at the end. Also show a vertical number line with one endpoint at zero. Then show these two lines together so that the zero points coincide.



Fill in the lines to form a grid as shown below.



Then direct the children to the investigation in their text.

Investigation

Have the children work in groups of two or three so that they can discuss their choice of co-ordinates for each location. Encourage them to record their results in a table such as the following:

♦	0 over, 4 up	(0,4)
➡	1 over, 1 up	(1,1)
▲	2 over, 3 up	(2,3)
★	3 over, 4 up	(3,4)
○	4 over, 2 up	(4,2)
✕	4 over, 3 up	(4,3)
■	5 over, 4 up	(5,4)

13 Geometry and Graphing

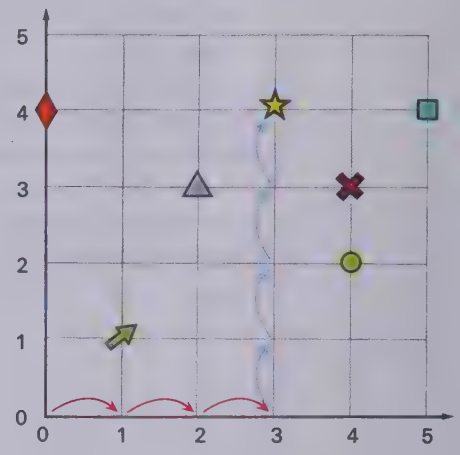
How can number pairs show locations?

Investigating the Ideas

Where is the star?

"3 **over** (→) and 4 **up** (↑)"

Its **co-ordinates** are (3, 4).



? Can you give the co-ordinates to show the locations of the other figures?

See Investigation.

Discussing the Ideas

1. To give co-ordinates, we write the "over" number first and the "up" number second. Explain how the location (4, 3) is different from the location (3, 4).
 (4,3) is 4 over and 3 up; (3,4) is 3 over and 4 up.
2. If someone said, "The co-ordinates of the **triangle** in the Investigation are (3, 2)," would he be correct? Explain.
 No, because (3,2) means 3 over and 2 up; the triangle is 2 over and 3 up (2,3).
3. A What are the co-ordinates of the figure that is farthest from (5, 5)? (1,1)
 B What figure is farthest from (0, 0)? Square (5,4)

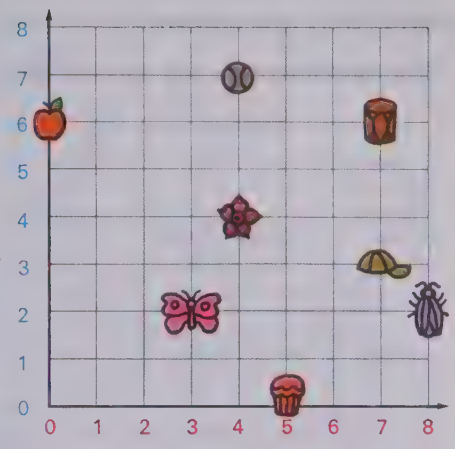
296

Discussion

One of the most important points of this lesson is stressed in both discussion exercise 1 and discussion exercise 2, namely, the order in which the co-ordinates are given. For example to describe the star, a child might carelessly say "4 up and 3 over" and then incorrectly write (4,3). For exercise 2, expect the children to explain that the first number of the co-ordinate pair gives the *over* direction, and the second number gives the *up* direction. Ask the children to give the co-ordinates which they found for the figures in the investigation. Make certain the children give the co-ordinates in the correct order. It would also be helpful to use a grid on an overhead projector or on the chalkboard and ask children to find the co-ordinates of points you have marked on the graph.

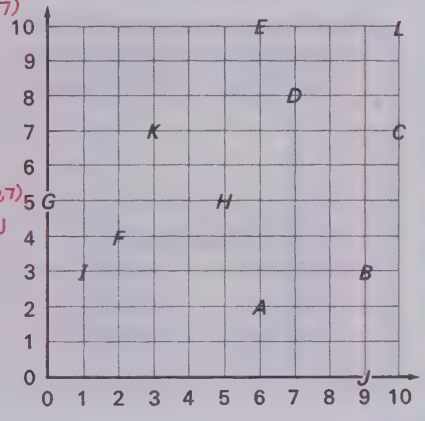
Using the Ideas

1. Give the missing numbers. Then give the co-ordinates.



- **A** The butterfly is 3 over and ▨ up. 2
 The co-ordinates for the butterfly are ___ ? ___ (3,2)
- **B** The cap is ▨ over and 7 3 up. The co-ordinates for the cap are ___ ? ___ (7,3)
- **C** The beetle is 8 over and ▨ up. 2
 The co-ordinates for the beetle are ___ ? ___ (8,2)
- **D** The drum is ▨ over and 6 up. 7
 The co-ordinates for the drum are ___ ? ___ (7,6)
- **E** The cupcake is 5 over and ▨ up. 0
 The co-ordinates for the cupcake are ___ ? ___ (5,0)
- **F** The apple is 0 over and ▨ up. 6
 The co-ordinates for the apple are ___ ? ___ (0,6)
- **G** What are the co-ordinates for the flower ? for the ball ?
 (4,4) (4,7)

2.



- **A** Give the co-ordinates for *K*. (3,7)
- **B** What letter is 9 over and 3 up ? B
- **C** What letter has co-ordinates (5, 5) ? H
- **D** Give the co-ordinates for *C*. (10,7)
- **E** What do you find at (9, 0) ? J
- **F** What letter has 2 as its first co-ordinate ? F
- **G** What letter is at (0, 5) ? G
- **H** Give the co-ordinates for *A*, *D*, and *L*. (6,2); (7,8); (10,10)



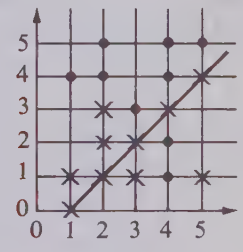
Using the Exercises

On page 297, you may choose to use parts of exercise 1 orally before you assign the page as independent work. When the children have finished the exercises, carefully check them together, again emphasizing the importance of giving the "over" number first.

Follow-up

For an entertaining class activity, adapt tick-tack-toe to a co-ordinate system by preparing a grid using only the positive integers 1 through 5 and 0, either on a large chart that can be covered with acetate or on a separate transparency that can be slipped under the roll of an overhead projector. Divide the class into two evenly matched teams. Designate one as the X team and the other as the dot (●) team.

The object of the game is to get five X's or dots in a row, column, or diagonal by naming the co-ordinates of each X or ●. The members of each team take turns naming any number pair they wish. Insist that each child make his own choice, whether it is the best choice or not, without any help from his team. The children will learn strategies to block the other team while continuing to build a string of X's or dots as they play. Eventually, they should become skillful enough to make every contest a tie game. You can then make the game more difficult by extending the playing board to an 8-by-8 or a 10-by-10 grid and requiring six X's or dots in a row. (Let the children discover how to name points on either axis.)



A Sample Winning Game of Tick-Tack-Toe

Resources for Active Learning

Mathex: Matching and Graphing No. 1, "Co-ordinates," pp. 26–29 (pupil pages 43–52), Encyclopaedia Britannica Publications Ltd. [For this and the next two lessons]

Assignments (page 297)* _____
Minimum: 1–2. Average: 1–2.
Maximum: 1–2.

PAGES 298–299

Objective

Given co-ordinates of a point and an appropriate grid, the child will be able to graph the point.

Preparation

Materials

graph paper, 1-cm grid (*Duplicator Masters*, page 67)

You might use a demonstration grid on which points have been marked and have children name the co-ordinates for each point. However, spend only a few minutes on such a review; use it to lead into the discussion section of the text.

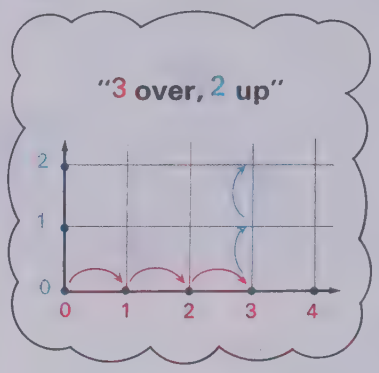


Can you graph a point?

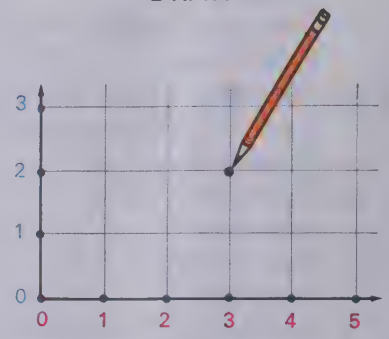
Discussing the Ideas

To **graph** the point with co-ordinates (3, 2)

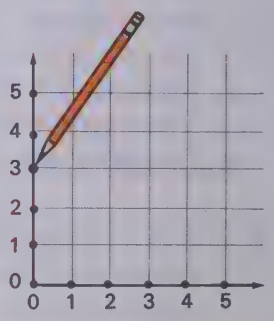
THINK



DRAW



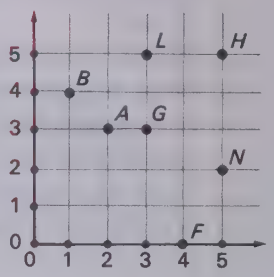
1. How would you explain to someone how to graph the point (5,2) ?
 Sample answer: Count 5 over and 2 up.



2. This picture shows a point being graphed. Explain how you could think about it to figure out the co-ordinates of the point. Sample answer: Think of counting 0 over and 3 up, so co-ordinates would be (0,3).

3. Match the letters of the points with the co-ordinates.

(1,4) B	(4,0) F
(5,2) N	(3,3) G
(2,3) A	(5,5) H



298

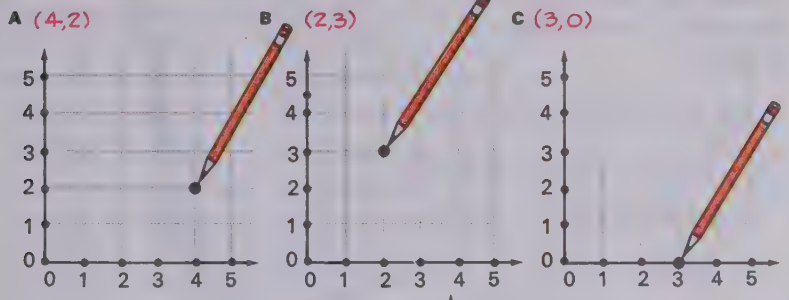
Discussion

While you are using a demonstration grid as suggested above, ask for a volunteer to graph the point with co-ordinates (3,2). Then ask a volunteer to graph the point with co-ordinates (2,3). Compare the co-ordinates of the two points and lead into a discussion of the text material on page 298. Continue to stress the importance of the order of the co-ordinates. Also help the children to realize how the two labelled co-ordinate axes enable them to locate a point. Teach the words *axis*, *co-ordinate axes*, and *graph* as discussed in the mathematics section of this lesson. It would be helpful for each child to use graph paper to practice graphing points by matching co-ordinates you specify. Have some children graph these points on the demonstration grid so that all may check their work.

Exercise 2 stresses the case when the "over" co-ordinate is zero. Use other examples of this kind to help children understand how to treat zero co-ordinates.

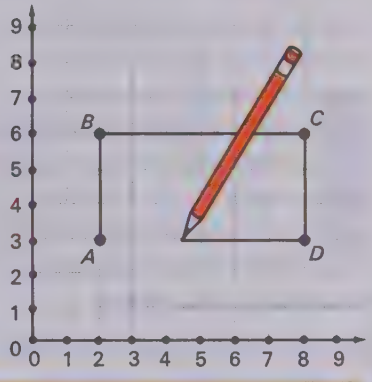
Using the Ideas

1. Each figure below shows a point being graphed. Give the co-ordinates of that point.



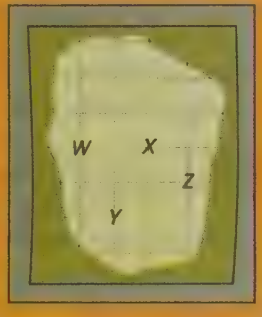
2. A The figure shows points *A*, *B*, *C*, and *D* being connected to form a geometric figure. What is this figure? Rectangle

 B Label your graph paper as shown here. Then graph points and connect them to form each of these figures: **triangle, right triangle, square, parallelogram.** Answers will vary.



think

1. If the co-ordinates of *W* are (5,6), what are the co-ordinates of *X*? of *Y*? of *Z*? (7,6) (5,3) (8,4)
2. If the co-ordinates of *Z* are (7,5), what are the co-ordinates of *W*? of *X*? of *Y*? (4,8) (3,6) (3,4)



Using the Exercises

In the exercises on page 299, children continue to match points and their co-ordinates. You may want to work with them in labelling co-ordinate axes on their graph paper. Make sure the zero point for both the horizontal axis and the vertical axis is at the same point.

Let those who try the *Think* problem explain how they figured out their answers. For example, once a pair of horizontal and vertical grid lines is identified, all of the other lines may be numbered accordingly.

Assignments (page 299)*_____
Minimum: 1–2. Average: 1–2.
Maximum: 1–2.

Mathematics

Each number ray is an *axis*. Using a pair of axes that intersect in a right angle at (0,0), we locate points on the plane. Such pairs of axes are called *co-ordinate axes*. On a grid composed of lines parallel to each axis, we *graph*, or picture, *points* (locations) in space.

The word *graph* is both a noun and a verb. As a noun, it indicates (a) the picture of a point or of a set of points or (b) the entire co-ordinate system showing the number lines and the points involved. As a verb, it means to mark a point either for each number or each pair of numbers in a given set.

Follow-up

The children might enjoy graphing and connecting the following points. Then encourage them to answer the related questions. Answers appear in parentheses, for your convenience.

(5,2), (7,2), (9,4), (9,6), (7,8), (5,8), (3,6), (3,4)

Which of the following points are inside the figure you have formed?

(1,1)	(*no*)	(5,5)	(*yes*)
(4,4)	(*yes*)	(7,5)	(*yes*)
(8,4)	(*yes*)	(9,2)	(*no*)
(2,6)	(*no*)	(0,0)	(*no*)
(3,3)	(*no*)	(6,7)	(*yes*)

Resources for Active Learning

Developmental Math Cards, "Squares" (game), H[4]12, Addison-Wesley.

Math Activity Cards (plotting points), B9,12, Macmillan.

Duplicator Masters, page 56
Workbook, page 99

PAGES 300–301

Objective

Given co-ordinates listed in a particular order, the child will be able to make a picture by graphing and connecting the points.

Preparation

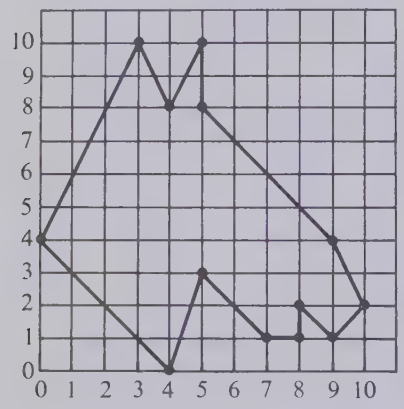
Materials

graph paper, 1-cm grid (*Duplicator Masters*, page 67)

Unless children have experienced difficulty with the ideas developed in the two previous lessons, it would be appropriate to begin immediately with the investigation.

Investigation

Study with the children the picture at the top of page 300. Help them understand what is meant by "connecting these points in order." Then direct them to try the investigation activity on their own. Their picture should be as indicated below.

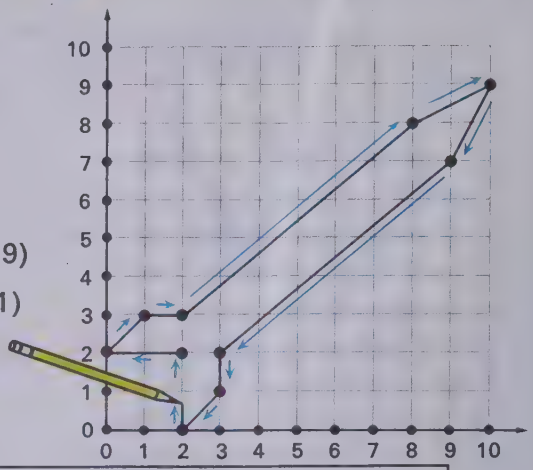


Can you graph a figure?

Investigating the Ideas

This picture was made by connecting these points in order:

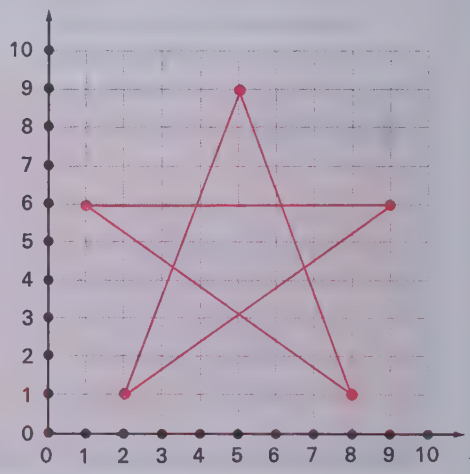
(2, 2) → (0, 2) → (1, 3)
START
→ (2, 3) → (8, 8) → (10, 9)
→ (9, 7) → (3, 2) → (3, 1)
→ (2, 0) → (2, 2)
END



Can you make a picture by connecting these points in order? (4, 0) → (5, 3) → (7, 1) → (8, 1) → (8, 2) → (9, 1) → (10, 2) → (9, 4) → (5, 8) → (5, 10) → (4, 8) → (3, 10) → (0, 4) → (4, 0) See Investigation.

Discussing the Ideas

1. The first 3 points have been graphed and connected. How would you complete the picture?
 (2, 1) → (5, 9) → (8, 1) → (1, 6) → (9, 6) → (2, 1)



2. What is the completed figure in Question 1? Star

3. Give the co-ordinates of four points that could be connected to form a
 A square. B rectangle.
 Answers will vary.



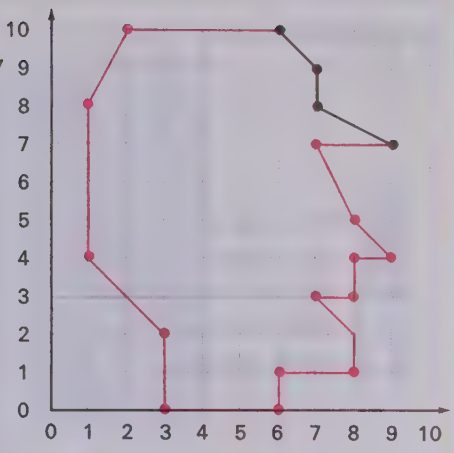
300

Discussion

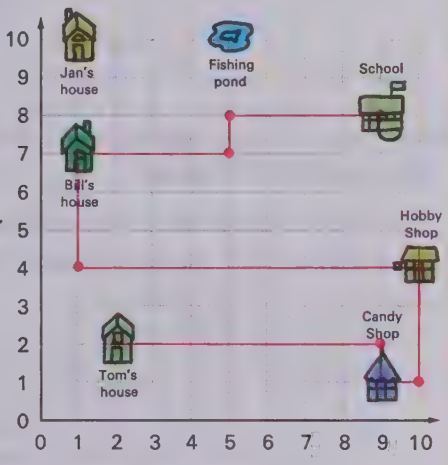
If you think it necessary, have a child graph his figure on a demonstration grid. Then work through the discussion exercises with the children. Have children show a variety of the possible answers for discussion exercise 3. Continue to observe whether or not the children are remembering that the first number of each pair means "over" and the second number means "up."

Using the Ideas

1. The first 4 points have been graphed and connected. Copy and complete the picture on your graph paper.
(6, 10) → (7, 9) → (7, 8)
→ (9, 7) → (7, 7) → (8, 5)
→ (9, 4) → (8, 4) → (8, 3)
→ (7, 3) → (8, 2) → (8, 1)
→ (6, 1) → (6, 0) → (3, 0)
→ (3, 2) → (1, 4) → (1, 8)
→ (3, 10) → (6, 10)



2. Make a graph like this and label the spots shown. (You may want to use large dots • instead of pictures.)



 A Connect these dots in order to show the route Tom took on his way to school.
(2, 2) → (9, 2) → (9, 1)
→ (10, 1) → (10, 4) → (1, 4)
→ (1, 7) → (5, 7) → (5, 8)
→ (9, 8)

 ★ B Give the points for the path you would take to school if you were Tom or Jan. (Choose one.)

 ★ C Give the points for one of the shortest paths (along the gray lines) from Tom's house to school.

Routes may vary.

Answers may vary. Sample: (2,2)→(2,8)→(9,8)

★ 3. Invent a picture and list the co-ordinates for the picture. Then see if a classmate can draw the picture.

See Using the Exercises.



Follow-up

Encourage children to display their graphs around the classroom.

You might also suggest that they graph a "picture" of the classroom, as if the floor were made of graph paper and viewed from above. Some children might realize that an accurate graph might best be made by measuring the length and width and labelling their axes to include these dimensions. Let them use their ingenuity to figure out how they might determine the position of the classroom tables, bookshelves, desks, etc. Some might actually walk out the steps along imaginary grid lines on the floor.

Using the Exercises

Be sure an adequate supply of graph paper is available for the exercises on page 301. As the children work, stress how important it is for them to connect the dots in the order in which they are given in the book. When they complete exercise 1, you might suggest that they try positioning an eye on the profile and then give this character a name.

Encourage children to try the starred problems. Be sure they understand that for exercise 3 they should give their classmate only a list of the co-ordinates for their picture, not the picture itself.

Assignments (page 301)

Minimum: 1–2A. Average: 1–2. Maximum: 1–3.

PAGES 302–303

Objective

Given half of a symmetric figure, the child will be able to draw the other half.

Preparation

Materials

graph paper

To prepare for this lesson, review the idea of symmetry and symmetric figures. For example, display a symmetric figure and fold it along the line of symmetry to show both halves. Have the children recall that in a symmetric figure each half is a mirror image of the other. If possible, help children realize this by holding a mirror along the line of symmetry. The reflection on the mirror will be the other half of the figure.

Investigation

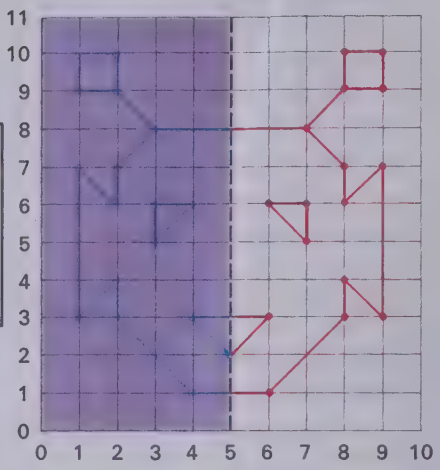
Encourage children to try any method they wish to draw the other half of the investigation figure. Of course, they must first accurately copy on their graph paper the half shown in the text. Then they might try folding along the line of symmetry and pushing the tip of their pencil through the paper to find the opposite points. Others might use the grid to count the position of the points in the direction opposite to the points given. If small unbreakable mirrors are available, some children may find these helpful. Some may try to trace the half in the text and then transfer that half by turning over the tracing paper and using it to complete their picture. Encourage diversity of approach to this activity; children will benefit from sharing explanations of the various methods they used.

Can you draw the other half of a symmetric figure?

Investigating the Ideas

Can you copy this half of a symmetric figure on your graph paper, and then draw the other half?

See Investigation.





Discussing the Ideas

1. Explain the method you used to complete the figure in the Investigation.
 See Investigation.
2. In the picture above:
 - A What are the co-ordinates of the point at the corner of the "Martian's" mouth ? (4,3)
 - B How can you find the co-ordinates of the point that matches this point in the other half of the picture ? Methods may vary. See Investigation and Discussion.
 - C What are the co-ordinates of the point at the upper tip of the "Martian's" ear ? (1,7)
 - D Explain how to find the co-ordinates of the point that matches this point in the other half of the picture. Methods may vary.

302

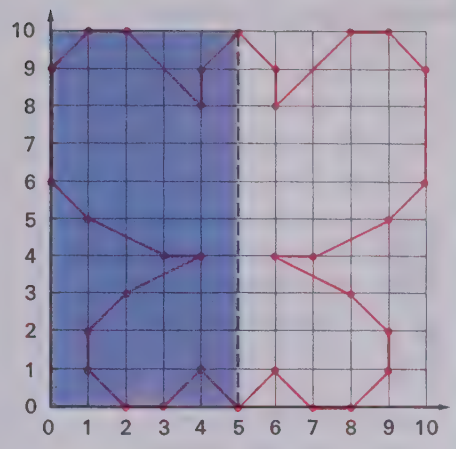
Discussion

Aside from learning about various ways to draw the other half of a symmetric figure, one of the important points of this lesson is for children to realize the role of the graph paper and of the co-ordinate axes. It is important for children to realize that every point in the text has a corresponding "mirror image" point on the other side of the line of symmetry. A discussion of exercise 2 should help children understand this. If necessary, continue such a discussion with reference to other particular points of the Martian's face, such as the top points of his antenna, or the lower tip of his ear.

Using the Ideas

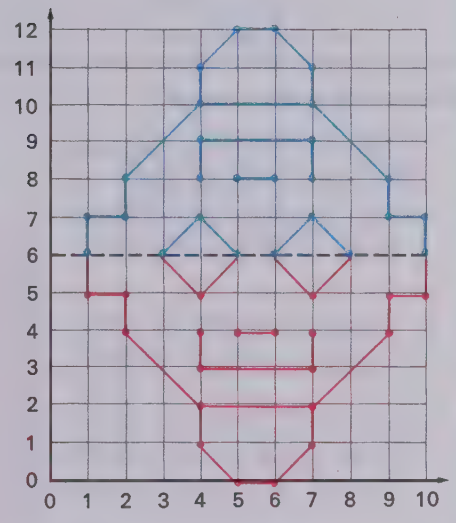
1. A Copy this half of a symmetric figure on your graph paper and draw the other half.

B What point in the other half matches the point (2, 10)? (8,10)



2. A Copy this top half of a symmetric figure on your graph paper and draw the other half.

B What point in the other half matches the point (4, 9)? (4,3)



★ **3.** Draw half of a symmetric figure on a piece of graph paper. Then have a classmate try to draw the other half.
See Using the Exercises.



Follow-up

If some children have difficulty thinking of ideas for a half of a symmetric figure, you might give them one of the following sets of co-ordinates. When the points for these co-ordinates are connected, one half of the indicated symmetric figure will appear.

Right Triangle Man
head: (6,7) → (1,7) → (6,1)
eye: (4,5) → (5,5) → (5,6) → (4,5)
mouth: (6,3) → (5,3) → (6,2)

Hexagon
(4,6) → (2,5) → (2,3) → (4,2)

Barn
(6,3) → (4,3) → (4,0) → (2,0) → (2,6) → (5,9) → (5,10) → (6,11)

(*Barn Window*) (6,8) → (5,8) → (5,6) → (6,6)

Resources for Active Learning

Franklin Series: *Mirror Magic*, "Symmetry: How can you get it?" pp. 73–83, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Freedom to Learn (similarity, symmetry, translation), p. 94, Addison-Wesley.

Using the Exercises

Supply the children with a sufficient amount of graph paper. Have them do the exercises on page 303 independently. Encourage them to extend exercise 3 by drawing halves of two or three different symmetric figures. The children's work will again be suitable for display.

Assignments (page 303)
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

PAGES 304–305

Objective

Given simple pictures on a graph, the child will be able to make a smaller or larger similar *figure by using graph paper.*

Preparation

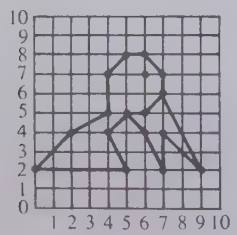
Materials

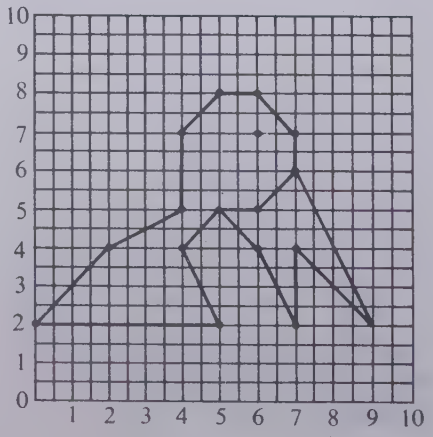
graph paper, 1-cm grid

To prepare for this lesson, display two or three sets of figures which are alike in every way except size. Simply exhibit each set and ask children what they notice about them. For example, if you have two regular hexagons, one with side length 15 cm and the other with side length 25 cm, the children should realize that they are different only in size. This, of course means that the areas as well as the side lengths of both figures are different. You might ask for suggestions as to how a smaller figure may be enlarged or how a larger figure may be made smaller without losing the shape of the original figure.

Investigation

In this investigation, children have an opportunity to try to enlarge a figure in such a way that it retains its shape. They are given no directions other than that they are to use graph paper. If the children's graph paper consists of units larger than those pictured in the text, a simple copying of the goose will result in a larger figure. However, a more instructive procedure would be for children to number their co-ordinate axes so that the spaces between each number are greater than on the original axes. For example, to enlarge the goose, a child might number every other line on the axes and then plot the same points according to the new numbering, as pictured below and at the right.



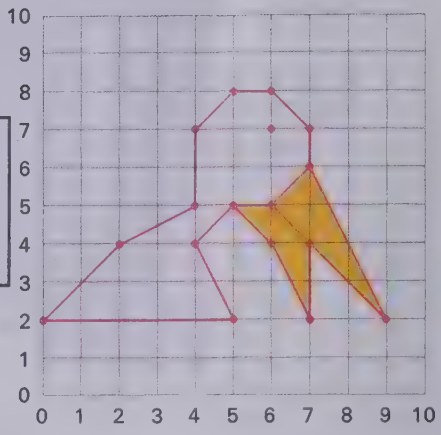


Can you stretch and shrink figures?

Investigating the Ideas

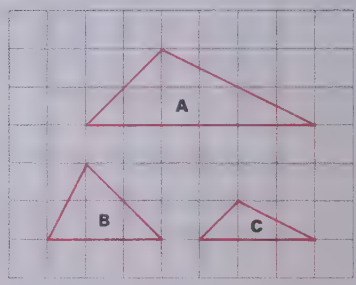
Can you use graph paper and make a picture of "Gorgeous Goose" like this, but larger?

See Investigation.



Discussing the Ideas

1. When two figures differ only in size, the figures are similar to each other. Which two triangles are similar?
 A and C



2. Can you find two shapes in your classroom that are similar?
 Answers will vary. See Discussion.

3. Jack said, "Any two squares that I draw will be similar to each other." Do you think Jack is correct? Yes



4. How might you make a picture like the one in the Investigation, but smaller? See Discussion.

304

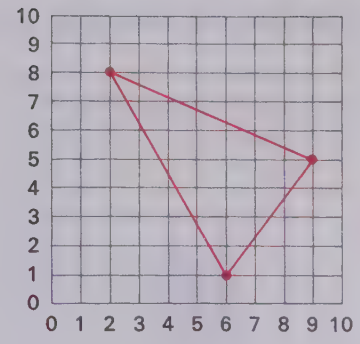
Discussion

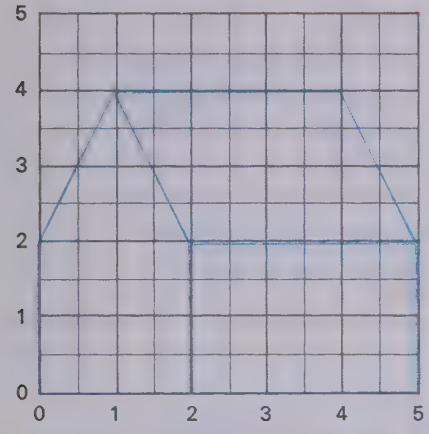
One of the principal goals of this lesson is to develop an understanding of *similar* figures. Use the results of the investigation and discussion exercise 1 to emphasize that similar figures are the same in every way except size, and sometimes in size also. As you discuss exercise 2, note such examples as the regions enclosed within windows and window frames, pictures and picture frames. All circles are similar, and, as exercise 3 points out, all squares are similar. (Circles which have radii of the same length and squares which have sides of the same length are both congruent and similar but this

Using the Ideas

1. Use graph paper to make a larger figure similar to this one.

Methods may vary, as in Investigation.





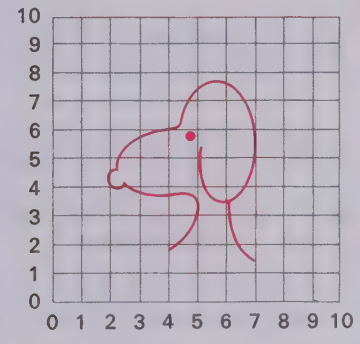
2. A Use graph paper to make a smaller figure similar to this one.

B Make a larger figure similar to this one.

Methods may vary.

★ **3.** Can you use graph paper to help you make a larger picture of this little dog?

See Using the Exercises.



think

Start with:	2
Double it:	4
Double again:	8
Double again:	16

After 3 doubles, you're at 16.
Guess where you'll be after 10 doubles. Were you correct?
2048



point should not be brought out to the children; simply speak of such figures as similar figures.) To reduce the size of the goose, children might use graph paper with smaller grids or they might number their axes by twos instead of by ones.

Using the Exercises

Supply the children with sufficient graph paper to do the exercises on page 305. Encourage all to try exercise 3 and to use the same idea with other cartoon characters. For example, have them use tracing paper to trace a cartoon character, then transfer it to graph paper by laying the tracing paper figure over it and drawing over the figure again. Then the task of enlarging the cartoon character may be treated.

Assignments (page 305)*

Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Mathematics

Intuitively, *similar figures* are figures which have the same shape. Recall that congruent figures not only have the same shape but must also have the same size. In general, similar figures do not have the same size; however, when two figures are congruent, it is true that they are also similar.

The mathematical definition of similar polygons is as follows:

> Two polygons are similar if there is a one-to-one correspondence between their vertices such that the corresponding angles are congruent and the lengths of the corresponding sides have the same ratio.

Since corresponding sides of similar figures must have the same ratio, we may think of one of a pair of similar figures as being either an enlargement or a reduction of the other. The co-ordinate plane affords a convenient grid on which children can easily display pairs of similar figures.

Resources for Active Learning

Developmental Math Grids, G²17, Addison-Wesley.

Freedom to Learn, "Geometry—Similarity," pp. 154–157, Addison-Wesley.

Measure and Find Out, Book 1 (shadows and graphs), Activities 18, 19, Scott Foresman. (Available from Gage Educational)

PAGES 306–307

Objective

Given a figure in the co-ordinate plane, the child will be able to show the position of the figure after a given translation.

Preparation

Materials

graph paper

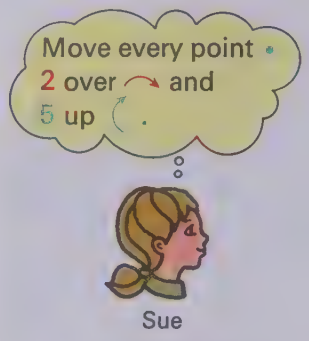
The nature of this investigation makes it appropriate to omit specific preparation and begin immediately with the investigation.

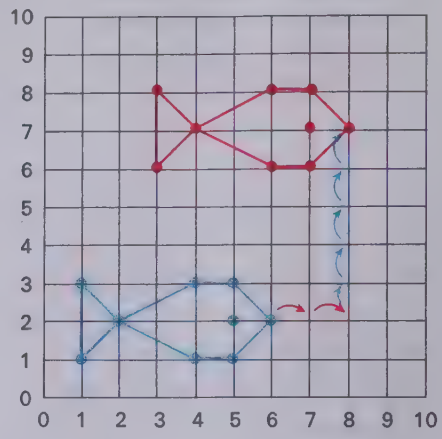
Investigation

In this investigation, children are directed simply to move each point according to a particular pattern. If they move each point correctly, their graph should show the same picture located in a different position on the graph. If some children finish quickly, you might suggest that they follow the same pattern and move the "Gorgeous Goose" of the previous lesson according to the same pattern.

Can you change the positions of figures?

Investigating the Ideas





? Can you show on your graph where the fish will be after Sue moves all the points and connects them?
See graph and Investigation.

Discussing the Ideas

1. A The eye of the fish is at what point on the graph? (5,2)
 B At what point will the eye be after it is moved? (7,7)

2. A At what point on the graph is the top tip of the tail? (1,3)
 B Where will this tip be after it is moved? (3,8)

3. Is the fish larger or smaller after the move? Explain.
 Same size. See Discussion.

4. Is the fish a different shape after the move?
 No. See Discussion.

5. Choose another move like the one Sue used, and show where it takes the fish. Answers may vary.

306



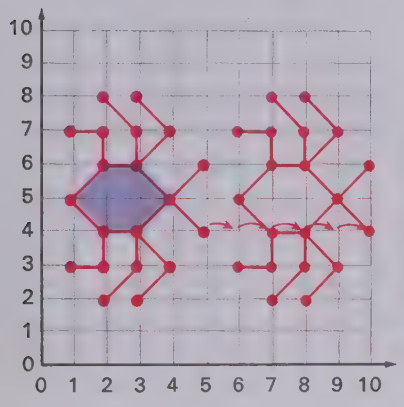
Discussion

One of the main points of this lesson is that if every point of a figure is moved according to the same pattern, the figure will retain its shape and size, but will have been translated a given distance in a given direction. Work carefully through the discussion exercises; they progressively develop the main idea. In exercise 5, be sure the children understand that they may choose any move "over" and "up," but it should not be the same as that done for the investigation. However, if their move includes very many steps, they will have a more awkward task of labelling their graph paper and locating each new point. Patterns such as "move every point 4 over and 2 up" or "3 over and 1 up" or "5 over and 4 up" should cause little or no difficulty.

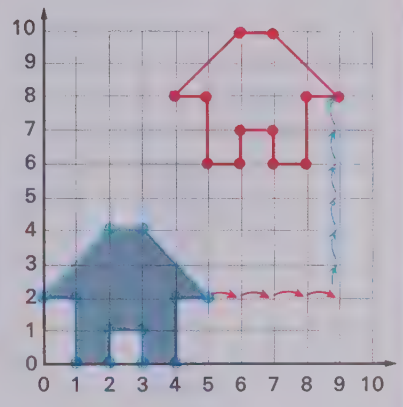
Using the Ideas

1. Use the move given and show the final position of the picture on your graph.

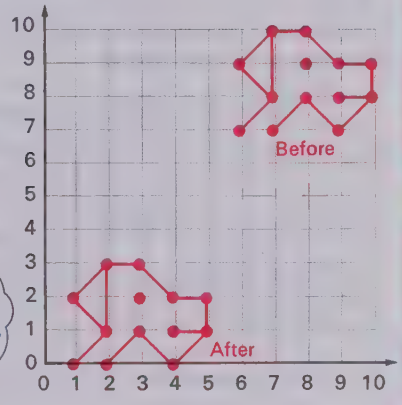
A Move every point 5 over and 0 up.



B Move every point 4 over and 6 up.



★ **2.** The picture before and after the move is shown. Can you complete the description of the move?



Every point was moved 5 over and 7 down.

Using the Exercises

Have the children do the exercises on page 307. Again, be sure there is an ample supply of graph paper. You might show the children how to gauge the placement of their axes so that they can fit more than one picture on each sheet of graph paper.

Although exercise 2 is starred, all the children will benefit from a discussion of it. Note that the position of the before and after pictures requires that the move be *over to the left* and *down*, rather than the usual *over* (to the right) and *up*.

Assignments (page 307)

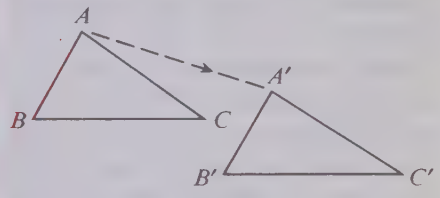
Minimum: 1. Average: 1.
Maximum: 1–2.

Mathematics

In this lesson, the idea of a translation is introduced by means of the co-ordinate plane by having the student "move" every point of a given figure by the same amount, such as "over 5 and up 2."

Mathematically, a translation is an example of a kind of motion in geometry that is called a *rigid motion* because under a given translation a given figure and its image (think of image as the final position of the figure) are congruent. Often a translation is described as a motion which preserves distances; that is, the figure is not distorted or twisted by the translation. Other kinds of rigid motions in geometry are *rotations* and *reflections*.

The figure below shows a translation in which every point of $\triangle ABC$ is translated in the same direction and distance, as shown by the arrow from point A to point A'.



Notice that AB is parallel to $A'B'$, BC is parallel to $B'C'$, and AC is parallel to $A'C'$. Similar statements must hold for any translation.

Follow-up

Encourage children to graph pictures of their choice on carefully labelled graph paper. Have them write directions for a move underneath the picture. Then suggest that they exchange papers with a classmate and move his picture according to the directions given.

Resources for Active Learning

Nuffield Project: *Shape and Size* 3, "Transformations," pp. 60–62, Wiley.

Workbook, page 100

PAGES 308–309

Objective

The child will be able to make a graph to show simple data that he or his classmates have collected.

Preparation

Materials

graph paper

To prepare for this lesson, you might spend a few minutes asking the children to mention different symbols people use to share their ideas or tell one another something. Let the children briefly discuss any form of communication which they find interesting. Then eventually lead them to consider the question which introduces this lesson, "Can graphs tell a story?"

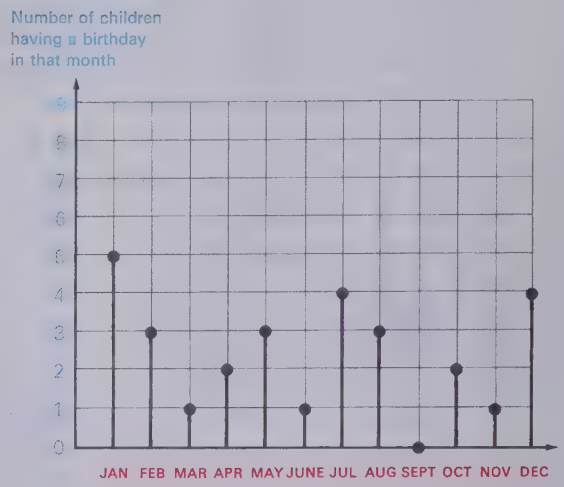
Investigation

In order for the class to be able to do this investigation, it will be necessary for them to collect data regarding the birthdays of all the class members. You might make the collection of this data a class project assignment and have the children figure out the best way to collect the data. For example, someone might list the months of the year on the chalkboard, call out each month, and tally the number of children who have a birthday in that month.

After the data is collected, you might have children work in groups of three or four and give them an opportunity to discuss how to make the bar graph. However, even if children work in groups, each should make his own bar graph.

Investigating the Ideas

This graph tells a story about birthdays of Jack's classmates.



? Can you make a graph like this one to show the birthdays of your classmates?
See Investigation.

Answers will vary from class to class.



Discussing the Ideas

1. A In what month does Jack's class have the most birthdays?
 January
 B In what month does your class have the most birthdays?
 Answer will vary with data collected.
2. Can you answer questions 1A and 1B for **fewest** birthdays?
 A September B Answers will vary.
3. How can you use the graph to find how many students are in Jack's class? Sample answer: Find the sum of the birthdays for all twelve months. (29)
4. What other information can you find in the graph?
 See Discussion.

Discussion

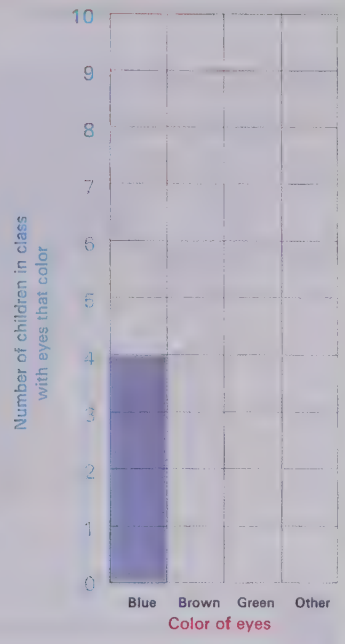
After the children have had ample time to complete their graphs for the investigation, direct them to the discussion exercises. These exercises focus on reading and interpreting the graph. As you discuss exercise 3, children might check their method of finding how many students are in Jack's class by using it on the graph for their own class, for which they know the correct total. Exercise 4 might be explored in greater detail if you ask some specific questions such as: "Are there more birthdays in the summer months than in the winter months? in the spring months than in the fall? Is there any month with no birthdays? What is the total number of birthdays in September, December, March, and June? in October, November, December? Does the graph tell anything about how old each person is?"

Questions such as these will help children realize that a graph is a very useful tool for communicating specific ideas if it is properly made and read.

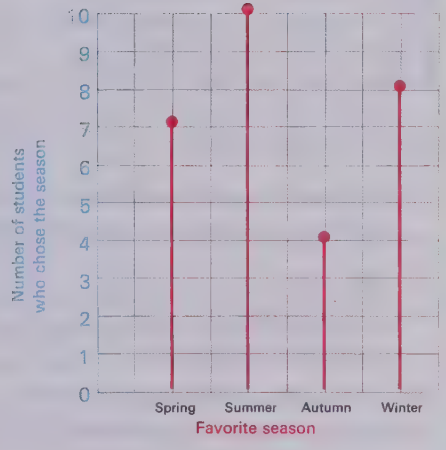
Using the Ideas

Answers for these exercises will vary from class to class.
See Using the Exercises.

1. Bob counted four people in his class with blue eyes. He started a graph like this.
 - A Make a graph like this to show the colors of the eyes of the children in your class.
 - B What does your graph tell you?



2. Val asked each of her classmates to name his favorite season of the year. Then she made a graph like this.
 - A Make a graph like this one to show your classmates' favorite seasons.
 - B Which season was chosen most often in your class?
 - C Which season was chosen least often in your class?



309



Using the Exercises

Both of the exercises on page 309 require that children gather information about their classmates. Again, you might use the problem of collecting information as a class project. For example, children may decide to find out about the color of eyes of their classmates by looking at each person's eyes individually and keeping tally. For exercise 2, they might decide to group the information together on a chart on the chalkboard. In either case, the importance of collecting data carefully should be stressed. Continue to help any children who seem to have difficulty in setting up their graphs. For those who finish quickly or who show special interest, suggest the activity in the follow-up section.

Assignments (page 309)*
Minimum: 1–2. Average: 1–2.
Maximum: 1–2.

Follow-up

Children might enjoy the challenge of making their own graph without a model to follow. You might suggest that they gather and graph information about one of their special interests. Or suggest that they make a graph to show how many children in their class have brown, blond, black, or red hair; how many children in their class (or school) come to school in cars, buses, by walking, by bicycling; or how many children watch certain TV programs.

Resources for Active Learning

[Graphs that tell stories]

Applied Mathematics Cards, Group 1/22–26, Schofield and Sims. (Available from Mafex Associates, Willowdale)

Developmental Math Cards, H¹6, H²11, Addison-Wesley.

Math Activity Cards, B7, 8; C6, 8, Macmillan.

Mathex: Graphing and Probability No. 6, pp. 1–3, Encyclopaedia Britannica Publications Ltd.

Measure and Find Out, Book 1, Activity 22, Scott Foresman. (Available from Gage Educational)

Nuffield Project: *Pictorial Representation* 1, Wiley

PAGES 310–311

Objective

Given a set of number pairs corresponding to a particular function rule, the child will be able to graph the function.

Preparation

Materials

graph paper

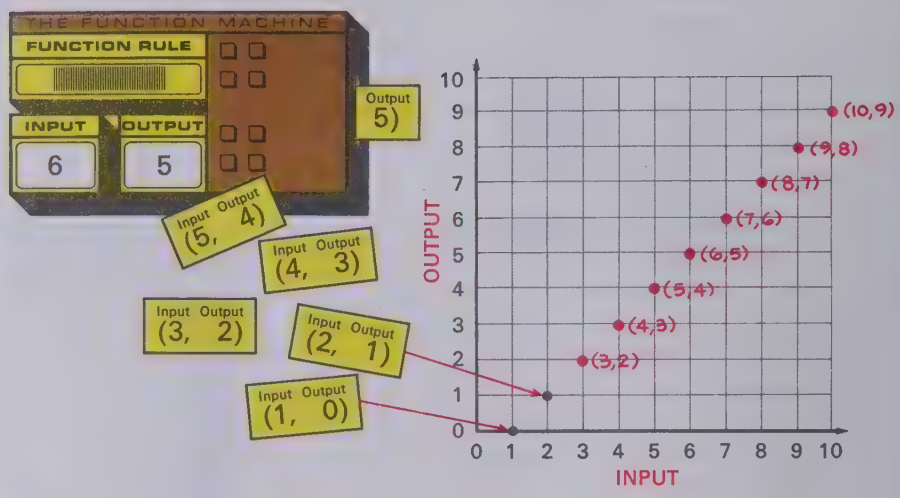
You might want to review several function rules before children begin the investigation. You could do this by using the "What's My Rule" game described on page 65. However, spend only a very few minutes on this preparation; the children will need considerable time to do the activities in the lesson itself.

Investigation

This investigation might be handled by the children either in groups or individually. Children's familiarity with the function machine should enable them to discover the rule (*subtract one*). Encourage them to find and graph as many number pairs as will fit on their graphs. Note that they are directed to graph the points for each given number pair, and not to connect these points into a line. One of the important ideas for them to discover is that the input and output numbers may be thought of as a pair of co-ordinates for a point.

Can you show function machine pairs on a graph?

Investigating the Ideas



Can you discover the function rule and give some other input-output pairs for the function machine cards? Function Rule : Subtract 1	Mark the points for all input-output pairs on a graph as shown. See above.

Discussing the Ideas

1. What pair is on the card coming out of the function machine? Where will you mark the point for this pair? (6,5) See graph above.
2. What are some other pairs on the cards that might come from the machine? (7,6), (8,7), (9,8), etc.
3. Do you think the points on the graph have a pattern? Describe it. Yes, they lie on the same straight line.

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Discussion

The discussion exercises should help children realize that each pair of input and output numbers may be matched to a point, just as the number pairs they worked with previously were co-ordinates of a point on a graph.

Exercise 3 develops the idea that if the points plotted in the investigation were connected they would all be on the same straight line. However, do not have the children connect the points; the pattern of points in a row should suggest the idea of a straight line well enough to enable children to describe this pattern.

If necessary, use a demonstration grid or an overhead projector to show the graphs of other function rules. Use a different pair of co-ordinate axes for each graph of a function, or use a different colored pen to distinguish each set of co-ordinates. Continue to stress with the children that they can think of the input and output as a pair of co-ordinates.

Using the Ideas

1. Think about the function machine. Copy and complete the number pairs for each set of 8 input-output cards. Then graph the points for each set of cards. For graphs, see Answers, T.E. page 311.

A Function Rule: Subtract 2

Input	Output
(2, ▮)	0
(3, ▮)	1
(4, ▮)	2
(5, ▮)	3
(6, ▮)	4
(7, ▮)	5
(8, ▮)	6
(9, ▮)	7

B Function Rule: Divide by 2

Input	Output
(0, ▮)	0
(2, ▮)	1
(4, ▮)	2
(6, ▮)	3
(8, ▮)	4
(10, ▮)	5
(12, ▮)	6
(14, ▮)	7

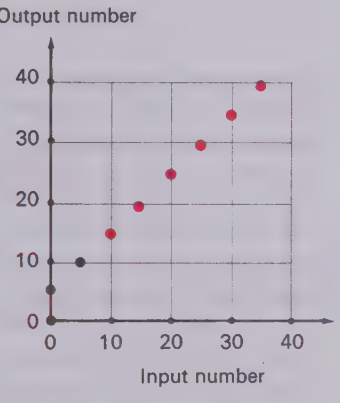
C Function Rule: Subtract from 9

Input	Output
(0, ▮)	9
(1, ▮)	8
(2, ▮)	7
(3, ▮)	6
(4, ▮)	5
(5, ▮)	4
(6, ▮)	3
(7, ▮)	2

★2. Copy and complete the number pairs. List the co-ordinates. Copy and complete the graph of this function.

Function Rule: Add 5

Input	Output	Co-ordinates
(0, 5)		(0, 5)
(5, ▮)		(5, ▮) 10
(10, ▮)		(10, ▮) 15
(15, ▮)		(15, ▮) 20
(20, ▮)		(20, ▮) 25
(25, ▮)		(25, ▮) 30
(30, ▮)		(30, ▮) 35
(35, ▮)		(35, ▮) 40



★3. Make your own list of number pairs for this function rule:

Multiply the number by itself.

Use input numbers 0, 1, 2, 3, 4. Draw the graph for this function.

Input	0	1	2	3	4
Output	0	1	4	9	16

For graph, see Answers, T.E. page 311.

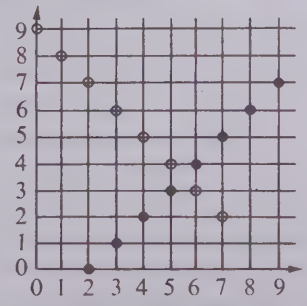


Using the Exercises

Assign the exercises on page 311 as independent work.

Suggest that the children either make a separate graph for each part of exercise 1 or graph each part in a different color to make it easy to distinguish them. Starred exercises 2 and 3 are intended to challenge more able children. For exercise 2, they are required to estimate the location of the points, but since the grid is large, they should not have much difficulty.

Answers, exercises 1A, C, page 311



1. **A** Points graphed by ●.
 C Points graphed by ○.

Assignments (page 311)*

Minimum: 1. Average: 1.
Maximum: 1–3.

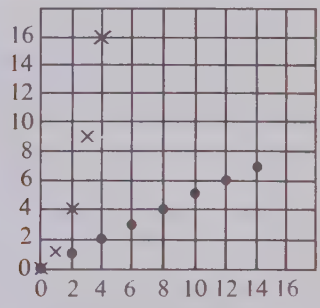
Mathematics

Each function rule produces a set of number pairs (the input-output numbers), and the graph of these pairs reveals a unique pattern. Picturing, or graphing, a special set of number pairs associated with a function is quite important in mathematics. Notice that except for exercise 3, the pattern displayed in the graph of each function indicates that each point would be on the same straight line. Such functions are called linear functions. More complicated functions and relations may give patterns such as parabolas, hyperbolas, etc.

Follow-up

Suggest that the children combine the "What's My Rule" game with graphing points. For this game, pairs of children can work together using the same grid. One child thinks of a rule and asks for an input number. Instead of recording the output number, he simply notes the number mentally, applies his rule, and graphs the appropriate point. His partner gives him another input number and he again plots the appropriate point. The goal is for the partner to guess the rule the other child is using.

Answers, exercises 1B, 3, page 311



1. **B** Points graphed by ●.
3. Points graphed by X.

Resources for Active Learning

Discovery, Section II, Units 21/2–4; 22/2,3, Encyclopaedia Britannica Educational Corp.

Inquiry in Mathematics via the Geoboard, "Linear Graphs," Geo-Cards 25/1–3; 26/1–6, Walker. (Available from Fitzhenry & Whiteside)

Mathex: Graphing and Probability No. 6, "Graphing Equations," pp. 7–10, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 57
Workbook, page 101

PAGES 312–313

Objective

Given negative numbers, the child will be able to read them and interpret their use on a graph.

Preparation

To prepare for this lesson, you might introduce the concept of negative numbers by suggesting an imaginary situation of receiving money and paying bills. For instance, ask the children to suppose that all of them have received a fifteen-dollar cheque. Continue with the idea that some receive another cheque, some receive a bill, some a few bills, and so on. Use a sample name and record an imaginary situation. Lead children to the use of the positive and negative numbers. Thus, if Tim receives the fifteen-dollar cheque, a bill for ten dollars, another fifteen-dollar cheque, and a bill for twenty dollars, his record would look like this:

15
$^{-}10$
15
$^{-}20$

Children might figure out how much money he has after each step:

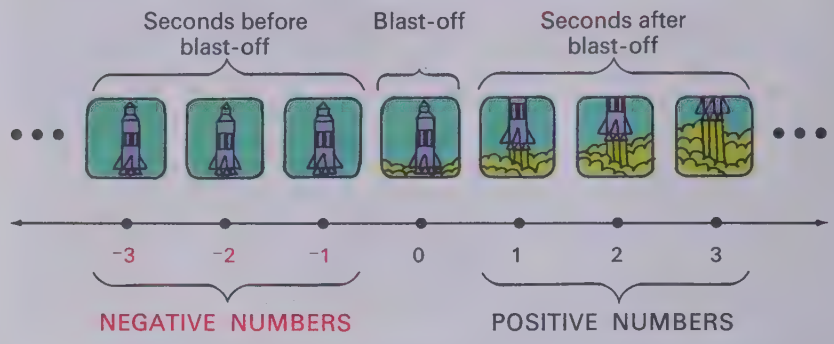
(1) $15
(2) $5
(3) $20
(4) 0



Can you graph negative numbers?

Discussing the Ideas

Eric watched as a moon rocket launching was shown on television. He drew a number line to show how **negative** and **positive** numbers can be used to describe the seconds **before** and **after** blast-off.



We read $^{-}3$ as "negative three." We read $^{-}2$ as "negative two."

1. Can you write and say at least five more negative numbers?
 Answers will vary. Samples: -4, "negative four"; -5, "negative five"
2. Can you think of some other times when you could use negative numbers? Sample answers: To show temperatures below zero; to show amounts of indebtedness; to describe topographical features below sea level
3. Maria and Nancy played a game. Maria's score was $^{-}7$ points, and Nancy's score was 5. Who do you think won the game? Why? Nancy, because any positive number is greater than a negative number.
4. If you use positive numbers to show an amount of money saved, what will negative numbers show? Amount of money spent.
5. If you use negative numbers to show the number of steps you walk toward the north, what will positive numbers show? Number of steps toward the south.

312

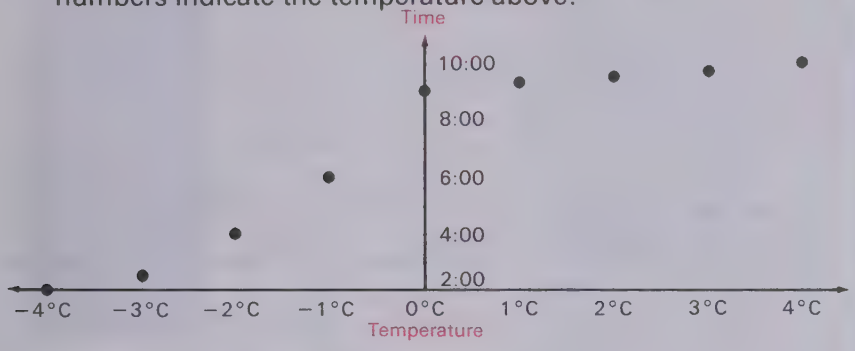
Discussion

Your introduction of negative and positive numbers should be clarified by a study of the text material. In this section the child learns to think of a negative number as the opposite of a positive number, as illustrated in the number line. Point out that $^{-}3$ is the same distance from zero as 3, but the direction of its location is opposite that of 3.

The exercises apply the concept of negative number to a variety of situations. Expand each of these according to the need and interest of the children. For example, exercise 5 might be expanded into a classroom game.

Using the Ideas

1. The graph below shows the approximate changes in temperature on a cold January morning. Negative numbers indicate the temperature below freezing and positive numbers indicate the temperature above.



A What was the temperature at 4:00?
What are the co-ordinates of that point? −2° c, (−2°c, 4:00)

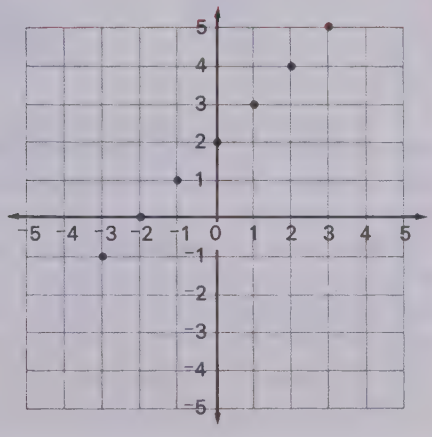
B What was the temperature at 9:00?
What are the co-ordinates of that point? 0°c, (0°c, 9:00)

C What do co-ordinates (−1°C, 6:00) tell you?
That it was 1° below zero at 6:00.

★ 2. Complete the function table. The graph may help you.

Function Rule: Add 2

Input	Output
3	5
2	4
1	
0	
−1	
−2	
−3	





Using the Exercises

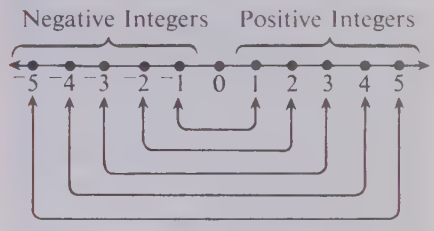
You might choose to use exercise 1 on page 313 as the basis for further discussion. You might also have children work in groups of two or three on both exercise 1 and exercise 2, and have them record their answers so that they can be shared when all have finished. When the exercises have been completed and the children discuss them, continue to stress that each negative number is the opposite of a positive number. Use the graphs of the negative numbers to emphasize this point.

Assignments (page 313)*
Minimum: 1. Average: 1.
Maximum: 1–2.

Mathematics

The two important mathematical concepts presented in this lesson are the introduction of negative integers and the use of these integers to extend graphing to the entire plane. Until now, graphing has been confined to one fourth of the plane.

A convenient way to think of the negative integers is to consider them "reflections" of the whole numbers on the number line.



The diagram shows how to pair each positive integer (whole number) with a corresponding negative integer. As shown on the number line, we consider the negative integers to be less than zero, which immediately suggests many applications for their use. In mathematics, negative numbers are solutions for equations of the following type:

$$4 + n = 0$$
$$n = {}^{-}4$$

Integers are employed in this lesson as co-ordinates which indicate various locations on the plane.

Resources for Active Learning

Madison Project: *Discovery in Mathematics: A Text for Teachers*, "Numbers with Signs," pp. 71–74, Addison-Wesley.

Math Activity Cards, "Addresses," B1, C1, Macmillan.

Mathex: Graphing and Probability No. 6, "Whole Numbers to Integers," pp. 10–11; "Positive and Negative Numbers," pp. 13–15, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Graphs Leading to Algebra* 2, "Coordinates Using the Integers," pp. 37–43, Wiley.

Teaching Aids for Elementary Mathematics, "Graphing Pictures," pp. 132–137, Holt, Rinehart and Winston.

PAGES 314–315

Objectives

The child will demonstrate his ability to work with the concepts developed in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

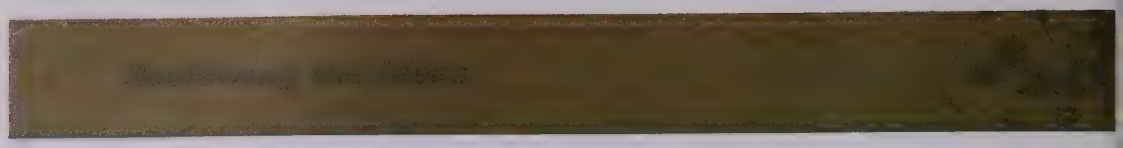
Preparation

Materials

graph paper

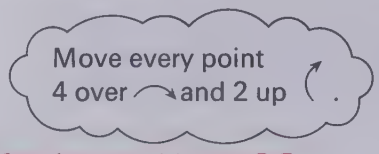
Depending on the needs of your class, you might use this preparation time to review algorithmic skills of multiplication and division or equivalent fractions. For example, write pairs of fractions on the chalkboard and ask children to tell whether or not they are equivalent.

$(\frac{1}{2}, \frac{2}{3})$	*(no)*	$(\frac{3}{12}, \frac{1}{4})$	*(yes)*
$(\frac{2}{4}, \frac{4}{8})$	*(yes)*	$(\frac{8}{14}, \frac{1}{2})$	*(no)*
$(\frac{3}{5}, \frac{3}{6})$	*(no)*	$(\frac{7}{21}, \frac{1}{3})$	*(yes)*
$(\frac{3}{9}, \frac{1}{3})$	*(yes)*	$(\frac{8}{24}, \frac{1}{6})$	*(no)*

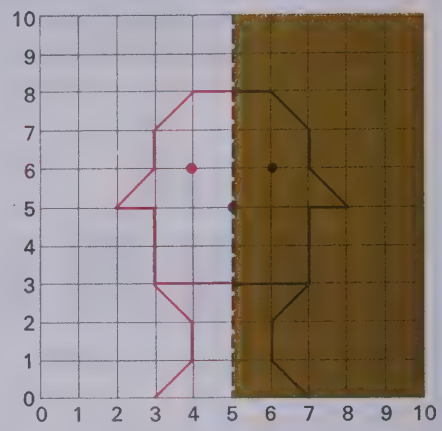


1. A Copy this half of the picture on your graph paper. See graph at right. Then graph the other half.

 B On your graph paper, show the final position of the complete picture after this move:

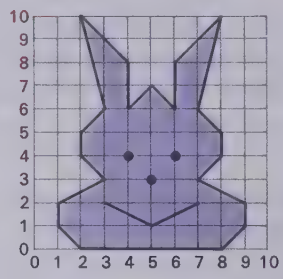


See Answer, T.E. page 315.



2. Use your graph paper to make a larger figure similar to this one.

Graphs will vary.



3. Group the children in your class according to color of hair — blond, brown, black, or red. Make a graph to show the number of children in each group.

Graphs will vary from class to class.

4. Graph these points and connect them in order to form a very common message. See Answer, T.E. page 315.

(1, 4) → (3, 7) → (3, 9) → (2, 7) → (3, 4) → (4, 5) → (3, 6) → (5, 6) → (6, 4) → (7, 6) → (7, 2) → (6, 1) → (6, 2) → (9, 5) → (8, 6) → (8, 5) → (9, 4) → (10, 4)

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Discussion

Before you assign page 314, you might read through the problems to be sure children understand the directions. Point out the significance of the "over" arrow in exercise 1B; it indicates a move to the right. Note that exercise 3 requires that the children gather information before they make the graph. A child may easily do this independently, from personal observations, but unless prior agreement is reached, you can anticipate some varying data regarding the number of children who have hair of a given color, depending on interpretation of the terms "blond, brown," etc. Have an adequate supply of graph paper available and give the children ample time to complete their work. Since both this page and page 315 include a large amount of material, you might plan to spend at least two days on this lesson.

Keeping in Touch with

Multiplication
Division
Fractions
Number theory

1. Find the products.

A $72 \times 9 = 648$ B $86 \times 8 = 688$ C $43 \times 12 = 516$

D $128 \times 27 = 3456$ E $214 \times 35 = 7490$ F $436 \times 124 = 54064$

2. Find the quotients and remainders.

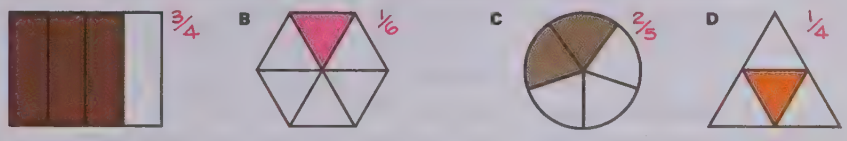
A $7\overline{)86}$ 12 R2 B $9\overline{)388}$ 43 R1 C $6\overline{)259}$ 43 R1

D $32\overline{)485}$ 15 R5 E $84\overline{)1492}$ 17 R64 F $69\overline{)2346}$ 34

3. Which number in the set {1, 2, 3, 4, 6, 9} is **not** a factor of A 12? 9 B 18? 4 C 24? 9

4. Write a fraction that shows the part of each region that is shaded.

A $\frac{3}{4}$ B $\frac{1}{6}$ C $\frac{2}{5}$ D $\frac{1}{4}$



5. Write three more fractions in each set of equivalent fractions.

A $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \ldots\}$ $\frac{8}{12}, \frac{10}{15}, \frac{12}{18}$ B $\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \ldots\}$ $\frac{4}{20}, \frac{5}{25}, \frac{6}{30}$ C $\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \ldots\}$ $\frac{28}{40}, \frac{35}{50}, \frac{42}{60}$

6. Give the lowest-terms fraction that is equivalent to each fraction.

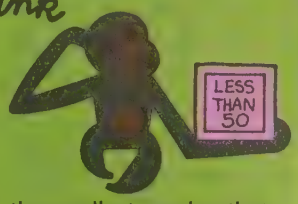
A $\frac{5}{10}$ $\frac{1}{2}$ B $\frac{2}{8}$ $\frac{1}{4}$ C $\frac{9}{12}$ $\frac{3}{4}$ D $\frac{14}{16}$ $\frac{7}{8}$ E $\frac{75}{100}$ $\frac{3}{4}$ F $\frac{10}{15}$ $\frac{2}{3}$ G $\frac{15}{25}$ $\frac{3}{5}$



You are invited to explore

ACTIVITY CARD 16
Page 355

think



I'm the smallest number that
Will get you less than fifty
If you want to take away
From seven more than sixty.

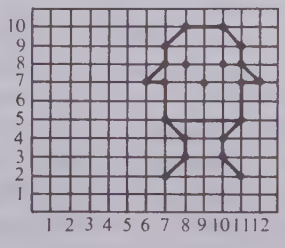
WHO AM I? 18

Using the Exercises

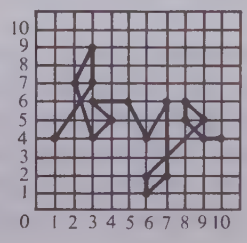
Page 315 is a fairly complete review of multiplication and division of whole numbers and of fractional concepts. When the children have finished these exercises, discuss any topics which caused difficulty. If some children show a lack of understanding of a particular topic, arrange to give them special help.

Let those who solve the *Think* problem explain it to the whole class. Once children interpret the rhyme, they should not have difficulty in finding the answer.

Answer, exercise 1B, page 314



Answer, exercise 4, page 314



Follow-up

Exercise 4 on page 314 might inspire children to write and graph messages of their own. Some may choose to print letters of a message on graph paper and then write out the co-ordinates for a classmate to graph in order to find the message. Encourage children to conclude this chapter with this activity or any other graphing activity which they found particularly interesting. For example, you might find that children would enjoy imagining a grid on the classroom floor and walking from one location to another according to the directions of a classmate. You could use a roll of adding machine paper to form the horizontal and vertical axes in the centre of the classroom. If your classroom floor is covered with square tiling, you might agree to use a certain number of tiles for every number. Children might give directions such as: take 3 steps north and 4 steps east; or, take 1 step south and 0 steps west; and so on. You might make this into a competitive game by dividing the class into teams, pointing out certain locations (at the door, by the closet, etc.) which a member must touch to score a point. If you use this idea, you will probably want each team to give directions to their own team member, trying to get him to the location as quickly as possible, using only the north or south and east or west instructions.

Resources for Active Learning

Developmental Math Cards, I[4]12, Addison-Wesley.

Freedom to Learn, pp. 50–52, Addison-Wesley.

Inquiry in Mathematics via the Geoboard, Geo-Cards 24/1–7, Walker. (Available from Fitzhenry & Whiteside)

Math Activity Cards, B4,5; C10, Macmillan.

Mathex: Graphing and Probability No. 6, p. 3 (pupil pages 4–13); pp. 3–7, Encyclopaedia Britannica Publications Ltd.

Workbook, page 102 (Use with text page 314.)

General Objectives

To introduce fractional numbers

To provide a transition from sets of equivalent fractions to fractional numbers

To introduce the fractional number line

To establish the fact that each whole number is also a fractional number

To introduce equality and inequality for fractional numbers

To informally introduce addition of fractional numbers

To introduce mixed numerals

To provide experience with word problems containing fractional numbers

The early lessons of the chapter are devoted to material which helps the children begin a linear left-to-right orientation in fraction concepts as preparation for locating fractional numbers on the number line. The concept of relating each set of equivalent fractions to exactly one length of a strip or one point on the number line, and thus to one fractional number, is introduced and stressed. The number line is the chief tool for providing experiences with these new fractional-number concepts.

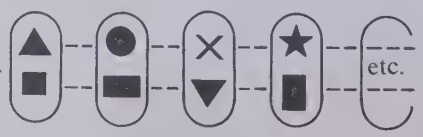
After relating fractional numbers to points on the number line, the concept of equality is introduced. We emphasize that any fraction from the set of equivalent fractions for a given fractional number can be used to name the number; thus the statement $\frac{3}{9} = \frac{1}{3}$ means that $\frac{3}{9}$ and $\frac{1}{3}$ are symbols for the same fractional number.

Following the work on equality of fractional numbers, the children learn to relate fractional numbers to the number line, and to lengths of the strips associated with a particular unit. Their attention is focussed on the lowest-terms fraction as the name most often used for a fractional number.

Inequality concepts for fractional numbers are followed by lessons concerning fractional numbers that are greater than one. Then, addition of fractional numbers is introduced intuitively through experiences with the strips and the number line, followed by an introduction to fractional numbers that are whole numbers and to mixed numerals. The final part of the chapter is concerned with fractional numbers in measurement, more material on addition, and finally, some fractional-number story problems.

Mathematics

Building the fractional-number concept from fractions (number pairs) is much like building the cardinal-number concept from sets. In the earlier texts of this series, we began to develop the concept of cardinal numbers by establishing an understanding of *equivalent sets*. Next, children were led to think about classes of equivalent sets, and then about the *cardinal number* associated with each class. For example, they learned to associate cardinal number 2 with the *class* of sets that are equivalent to:



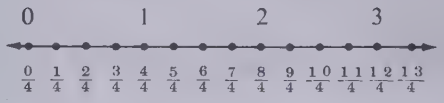
The equivalence of the sets in this class is illustrated as follows:



Just as we abstracted cardinal number 2 from the class of sets illustrated above, so can we abstract fractional number one half from the class of equivalent fractions.

$$\{\tfrac{1}{2}, \tfrac{2}{4}, \tfrac{3}{6}, \tfrac{4}{8}, \tfrac{5}{10}, \tfrac{6}{12}, \tfrac{7}{14}, \ldots\}.$$

As you saw in Chapter 12, we can start with any lowest-terms fraction and imagine an infinite set of equivalent fractions with which we associate a fractional number. The number line is very effective for illustrating these ideas. For example, the number line charts on page 323 of the text demonstrate that for each set of equivalent fractions there is one and only one fractional number. The following number line, which shows fourths, helps give meaning to improper fractions and corresponding fractional numbers.



The number line also helps illustrate that each whole number can be considered a fractional number. That is, the whole numbers are a subset of the fractional numbers.

Although we think of exactly one number for a given point on the number line (and exactly one point for a given fractional number), we may name the point with any fraction from the set of equivalent fractions for that point.

The fact that each fractional number has many different names leads to the concept of equality for fractional numbers. When we write

$$\tfrac{1}{2} = \tfrac{4}{8},$$

we are indicating that $\frac{1}{2}$ and $\frac{4}{8}$ name the same fractional number.

> Two fractions name the same fractional number if and only if they are equivalent.

The use of the strips to represent length in the first lesson of this chapter involves a very subtle transition from the idea of fraction to that of fractional number. Previously, in the chapter on fractions, the strips were used exclusively with ordered pairs of whole numbers; that is, the ratio concept was very much in evidence when the strips were used to provide experiences with fractions. In the first lesson of this chapter, the strips are used in much the same way as they were used to introduce fractions, with the exception of introduction of the concept of length. We take the point of view that length is a number; hence, when we say, "The fraction

one half compares the red strip to the unit" (in this case, the purple strip), we are working with the fraction concept. When we use the language, "The length of the red strip is one-half unit," we are moving from the concept of one-out-of-two parts, to the concept of a number which indicates the length of the red strip. Experiences are then provided to develop the idea that the red strip has a definite length and that this length can be named by a complete set of equivalent fractions, generated from imagining the purple strip partitioned in different ways.

Having introduced the idea of using fractions to represent length, we can move to the slightly broader concept, on page 318, of looking at sets of equivalent fractions and imagining a given length on a ruler. Again, the purpose is to provide a smooth transition from the concept of a set of equivalent fractions to that of fractional number.

Teaching the Chapter

Materials

Colored strips
Demonstration number line
Maps of Canada, one showing time zones and one showing distances between major cities
Measuring cup showing graduated fractional parts
Metre stick
Objects to group or cut apart, to illustrate fractional-number concepts
Overhead projector and transparencies (if available)
Rulers (1 per child)
Tagboard
Tape measure, 2 m, steel or cloth
Toothpicks (1 box)
Wooden cubes, 2 cm on an edge

Vocabulary

mixed numeral fractional number

The use of materials in this chapter centres on the colored strips and the number-line pictures. It would be convenient to keep a large display number line in a prominent location, since you will want to refer to the number line frequently. The basic materials for the children to manipulate are their sets of colored strips. The strips provide a great variety of examples of situations which relate to fractional-number concepts. At times you may feel the need to use demonstration materials of cutout objects to illustrate fractional-number concepts. The maps suggested in the Materials list are to be used with the time zones on page 321 and the airline distances on page 346.

You will notice that we use the term *mixed numeral* rather than the term *mixed number* to refer to numbers represented by symbols such as $3\frac{1}{2}$, $4\frac{1}{4}$. We make this distinction because it is the symbol that is "mixed," not the number.

Lesson Schedule

Plan to cover this material in about three weeks. Naturally, you will want to adjust your time schedule to the needs and abilities of your children and according to the time remaining in the school year.

Evaluation of Progress

Evaluating the children's achievement may be more difficult for this chapter than for any other chapter in the text. We do not expect the children to achieve thorough mastery of any particular skill in this chapter. Your primary consideration should be the children's understanding of the general concept of fractional numbers, their relation to whole numbers, and their location on the number line. Associated with these are concepts of equality and inequality for fractional numbers.

This kind of evaluation may be made best by considering the children's responses and participation during a given discussion.

While teaching this chapter, keep in mind that we do not expect all children to master the fractional-number concepts completely on this first exposure. Do not be overly concerned if some children have difficulty grasping the idea that there is exactly one number and exactly one point on the number line associated with each complete set of equivalent fractions. These children can continue to study fractional numbers without fully abstracting the idea from the class of equivalent fractions.

Resources for Active Learning

General Activities

[The resources listed in Chapter 12 will also be useful throughout this chapter. A few are repeated for emphasis.]

Developmental Math Cards, I[25], Addison-Wesley

Mathex: Numeration No. 7, "Meaning of Fractions," pp. 34–43 (pupil pages 27–41), Encyclopaedia Britannica Publications Ltd. Activities and assignment cards using the body, shapes, charts, and linear, liquid, and area measurement.

Nuffield Project: *Computation and Structure* 3, "Fractions," pp. 70–78, Wiley

Manipulative Devices

Modulated Cut-outs (Encyclopaedia Britannica Publications Ltd.)

Unifix Fractions Kit (Educational Teaching Aids)

Commercial Games

Action Fractions (Math Media)

Come Out Even (Holt, Rinehart and Winston)

Competitive Fractions (Selective Educational Equipment)

Fraction Dominoes (Responsive Environments Corp.; Selective Educational Equipment)

Fractions Are as Easy as Pie (Milton Bradley; school supplier)

PAGES 316–317

Objective

Given a unit and an object pictured as a part of the unit, the child will be able to give the length of the object as a fractional number by giving a set of equivalent fractions.

Preparation

To prepare for this lesson, you might have the children review the use of fractions to make comparisons of strip lengths. For example, have the children use the strips and ask them to show the two strips which match the fraction $\frac{1}{4}$, or the fraction $\frac{3}{5}$. Or you might ask them to name a fraction which shows how the 4-strip compares to the 6-strip, or the 4-strip compares to the 5-strip, and so on. Use the preparation period simply to review how a comparison of two strips may be expressed by a fraction.

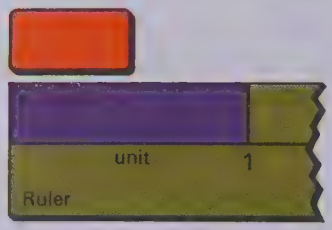
Investigation

Although the strips are referred to in this investigation, it is not necessary for the children to manipulate them. Read the investigation material with the children. It is important that children realize that the unit of measurement being discussed is the purple, or 4-strip, and that the red strip is being discussed in comparison to this purple-strip unit. Also, note that the ruler pictured is not a centimetre ruler but only indicates the purple strip as the unit. The challenge the children are given is to relate several fractions to the one length being discussed. The pictures guide them in building the set of equivalent fractions for the number one half. However, some children may realize that fractions such as $\frac{25}{50}$ or $\frac{50}{100}$ or $\frac{100}{200}$ are also names suitable for representing the length of the red strip in this case.

14 Fractional Numbers

● *Can fractions be used for length?*

Investigating the Ideas



The fraction $\frac{1}{2}$ compares the red strip with the unit. The **length** of the red strip is **one-half** unit.

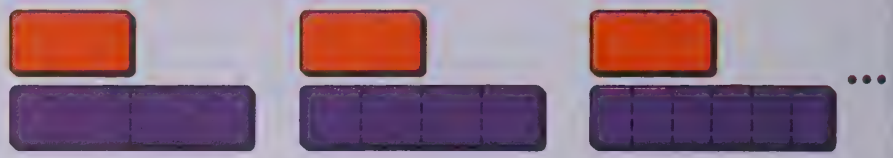
For a given unit, the length of the red strip does not change, but different fractions can be used to represent it.

? Can you find 8 fractions for the red strip when the purple strip is the unit?

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \ldots\}$

See Investigation.

These pictures may help you.





Discussing the Ideas

1. How many fractions are there that compare the red strip with the purple strip? Infinitely many; that is, the set $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\}$ continues without end.
2. How can this statement help you understand the Investigation?

 Equivalent fractions represent the same length.

See Discussion.

3. Can you find a set of equivalent fractions for the length of the light green strip (when the purple strip is the unit)?



$\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \ldots\}$

Discussion

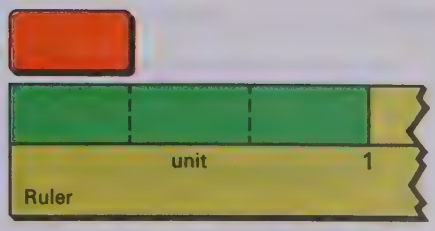
Have several children give the fractions which they listed in the investigation. Note that the number assigned to the red strip is determined by the given unit. Different fractions may be generated by thinking about this unit differently, as suggested by the investigation pictures. Hence, the number for the length may be represented by any fraction from a set of equivalent fractions. As you discuss exercise 1, therefore, children should begin to realize that there is an endless number of fractions we could use to name the comparison of the red strip to the purple strip.

A discussion of exercise 2 should focus on the idea that, once a unit is established, the length of an object is determined. This length may then be expressed with many different names. Just as $3 + 1$, 2×2, $12 - 8$ all name the number four, so the fractions $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\}$ all name the number one half.

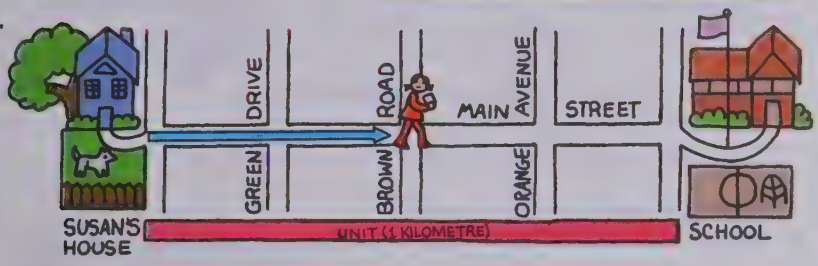
In exercise 3, develop the idea of how another length given in the same unit may be represented by many equivalent fractions. It would be helpful to present other examples. For example, ask the children to name some fractions for the length of the purple strip when the dark green strip is the unit ($\frac{4}{6}$ or $\frac{2}{3}$).

Using the Ideas

1. Give a set of equivalent fractions that tell the length of the red strip when the dark green strip is the unit.
$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \ldots\}$



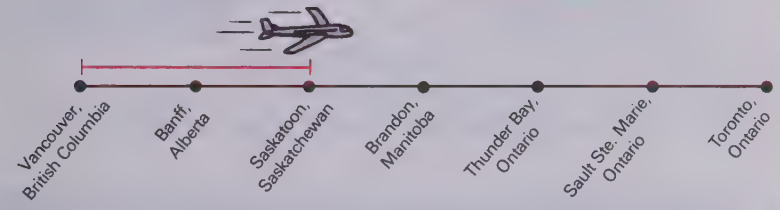
2.



A It is 1 kilometre from Susan's house to school. If the kilometre is the unit, give a fraction that tells the distance Susan has walked. $\frac{1}{2}$ (or equivalent)

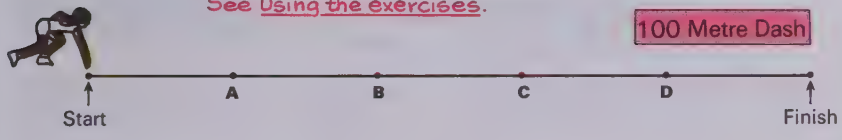
B Give another fraction for this distance. $\frac{2}{4}$ (or equivalent)

3.



If the complete trip from Vancouver to Toronto is the unit, give two fractions that represent the distance the plane has flown. $\frac{1}{3}, \frac{2}{6}$ (or equivalents)

4. Write a "fraction question" about this picture.
See Using the exercises.



317



Mathematics

In this book, we use the term *fractional numbers* to refer to the set of positive numbers and zero which can be represented by fractions. Each whole number n can be thought of as a fraction in the form of $\frac{n}{1}$. The set of *rational numbers* contains the fractional numbers and their negatives, or opposites. Specifically, the set of rational numbers consists of all the numbers of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. That is, we do not permit the denominator of a fraction to be zero, for this would be equivalent to dividing by zero.

Follow-up

Suggest that the children use a strip of paper about 10, 12, or 15 cm long as a unit to measure various objects in the classroom. Then ask them to give as many equivalent fractions as they can for the measure of each object. They can do this by folding the paper into various fractional parts. For example, if the desk top is 4 units long and the paper is folded into thirds, the length of 4 units may be called $\frac{1\,2}{3}$; if the paper is folded into fourths, the length of 4 units may be called $\frac{1\,6}{4}$.

Workbook, page 103

Using the Exercises

You might choose to use page 317 as a basis for class discussion or as a small-group activity. Note that these exercises continue to build on the idea of length, preparing the child for consideration of fractional numbers in relation to a number line. For exercise 4, encourage children to write three or four "fraction questions," such as, "When the runner is at point B, what fraction of the race has he run?"

Assignments (page 317)

Minimum: 1–4, oral. Average: 1–4.
Maximum: 1–4.

PAGES 318–319

Objective

Given a set of equivalent fractions, the child will be able to relate the fractional number the set represents to a point on a number line.

Preparation

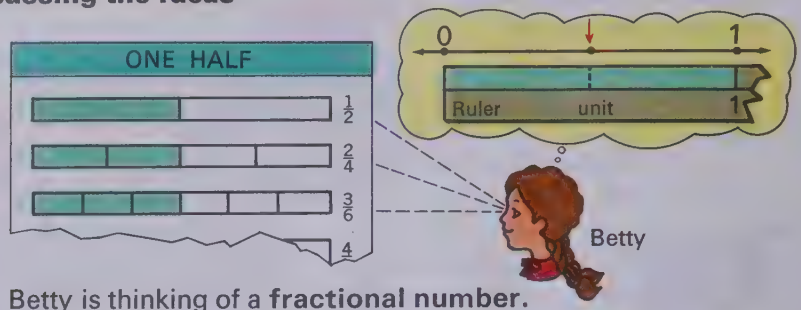
To prepare for this lesson, you might review building sets of equivalent fractions. For example, write the fraction $\frac{1}{4}$ on the chalkboard and ask children to give other fractions which may be used in place of $\frac{1}{4}$, such as $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, ... Continue similarly with other fractions, such as $\frac{1}{3}$, $\frac{3}{5}$, $\frac{3}{8}$, $\frac{2}{5}$, and so on. However, remember to keep the preparation time brief.



Let's explore fractional numbers.

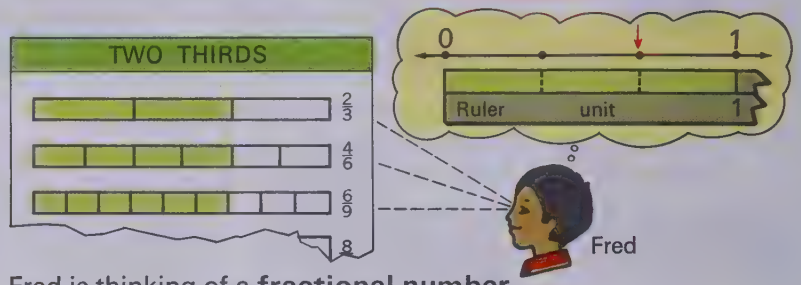
Discussing the Ideas

1.



Betty is thinking of a **fractional number.** How do you think she found the point on the number line for this number? See Discussion.

2.

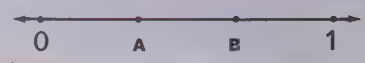


Fred is thinking of a **fractional number.** How do you think he found the point on the number line for his number? See Discussion.

3. For each set of equivalent fractions, we think of just one number.

We call this number a fractional number

For this set of equivalent fractions, → $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$ we think of **one fractional number** and **one point** on the number line. Which point? Explain.



Point A, because it is ⅓ of the unit shown.

318

Discussion

The main purpose of this lesson is to help children think of a set of equivalent fractions as a single number and to identify that number with a point on the number line.

Ask the children to study the one-half chart. Explain that the torn bottom edge of this chart means that they should think of the chart as going on indefinitely. Observe with the children that the girl is looking at the chart and thinking about a point on the number line.

Have them note that the point marks one half of the length of the unit. Point out that the girl is looking at the chart labelled "one half" and she is thinking about exactly one number.

Continue a similar discussion with exercise 2. Again, the boy is looking at a number chart for equivalent fractions and thinking about one number and one point on the number line.

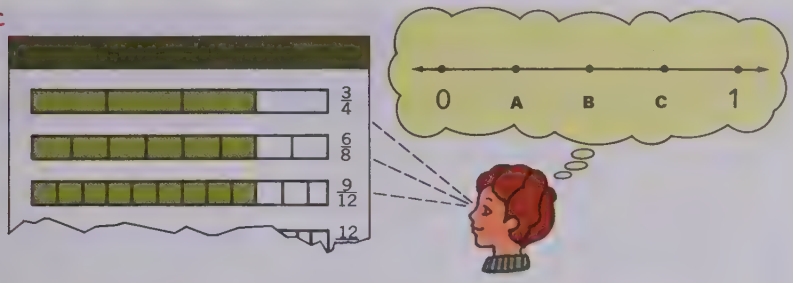
As you discuss exercise 3, remind the children that the numbers they studied previously are called *whole numbers* and that now they are going to study numbers that are called *fractional numbers.*

Study with the children the diagram showing the set of fractions $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$ and its location on the number line. Observe with the children that for this set of equivalent fractions they are to think of just one number and one matching location on the number line.

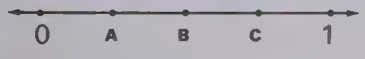
Using the Ideas

Answer A, B, or C to give the point on the number line for each set of equivalent fractions.

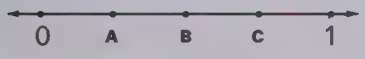
1. C



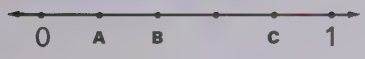
2. $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \ldots\}$ A



3. $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots\}$ C



4. $\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \ldots\}$ A



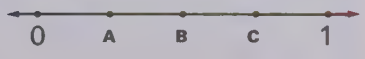
5. $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots\}$ C



6. $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \ldots\}$ C



7. $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\}$ B



8. $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \ldots\}$ B



9. $\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \ldots\}$ A



Using the Exercises

Assign the exercises on page 319 as independent work. When the children finish, discuss their answers with them. During this discussion, stress the idea that in each exercise one set of equivalent fractions gives many names for a single number and matches one point on the number line. This idea will be emphasized throughout the next three lessons.

Assignments (page 319)

Minimum: 1–9. Average: 1–9.
Maximum: 1–9.

Mathematics

This lesson contains the heart of the mathematical development of the fractional-number concept. The charts on page 318 illustrate the idea of a class of equivalent fractions. The fact that these fractions are equivalent is illustrated by showing that they represent the same part of a bar, and the fact that each class continues indefinitely is suggested by the torn-off bottom edge of each chart. The picture of a child looking at a chart and then thinking of exactly one point on the number line is intended to indicate the general idea of fractional number; that is, there is exactly one quantitative idea associated with each chart (set of equivalent fractions). We indicate this association with a point on the number line to show that we consider this idea as a number in much the same way as we have thought about whole numbers. You will note that up to this point we have reserved the number line for whole numbers; we have not put fractions on the number line. In future lessons we shall indicate numbers by using fractions on the number line.

Follow-up/"Match My Fraction"

If you wish to provide further review of familiar equivalent fractions, you might have children play "Match My Fraction." Direct the children to think of a fraction, preferably one with a denominator of two, three, four, or eight. Ask a child to give his fraction, and then call upon a second child to give another fraction for the same fractional number. He, in turn, gives a fraction from a different set and calls on someone else to name an equivalent fraction. This type of activity may be handled as a team competition, with a member from one team giving a member from another team a fraction to match. In this case, every time a team member responds with a correct match, his team scores a point. In order to keep the fractions simple enough, you might prepare cards to show lowest-terms fractions with simple denominators.

PAGES 320–321

Objective

Given a set of equivalent fractions and a number line divided appropriately with points, the child will be able to match a set of fractions with the corresponding point on the number line.

Preparation

On the chalkboard, draw a number line approximately 1 metre long, allowing 75 cm without labels between 0 and 1.

Ask the children to estimate the location of the point for a set of fractions you give them. For example, give them the set $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \ldots\}$ and ask a child to mark on the number line a point which he thinks would show this set of fractions. Do not expect a high degree of accuracy in these estimates, but help the children if the positions they mark are wholly unreasonable.

Can you match fractions with points?

For each set of equivalent fractions in the table below there is a fractional number. Give the point on the number line for that number.

	Set of fractions for the number	Which point on the number line goes with the number?
1.	$\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\}$ B	0 A B C D 1
2.	$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$ B	0 A B C 1
3.	$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$ C	0 A B C 1
4.	$\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\}$ B	0 A B C 1
5.	$\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$ C	0 A B C 1
6.	$\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \ldots\}$ C	0 A B C 1
7.	$\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \frac{12}{28}, \ldots\}$ B	0 A B C 1
8.	$\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\}$ C	0 A B C 1
9.	$\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$ B	0 A B C 1
10.	$\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$ B	0 A B C 1
11.	$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$ B	0 A B C 1

Discussion

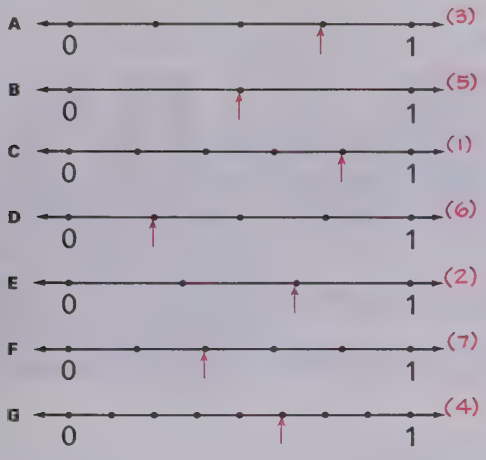
If necessary, work through one or two exercises with the children. Point out that they can find the number of parts into which the segment is divided between zero and one and then look for the fraction that corresponds to this number of parts. For example, in exercise 5 the children should observe that the segment is divided into eighths, and they should find the point for the fraction $\frac{6}{8}$. Or in exercise 6, they should see that the segment is divided into tenths, and they can locate the point for the fraction $\frac{8}{10}$.

Explain to the children that in exercise 12 they need to figure out a fraction for the point indicated by the arrow and then look for that fraction within one of the numbered sets. They can be assured that when they find the given fraction in the set, the entire set of equivalent fractions belongs to that fractional-number point.

In exercise 13, the children should observe that parts A, B, C, and D all indicate the same point. However, the children must give a different fraction for each. This exercise develops background for the idea that we can use many names for the same fractional number and the same point on the number line.

You might use the *Think* problem

12. Match the sets of fractions with the number-line pictures.



(1) $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots\}$

(2) $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \ldots\}$

(3) $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots\}$

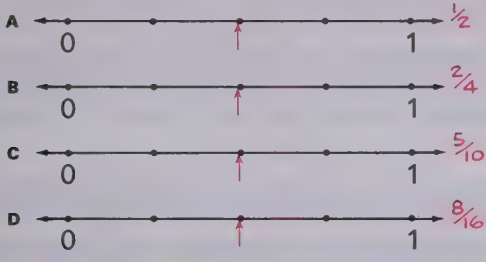
(4) $\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \ldots\}$

(5) $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\}$

(6) $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \ldots\}$

(7) $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \ldots\}$

13. Choose the fraction for the point over the red arrow.



$\frac{6}{8}, \frac{2}{3}, \frac{1}{2}, \frac{1}{4}$

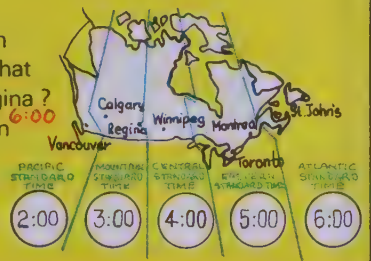
$\frac{1}{3}, \frac{2}{4}, \frac{5}{8}, \frac{7}{7}$

$\frac{5}{10}, \frac{3}{8}, \frac{1}{5}, \frac{6}{8}$

$\frac{4}{9}, \frac{3}{5}, \frac{8}{16}, \frac{7}{8}$

think

1. From the map, you can see that it is 2:00 in Vancouver when it is 4:00 in Winnipeg. What time is it in Montreal when it is 4:00 in Regina ? 6:00
2. When it is 7:30 in Calgary, what time is it in A Vancouver, B Toronto, C Winnipeg ? 6:30 9:30 8:30
3. What time zone are you in if you are eating lunch when the children in Vancouver are just starting school ? Eastern



as stimulus for a more detailed study of time and time zones. For example, the children might decide what time would be most convenient to call relatives who live in other time zones, or the hour at which they would need to board a plane or train in order to reach a distant city at a certain hour according to that city's local time.

Follow-up

To provide children with further practice similar to the exercises in this lesson, you might prepare a worksheet similar to the sample below.

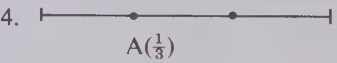
Match an equivalent set from Part B with a point marked A from Part A.

Part A

1. 

2. 

3. 

4. 

5. 

6. 

7. 

Part B

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$

$\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \ldots\}$

$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

$\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \ldots\}$

$\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\}$

$\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$

Workbook, page 104

Assignments (pages 320–321)

Minimum: 1–13. Average: 1–13.
Maximum: 1–13.

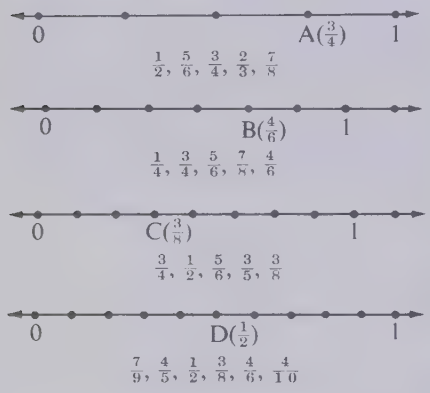
PAGES 322–323

Objective

Given the portion of the number line between 0 and 1 divided by points into parts of equal length, the child will be able to give one or more fractions that name the fractional number for a given point.

Preparation

Draw on the chalkboard three or four number lines and mark points of familiar fractions on them. Beside each, write four or five fractions, one of which matches the point that is labelled on that number line. Ask the children to choose which fraction matches the point (indicated in parentheses below).



Investigation

The preparation suggestions provide a review of identifying points and fractions. The investigation broadens this idea and develops the concept that *many* fractions may be names for the number that corresponds to a point and that such fractions all belong to the same equivalence class. The children should be encouraged to locate the student who gave the incorrect answer and then answer the investigation question independently.

It would be helpful to provide other exercises on the chalkboard such as:

"Give at least three names for the fractional number that matches each marked point."



Stress the fact that these three fractions are equivalent; this fact is central to the main point of the lesson.



How do we name fractional numbers?

Investigating the Ideas

Find the student who did not answer the teacher's question correctly. Tom



? Can you give some other fractions for this point? $\frac{4}{8}$, $\frac{7}{14}$, $\frac{8}{16}$, ...

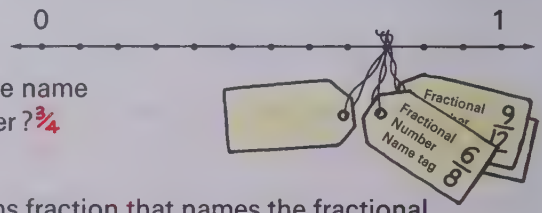
See Investigation.

Discussing the Ideas

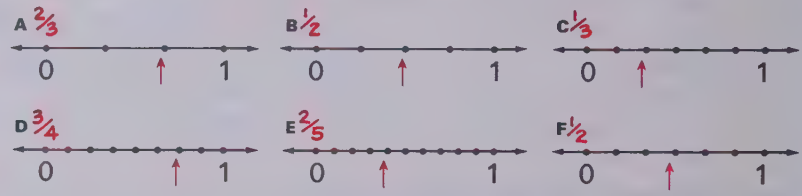
1. How can the statement below help you to find which student was wrong? See Discussion.

 Any fraction from a set of equivalent fractions can be used to name the fractional number for that set.

2. Although each fractional number has many names, we **usually** use the lowest-terms fraction to name the number. What do you think is on the name tag that is turned over? $\frac{3}{4}$



3. Give the lowest-terms fraction that names the fractional number for the point over the red arrow.

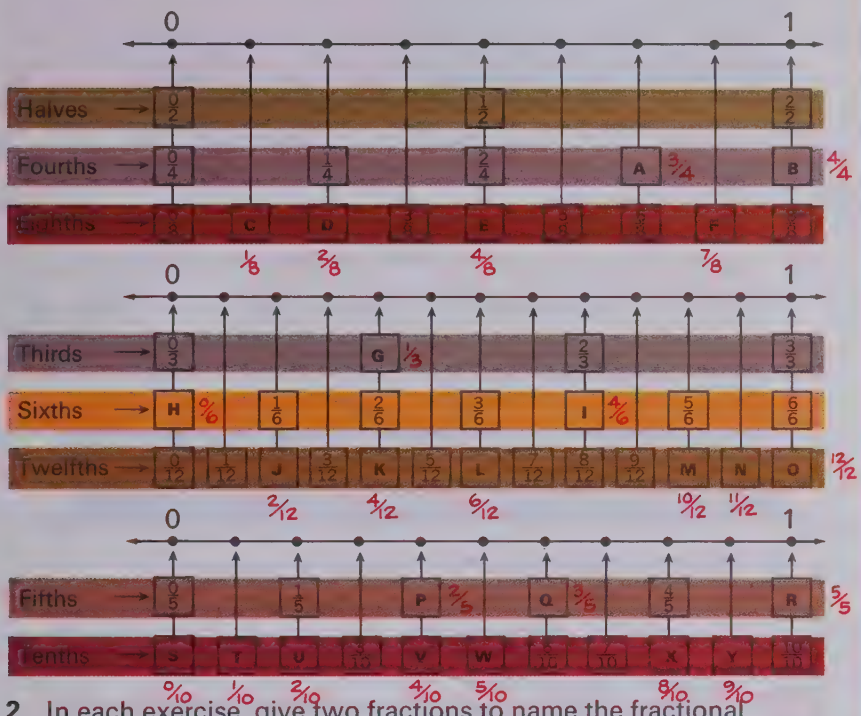


Discussion

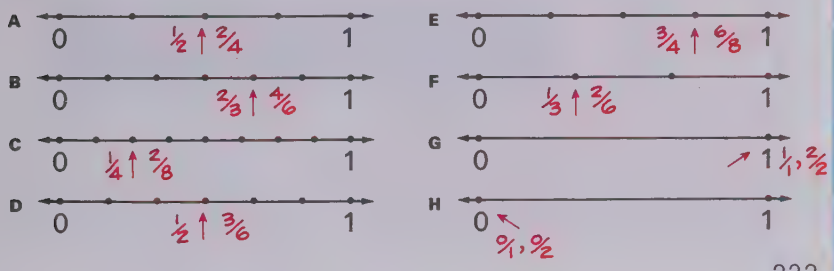
Write the fractions the children gave in answer to the investigation question. Ask them to name the lowest-terms fraction of the set, and to point out which child in the illustration gave the incorrect answer. As you discuss exercise 1, they should respond that since $\frac{3}{5}$ is not in the same set of equivalent fractions as $\{\frac{1}{2}, \frac{2}{4}, \ldots\}$, they know it does not name the same fractional number. Include in your discussion of exercise 2 other examples such as those suggested in the investigation, similar to exercise 3. Even though we want the children to become accustomed to using the lowest-terms fraction to designate a fractional number, make sure they realize that any fraction from the appropriate set of equivalent fractions is correct.

Using the Ideas

1. Just one fractional number goes with each point. Some of the fractions that name the fractional number are given. Give the missing fractions.



2. In each exercise, give two fractions to name the fractional number for the point over the red arrow.



323

Resources for Active Learning
Developmental Math Cards, H118, Addison-Wesley.

Workbook, page 105

Using the Exercises

On page 323 it may be helpful to work through parts A through G of exercise 1 with the children before having them complete the page on their own. This will give them an idea of what is expected in these exercises. Be sure to point out to the children the concept involved: they are using different fractions to name the points on the number line.

Following this discussion, have the children complete the exercises. When they have finished, allow time for checking papers and for further discussion. Take this opportunity to re-emphasize that they are learning to associate with a given point on the number line, not only a fractional number and a set of equivalent fractions, but any fraction from the set of equivalent fractions.

Assignments (page 323)
Minimum: 1, oral; 2. Average: 1–2. Maximum: 1–2.

PAGES 324–325

Objective

Given an equation such as $\frac{4}{8} = \frac{2}{4}$, the child will demonstrate his understanding that the fractions are names for the same fractional number by (1) giving the lowest-terms fraction, or (2) giving other fractions that are equivalent to them, or (3) relating both fractions to the same point on the number line.

Preparation

To facilitate children's work in this lesson, you might review the cross-product check for equivalent fractions. For example, write $\frac{2}{3}$ and $\frac{6}{9}$ with the loops



and review the procedure of cross multiplying to check equality. Give the children several examples for practice in using this check.

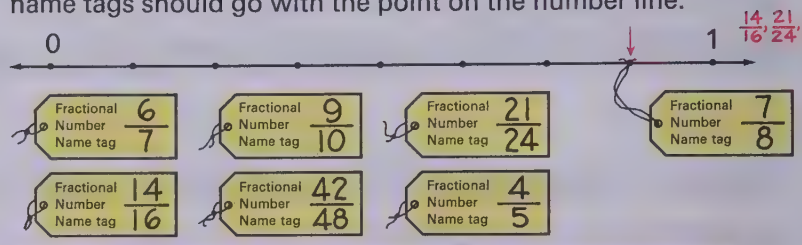


When do two fractions name the same number?

Discussing the Ideas

1. | If two fractions are equivalent, then they name the same fractional number. | If two fractions are not equivalent, then they do not name the same fractional number. |
|---|---|

Study the statements above. Then explain which other name tags should go with the point on the number line.



2. When we write "=" between two fractions, we mean that the two fractions name the same fractional number.

$\frac{2}{4}$ names this fractional number $\longrightarrow \{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$.

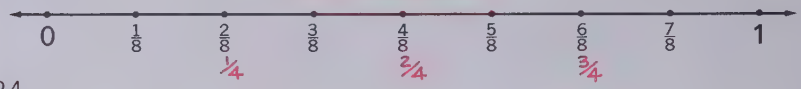
$\frac{3}{6}$ names this fractional number $\longrightarrow \{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$.

We write: $\frac{2}{4} = \frac{3}{6}$.

Study the explanation above. What equations could we write about the fractional number for the point in exercise 1? $\frac{7}{8}=\frac{14}{16}; \frac{7}{8}=\frac{21}{24}; \frac{7}{8}=\frac{42}{48}; \frac{14}{16}=\frac{21}{24}; \frac{14}{16}=\frac{42}{48}; \frac{21}{24}=\frac{42}{48}$

3. Study this statement. The lowest-terms fraction is the name often used for a fractional number.

How can you use this idea to relabel some of these points on the number line? See Discussion.

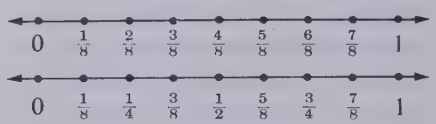


Discussion

In the previous lesson, children learned that a fractional number has many names, and each fraction which names a particular fractional number belongs to one and the same set of equivalent fractions. Though the basic concept is the same in this lesson, the presentation helps to give meaning to the concept of equality for fractional numbers.

As you discuss exercise 1, help the children realize that they should check each fraction to see whether it is equivalent to $\frac{7}{8}$. It would be helpful to have some children work out the equivalent-fraction check on the chalkboard. Exercise 2 introduces the use of the equal sign with equivalent fractions. Have children observe that they may write the equal sign only between *equivalent fractions*. In other words, only equivalent fractions name the same fractional number. Therefore, to indicate equality between two fractions is to indicate that the two fractions do name the same fractional number.

In exercise 3, make sure children realize that although the lowest-terms fraction is the usual way of naming a fractional number, it is not incorrect to label all of these points using eighths. You might draw the following number lines on the chalkboard for reference during the discussion of this exercise.



Using the Ideas

1. In each exercise, tell whether or not the two fractions name the same fractional number.

A $\frac{1}{2}$ $\frac{2}{4}$ Yes C $\frac{4}{16}$ $\frac{1}{3}$ No E $\frac{7}{10}$ $\frac{20}{30}$ No G $\frac{1}{8}$ $\frac{10}{80}$ Yes I $\frac{3}{10}$ $\frac{9}{20}$ No

B $\frac{2}{3}$ $\frac{8}{12}$ Yes D $\frac{5}{8}$ $\frac{15}{24}$ Yes F $\frac{9}{12}$ $\frac{12}{16}$ Yes H $\frac{0}{8}$ $\frac{0}{7}$ Yes J $\frac{4}{9}$ $\frac{12}{27}$ Yes

2. Give three other names for each fractional number.

A $\frac{4}{8}$ $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ B $\frac{5}{6}$ $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}$ C $\frac{1}{4}$ $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}$ D $\frac{3}{9}$ $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}$ E $\frac{8}{12}$ $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}$ F $\frac{3}{10}$ $\frac{6}{20}, \frac{9}{30}, \frac{12}{40}$

3. Find the missing numerators and denominators. Think about equivalent fractions.

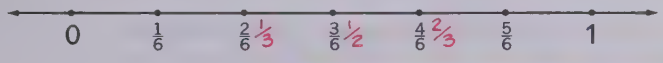
A $\frac{2}{8} = \frac{\square}{4}$ 1 E $\frac{3}{6} = \frac{1}{\square}$ 2 I $\frac{5}{10} = \frac{\square}{2}$ 1 M $\frac{3}{12} = \frac{\square}{4}$ 1

B $\frac{4}{8} = \frac{1}{\square}$ 2 F $\frac{4}{6} = \frac{\square}{3}$ 2 J $\frac{6}{10} = \frac{\square}{5}$ 3 N $\frac{4}{12} = \frac{\square}{3}$ 1

C $\frac{6}{8} = \frac{\square}{4}$ 3 G $\frac{2}{10} = \frac{\square}{5}$ 1 K $\frac{8}{10} = \frac{\square}{5}$ 4 O $\frac{6}{12} = \frac{\square}{2}$ 1

D $\frac{2}{6} = \frac{1}{\square}$ 3 H $\frac{4}{10} = \frac{2}{\square}$ 5 L $\frac{2}{12} = \frac{1}{\square}$ 6 P $\frac{8}{12} = \frac{2}{\square}$ 3

4. Give the lowest-terms fraction for each point on this number line.



5. Answer T (true) or F (false).

A $\frac{5}{10} = \frac{1}{2}$ T H $\frac{50}{100} = \frac{1}{2}$ T

B $\frac{3}{8} = \frac{9}{24}$ T I $\frac{3}{12} = \frac{1}{3}$ F

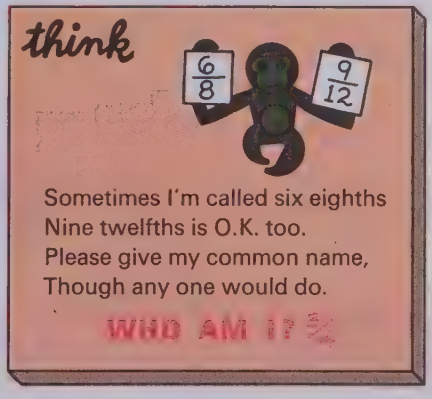
C $\frac{5}{6} = \frac{5}{7}$ F J $\frac{1}{4} = \frac{10}{40}$ T

D $\frac{6}{10} = \frac{3}{5}$ T K $\frac{0}{8} = \frac{1}{10}$ F

E $\frac{4}{8} = \frac{20}{40}$ T L $\frac{6}{12} = \frac{2}{4}$ T

F $\frac{2}{3} = \frac{8}{15}$ F M $\frac{2}{5} = \frac{6}{10}$ F

G $\frac{3}{15} = \frac{1}{5}$ T N $\frac{1}{7} = \frac{3}{21}$ T





Using the Exercises

Before you assign the exercises on page 325 as independent work, go over a few parts of exercise 3 with the children. Give them a few clues for finding the missing numbers. For example, from the check for equivalent fractions, they can see that in exercise 3A, 2×4 must be equal to 8 times some number (1). Also, they may remember that two eighths and one fourth belong to the same set of equivalent fractions, so the missing numerator must be 1.

Note that in the *Think* problem the term *common name* is used as another way of speaking of lowest-terms fractions.

Assignments (page 325)

Minimum: 1–2. Average: 1–4. Maximum: 1–5.

Follow-up/"E-Qual-ly-Cards"

Children would enjoy playing a game called "E-Qual-ly-Cards." Take an ordinary deck of cards for every four, five, or six players. Remove from this deck all face cards and consider the aces as ones. Deal four cards to each player. The object of the game is to form a pair of equivalent fractions. Each player takes a turn drawing from the deck or from the discard pile. He must always have only four cards in his hand, so after he draws a card he must discard a card face up. The next player in turn may take that discarded card or draw from the cards remaining in the original deck. When a player arranges a pair of equivalent fractions in his hand, he shows them and is declared winner. Note that some pairs may be thought of in a couple of ways; for example, if a player holds two fours and two eights he may arrange them as $\frac{4}{8}$ and $\frac{4}{8}$, as $\frac{4}{4}$ and $\frac{8}{8}$, or as $\frac{8}{4}$ and $\frac{8}{4}$.

Duplicator Masters, page 58
Workbook, page 106

PAGES 326–327

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Unless your class has found another concept particularly troublesome, briefly review comparisons of large numbers by writing several pairs of large numbers on the chalkboard and asking the children to tell which number in each pair is larger. Use pairs of numbers whose first few digits are the same (e.g., 24 108 and 24 180) so that the children must make some judgments concerning place-value concepts to arrive at the correct answer. You might also give the children an opportunity to review the reading of large numbers as part of this preparation.

If time permits, provide some review of measurement concepts, especially those regarding volume and measurement to the nearest half unit.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Measurement
Inequalities

1. Which number is larger?
 - A 362 807 or 359 968
 - B 4 027 340 or 3 928 643
 - C 78 076 or 78 100
 - D 9 284 316 or 9 283 978
 - E 640 000 or 98 000

2. Find the sums.
 - A 3287 + 642 + 93 216 + 49 97 194
 - B 43 826 + 925 + 8301 + 9640 62 692
 - C 37 064 + 85 + 79 + 3246 40 474
 - D 9324 + 657 + 8421 + 38 18 440

3. Find the differences.
 - A 9264 − 381 8883
 - B 731 − 285 446
 - C 6403 − 856 5547
 - D 7218 − 3451 3767
 - E 6000 − 87 5913
 - F 4006 − 198 3808

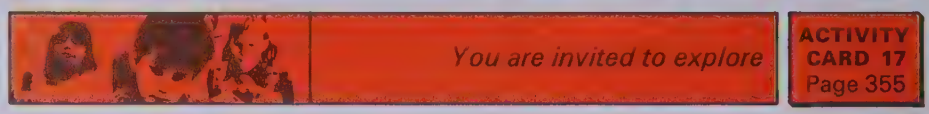
4. Find the products.
 - A 37 × 26 962
 - B 58 × 79 4582
 - C 283 × 46 13 018
 - D 349 × 38 13 262

5. Find the quotients and remainders.
 - A 348 ÷ 7 49 R5
 - B 625 ÷ 20 31 R5 ★
 - C 482 ÷ 36 13 R14 ★
 - D 1927 ÷ 51 37 R40

6. Measure each segment to the nearest half centimetre.
 - A ——— 8½ cm
 - B ——— 7½ cm
 - C ——— 10 cm
 - D ——— 9 cm

7. Find the volume for each figure.





Discussion

Assign page 326 as independent work. When the children are finished, allow time for checking and discussing answers. Note that exercises 5C and 5D are starred; though this indicates that they are intended for more capable children, the entire class would benefit from a discussion of them. For those who are interested in the problems in exercise 7, you might recommend the activity suggested in the follow-up section of this lesson.

It would be helpful to discuss the table on page 327 before assigning the word problems. Point out that the buildings listed in the table are not the eight tallest buildings in North America, but that the Sears Tower, the World Trade Center, and the Empire State Building were the three tallest buildings in the world at the time the data in this table were compiled. Have the children note that there are other buildings which are taller than some of those listed. If practical, have children do research regarding the heights of buildings in a city familiar to them. Remember that one of the objectives of such a word-problem set is to build children's interest in working with arithmetic problems.

When the children finish working the problems, spend ample time dis-



TALL BUILDINGS



Building	Location	Height	Stories
Sears Tower	Chicago, Illinois	435 metres	110
World Trade Center	New York City	405 metres	110
Empire State	New York City	375 metres	102
John Hancock	Chicago, Illinois	332 metres	100
Chrysler	New York City	314 metres	77
Rockefeller Center	New York City	255 metres	70
Commerce Court	Toronto, Ontario	235 metres	57
Toronto-Dominion Centre	Toronto, Ontario	219 metres	55

1. How much taller is the Empire State building than the Toronto-Dominion Centre ? 156 m

2. The CN Telecommunications Tower in Toronto is over 540 metres tall. How much taller is the CN Tower than the Sears Tower ? 105 m

3. About how tall is each story in these buildings ?
 - A Sears Tower 4 m
 - B Empire State Building 3⅔ m

4. A About how tall is a building if it has 58 stories and each story is about 4 metres tall ? 232 m
 B If this building has a 58-metre television tower on top, about how far is the top of the tower from the ground ? 290 m

5. A radio tower on top of the Commerce Court is about 38 metres high. How far is the top of the tower from the ground ? 273 m

6. How much taller is Commerce Court than the Toronto-Dominion Centre ? 16 m

cussing both the problems themselves and other related topics of interest to the children. Some children, for example, may be interested in measuring the height from the floor to the ceiling of the classroom, and of some rooms in their homes. (Such activities, however, should be undertaken only when adequate safety precautions are assured.)

Follow-up

To extend the ideas of exercise 7, page 326, ask the children to count the squares on the surface of figures A and B. Encourage the children to explain why the volume in figure B has increased by one even though the number of faces stays the same as in figure A. We would, of course, like to have children observe that adding a block to the figure in part B covers up three faces, but it also provides three additional faces. Finally, more able children should see that for the figure in part C the volume is one less than in part A, and the number of faces is two less than in part A. If possible, provide blocks so that those interested may arrange various stacks of blocks and count the number of blocks in each stack.

PAGES 328–329

Objective

Given a particular strip as a unit, the child will be able to use lowest-terms fractions to express the lengths of other strips in relation to the unit strip.

Preparation

Materials

colored strips

It would be appropriate to begin immediately with the investigation. However, if you prefer, review naming a fractional number in lowest terms. For example, write the following fractions and ask the children to give the lowest-terms fraction equivalent to each.

$\frac{4}{8}$	$\frac{5}{10}$	$\frac{15}{20}$	$\frac{6}{8}$
$\frac{6}{9}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{3}{9}$
$\frac{9}{12}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{2}{4}$

Investigation

Some children may need a bit of help in order to understand what this investigation requires of them. Remind them of how a fraction can be used to name the length of each strip in relation to the unit. For example, have them compare the light green strip to the brown (unit) strip and decide whether or not it is in lowest terms. Of course, this comparison gives the fraction $\frac{3}{8}$, which is in lowest terms. Suggest that they make a chart to show the fraction for each strip. You might put on the chalkboard a chart similar to the following (but without the answers).

Strip	*Unit*	*Lowest-Terms Fraction*
white	brown	$(\frac{1}{8})$
red	brown	$(\frac{1}{4})$
light green	brown	$(\frac{3}{8})$
purple	brown	$(\frac{1}{2})$
yellow	brown	$(\frac{5}{8})$
dark green	brown	$(\frac{3}{4})$
black	brown	$(\frac{7}{8})$
blue	brown	$(\frac{9}{8})$
orange	brown	$(\frac{5}{4})$

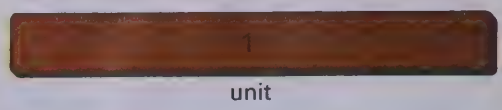
It would also be helpful to designate another unit, such as the blue strip, and have the children give the lowest-terms fractions for the lengths of the other strips in relation to the new unit.



● *Let's explore other fractional number lengths.*

Investigating the Ideas

Suppose the brown strip is the unit.



? Can you find the lowest-terms fraction for the length of each of your other strips? See Investigation.



Discussing the Ideas

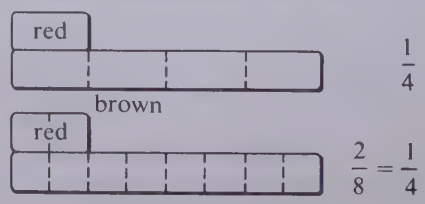
1. How are the fractions you found for the lengths of the blue and orange strips different from those for the other strips?
 They are improper fractions (numerators are greater than denominators).
2. Can you describe more than one way to find the lowest-terms fraction for the length of the red strip? See Discussion.
3. Can you describe more than one way to find the lowest-terms fraction for the length of the orange strip? See Discussion.

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Discussion

Have children write on the chalkboard the lowest-terms fraction for the length of each strip in relation to the brown strip as the unit. During your discussion of the text exercises, help the children realize that the fractions which express the lengths of the blue and orange strips are improper fractions. Note that the improper fraction $\frac{10}{8}$, which is the length of the orange strip, is not the lowest-terms fraction because it may be replaced by $\frac{5}{4}$. You might also discuss the length of the brown strip when compared to itself as the unit. Help children realize that the fraction $\frac{8}{8}$ is another name for 1.

In discussing exercises 2 and 3, some children may suggest finding the lowest-terms fraction by thinking of both strips (the strip being compared and the unit strip) in terms of eighths, and then finding the lowest-terms fraction from the appropriate set of equivalent fractions. Others may immediately think of the unit in fewer divisions when possible, such as in fourths when the red strip is being considered.

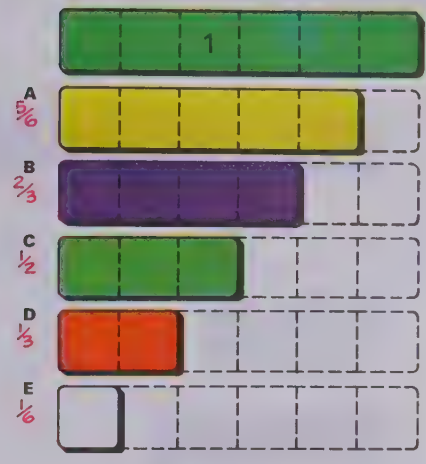


$\frac{1}{4}$

$\frac{2}{8} = \frac{1}{4}$

Using the Ideas

1. If the dark green strip is the unit, what is the lowest-terms fraction for the length of each of the other strips?



2. If the dark green strip is the unit, what is the lowest-terms fraction for the length of the blue strip? 3/2 the brown strip? 4/3

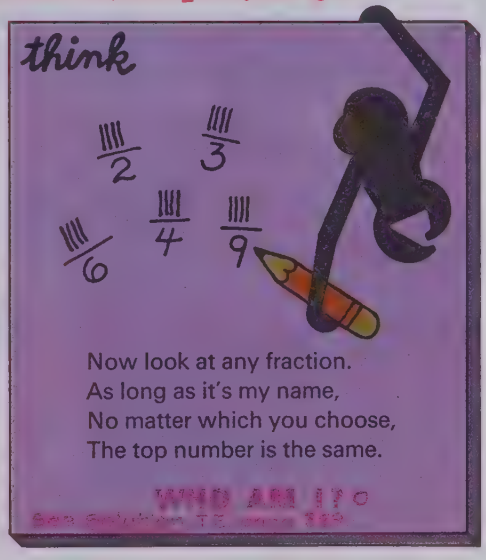
White 1/10, red 1/5, lt. green 3/10, purple 2/5, yellow 1/2, dk. green 3/5, black 7/10, brown 4/5, blue 9/10

3. If the orange strip is the unit, give the lowest-terms fraction for the length of each of the other strips.

4. If the yellow strip is the unit, what is the lowest-terms fraction for the length of the orange strip? 2/1

★ 5. If the purple strip is the unit, can you make a "train" of 2-strips that is $\frac{9}{2}$ units long?

See Using the Exercises.





Using the Exercises

Assign the exercises on page 329 as independent work, to be done by children working individually or in groups of two or three. Remind them that the length of each strip depends on the unit strip. Exercises 1 and 2 deal with the same unit, first with proper fractions and then with improper fractions. Encourage children to use their strips while they do exercises 3, 4, and 5. Though exercise 5 is designed to be challenging, all would benefit from an explanation: Since the 2-strip is the $\frac{1}{2}$ unit in relation to the purple strip, then nine 2-strips will be needed to make a train $\frac{9}{2}$ units long.

Assignments (page 329)

Minimum: 1–4. Average: 1–4.
Maximum: 1–5.

Solution, Think, page 329

After children have had sufficient opportunity to work with the *Think* problem on their own, explain the answer by listing the first ten fractions in the set $\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \ldots\}$. Observe with them that they have never seen another set of equivalent fractions in which all the numerators are the same. Then explain to them that zero is the only number which has this characteristic. This would also be a good time to explain to the children that zero may be expressed as a fractional number or as a whole number. Perhaps the best way for children to understand this is through their experience in working with the number line.

Follow-up

Encourage children to make charts showing the lengths of strips for different units. For instance, you might have them work in groups and have each group make a chart for a particular unit strip. They might draw the unit strip and each of the other strips, and label each strip according to its fractional length in terms of that unit. Remind them to use lowest-terms fractions for each.

Example:

unit strip: light green
white: $\frac{1}{3}$
red: $\frac{2}{3}$
light green: $\frac{3}{3}$ or 1
purple: $\frac{4}{3}$
yellow: $\frac{5}{3}$
dark green: $\frac{6}{3}$ or 2
black: $\frac{7}{3}$
brown: $\frac{8}{3}$
blue: $\frac{9}{3}$ or 3
orange: $\frac{10}{3}$

PAGES 330–331

Objectives

Given two different fractional numbers, the child will be able to write an inequality to express which fractional number is greater.

Preparation

Materials

colored strips

The children's familiarity with the strips should be sufficient to allow an oral review involving comparisons of their lengths. Ask children questions such as: "If the brown strip is the unit, what length has the purple strip?" ($\frac{1}{2}$) "If the yellow strip is the unit, what length has the red strip?" ($\frac{2}{5}$) "If the blue strip is the unit, what length has the light green strip?" ($\frac{3}{9} = \frac{1}{3}$) (If necessary, let those who wish to do so use their strips while you ask these questions.)

Also, review the inequality symbols ($<$, less than; $>$, greater than).

Investigation

Study the introductory text and illustrations with the children. Make sure they realize that, in relation to the brown strip, the purple strip has length $\frac{4}{8}$ or $\frac{1}{2}$. By comparing the lengths of the strips, children are able to see which fraction represents the greater number. Encourage children to record their results. For example, suggest a table like the one below.

Unit	*Strips*	*Inequality*
brown	light green ($\frac{3}{8}$)	$\frac{3}{8} < \frac{1}{2}$
	purple ($\frac{1}{2}$)	

Sample pairs of strips which the children may use are as follows:

black $>$ yellow $\frac{7}{8} > \frac{5}{8}$	red $<$ dark green $\frac{1}{4} < \frac{3}{4}$
blue $<$ black $\frac{3}{4} < \frac{7}{8}$	blue $>$ yellow $\frac{3}{4} > \frac{5}{8}$
red $<$ green $\frac{1}{4} < \frac{3}{8}$	blue $<$ orange $\frac{9}{8} < \frac{5}{4}$

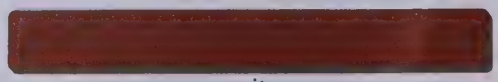
Note that all these fractions are given in lowest terms. It is not incorrect for children to use a fraction in higher terms, but encourage them to use lowest-terms fractions. Depending on the children's ability, you might suggest that they write inequalities using a different strip as the unit length, such as the dark green strip.



Which of two fractional numbers is greater?

Investigating the Ideas

If the brown strip has length 1,



unit

then these strips show the **inequalities**



$\frac{3}{8} < \frac{1}{2}$ ("$\frac{3}{8}$ **is less than** $\frac{1}{2}$")

and



$\frac{1}{2} > \frac{3}{8}$ ("$\frac{1}{2}$ **is greater than** $\frac{3}{8}$")

? Using the brown strip as the unit, what other inequalities can you write and show with your strips?

See Investigation.

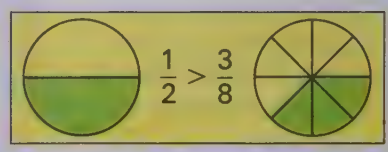
Discussing the Ideas

1. How can you tell that $\frac{3}{8}$ is less than $\frac{1}{2}$ by looking at the strips?

 See Discussion.

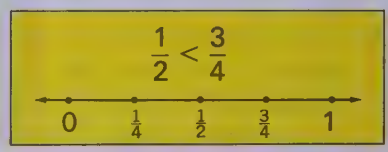
2. Explain how the shaded parts of the two circular regions help you see that $\frac{1}{2}$ is greater than $\frac{3}{8}$.

 Shaded area for ½ is larger than for ⅜.



3. Explain how the number-line picture helps you see that $\frac{1}{2}$ is less than $\frac{3}{4}$.

 ½ is a smaller portion of the unit than ¾.



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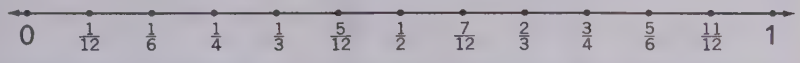
Discussion

Discuss the pairs of strips and inequalities which the children recorded. A comparison of the lengths of the light green and the purple strips shows that $\frac{3}{8}$ is less than $\frac{1}{2}$. Also, discuss this comparison in terms of $\frac{3}{8}$ and $\frac{4}{8}$. Point out how easy it is to compare fractions when their denominators are the same. For example, we may think three eighths is fewer than four eighths without using strips or materials because we know that three is less than four.

It would be helpful to use other demonstrations to show inequalities of fractions. For example, demonstrate with a measuring cup that $\frac{1}{2}$ is greater than $\frac{1}{4}$ or that $\frac{1}{3}$ cup is greater than $\frac{1}{4}$ cup. You might also use a metre stick to show that $\frac{1}{2}$ metre is less than $\frac{2}{3}$ metre. Shaded regions similar to those in exercise 2 would also be useful. Of course, a number line marked in various fractional segments is an excellent aid in comparing fractional numbers, and may more easily help the children recall that they are comparing not just lengths or quantities but fractional numbers represented by fractions.

Using the Ideas

1. Study the number line. Then give the correct sign (<, =, or >) for each ◯.



A $\frac{1}{4} \bigcirc \frac{1}{3}$ <

B $\frac{1}{2} \bigcirc \frac{5}{12}$ >

C $\frac{1}{12} \bigcirc \frac{1}{4}$ <

D $\frac{3}{4} \bigcirc \frac{5}{6}$ <

E $\frac{7}{12} \bigcirc \frac{2}{3}$ <

F $\frac{1}{2} \bigcirc \frac{6}{12}$ =

G $\frac{2}{3} \bigcirc \frac{3}{4}$ <

H $\frac{5}{6} \bigcirc \frac{11}{12}$ <

I $\frac{1}{12} \bigcirc 0$ >

J $\frac{7}{12} \bigcirc \frac{1}{2}$ >

K $\frac{5}{12} \bigcirc \frac{1}{2}$ <

L $\frac{1}{2} \bigcirc \frac{1}{3}$ >

M $\frac{2}{12} \bigcirc \frac{1}{6}$ =

N $\frac{1}{4} \bigcirc \frac{2}{12}$ >

O $\frac{1}{3} \bigcirc \frac{3}{12}$ >

2. Write an inequality for each exercise.

A



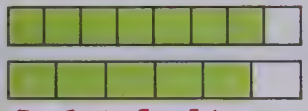
Answer: $\frac{1}{4} < \frac{1}{2}$ (or $\frac{1}{2} > \frac{1}{4}$)

B



$\frac{4}{6} > \frac{1}{3}$ (or $\frac{1}{3} < \frac{4}{6}$)

C



$\frac{7}{8} > \frac{5}{6}$ (or $\frac{5}{6} < \frac{7}{8}$)

D



$\frac{3}{5} < \frac{7}{10}$
(or $\frac{7}{10} > \frac{3}{5}$)

3. Answer T (true) or F (false).

A $\frac{1}{2} < \frac{5}{8}$ T

B $\frac{3}{8} > \frac{2}{2}$ F

C $\frac{1}{4} < \frac{1}{8}$ F

D $\frac{8}{8} = \frac{4}{4}$ T

E $\frac{8}{16} > \frac{4}{8}$ F ★

F $\frac{7}{8} < \frac{2}{2}$ T ★

G $\frac{0}{2} = \frac{0}{8}$ T

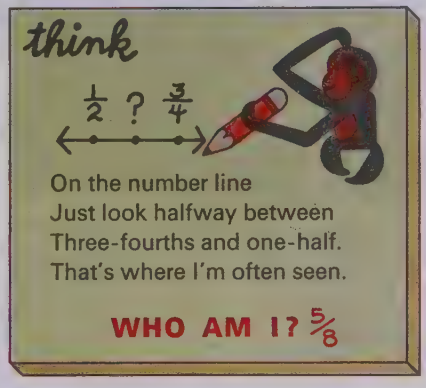
H $\frac{2}{3} = \frac{3}{4}$ F

I $\frac{4}{5} < \frac{5}{8}$ F

J $\frac{3}{4} < \frac{3}{8}$ F

K $\frac{4}{8} < \frac{4}{7}$ T

L $\frac{8}{14} < \frac{8}{15}$ F





Mathematics

If two fractional numbers, $\frac{a}{b}$ and $\frac{c}{d}$, are to be compared, we can express both fractions with a common denominator, bd. Then we would need to compare $\frac{ad}{bd}$ and $\frac{bc}{bd}$. Since the denominators are the same, we need but to compare the numerators, ad and bc. Therefore, for positive fractional numbers, we can state

$$\frac{a}{b} > \frac{c}{d} \text{ if and only if } ad > bc.$$

In this lesson, we present informally (through the use of the strips, regions, and the number line) the general concept of inequalities for fractional numbers. A precise definition of inequality for fractional numbers is not presented to the children at this time.

Follow-up

Children might enjoy a game which would help them compare fractions. Number the sides of each of a pair of small cubes with the even numbers from two to twelve. Each player takes a turn rolling the cubes to form a *proper* fraction. Before he rolls, he guesses whether his fraction will be less than $\frac{1}{2}$ or greater than $\frac{1}{2}$. If he guesses right, he earns one point and takes another turn. If he guesses wrong, the next player takes his turn. For example, the first player calls "less than" and rolls a 1 and a 3. Since $\frac{1}{3} < \frac{1}{2}$, he earns a point and another roll. Children could play this game in groups of two or three.

Workbook, page 107

Using the Exercises

Assign the exercises on page 331 and plan sufficient time for consequent discussion. In exercise 2, note that more than one correct answer is possible for each part, depending on the terms chosen and the inequality sign used. In exercise 3, call attention to parts C, J, K, and L, in which both numerators are the same but the denominators are different. Help the children recognize that, when two fractions have the same numerator, the fraction with the smaller denominator represents the greater fractional number. Children will be able to grasp this idea more readily perhaps if you suggest that they think, in part J for example: "Since $\frac{1}{4}$ of an object is more than $\frac{1}{8}$ of the same object, $\frac{3}{4}$ of that object must be more than $\frac{3}{8}$ of that object; therefore, the statement $\frac{3}{4} < \frac{3}{8}$ is false."

Assignments (page 331)

Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

PAGES 332–333

Objective

Using appropriately divided number lines, the child will be able to name fractional numbers greater than one by using improper fractions.

Preparation

To prepare for this lesson, review the strip activities of the previous lessons which treat fractional numbers greater than one. For example, ask the children to give the fractions which show the following cases:

If the dark green strip is the unit, what is the length of the blue strip? ($\frac{3}{2}$ or $\frac{9}{6}$)

If the red strip is the unit, what is the length of the brown strip? ($\frac{4}{1}$ or $\frac{8}{2}$) Make sure children realize that in this case the white strip is the half unit.

If the purple strip is the unit, what is the length of the brown strip? of the orange strip? ($\frac{2}{1}$ or $\frac{8}{4}$; $\frac{5}{2}$)

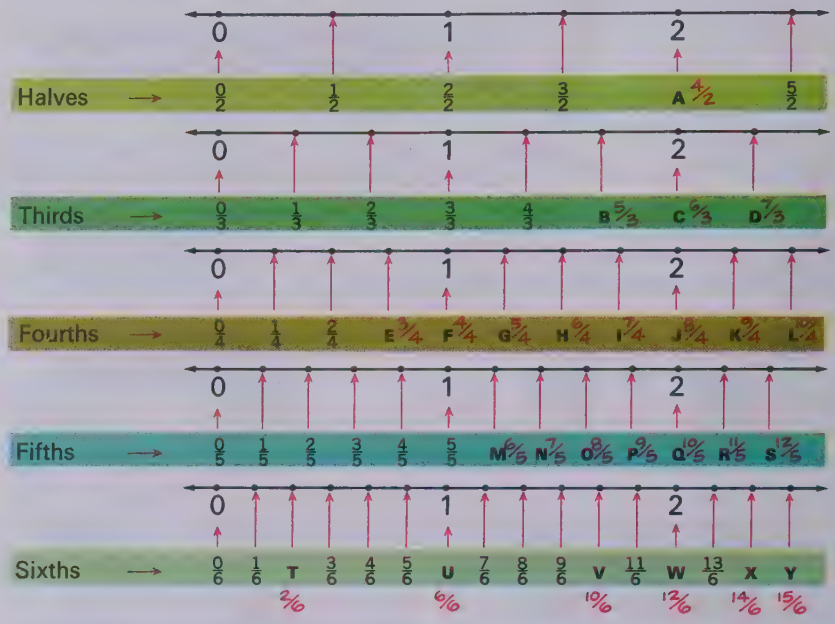
If the light green strip is the unit, what is the length of the blue strip? of the dark green strip? ($\frac{3}{1}$ or $\frac{9}{3}$; $\frac{2}{1}$ or $\frac{6}{3}$)



Can you name larger numbers on the number line?

Discussing the Ideas

1. Give the missing fractions.



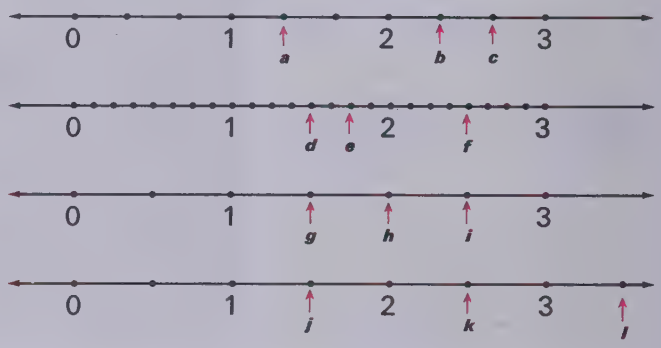
2. Choose the point for the number given.

A $\frac{7}{5}$ (Answer: b)

B $\frac{12}{8}$ d

C $\frac{16}{8}$ h

D $\frac{10}{4}$ K

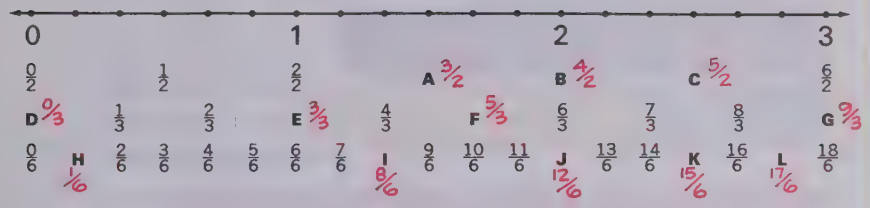


Discussion

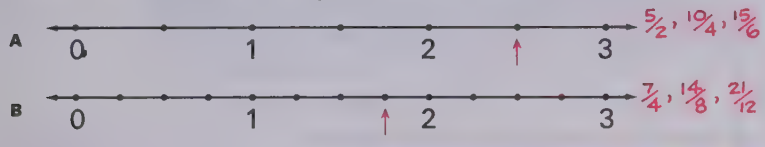
Show on your demonstration number line two or three of the fractions treated in the preparation section. Point out to the children how they can count the number of halves, thirds, fourths, or whatever the denominator might be. You might have the children do the exercises and discuss the ideas afterward. Help children realize where numbers such as $\frac{4}{2}$, $\frac{5}{2}$, and $\frac{6}{2}$ appear on the number line in relation to the number 1. Also, have them observe the association between $\frac{0}{4}$ and 0, $\frac{4}{4}$ and 1, $\frac{8}{4}$ and 2, etc. Have them note that each whole number is also a fractional number, and that each such number has an unlimited number of names; for example, all fractions of the set $\{\frac{4}{2}, \frac{8}{4}, \frac{16}{8}, \ldots\}$ are names for the number 2.

Using the Ideas

1. Give the missing fractions.

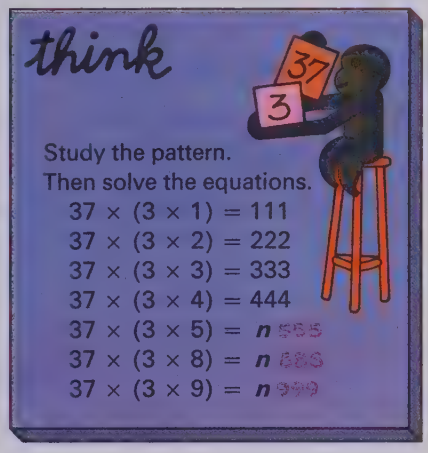


2. Give a set of three equivalent fractions that name the fractional number for the point above the red arrow.

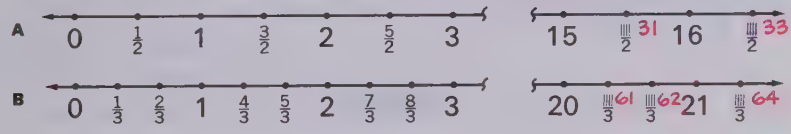


3. Answer **T** (true) or **F** (false).

A	$\frac{2}{2} > \frac{1}{2}$ T	H	$\frac{9}{4} < 2$ F
B	$\frac{2}{2} < \frac{3}{2}$ T	I	$\frac{11}{4} > \frac{5}{2}$ T
C	$\frac{3}{4} > \frac{3}{2}$ F	J	$\frac{12}{4} < 3$ F
D	$\frac{5}{2} > 2$ T	K	$\frac{8}{8} > \frac{4}{4}$ F
E	$\frac{6}{8} = \frac{9}{12}$ T	L	$\frac{6}{3} > \frac{5}{2}$ F
F	$\frac{5}{2} < 3$ T	M	$\frac{0}{4} < \frac{0}{8}$ F
G	$\frac{6}{4} = \frac{3}{2}$ T	N	$\frac{0}{4} > \frac{0}{8}$ F



★**4.** Find the missing numerators.





Using the Exercises

Have the children do the exercises on page 333 independently, and allow time for checking papers and for discussion afterward.

In starred exercise 4, call attention to the fact that a considerable portion of each number line is missing. In exercise 4A, for example, the break in the number line between 3 and 15 shows that something is missing. The number line has been divided into $\frac{1}{2}$-unit segments. By counting these segments, the children can see that from 0 to 2 there are four $\frac{1}{2}$-unit segments. The point labelled 2 on the number line could also be labelled $\frac{4}{2}$. By extending these ideas, they can see that there would be 30 segments from 0 to 15, meaning that another name for 15 is $\frac{30}{2}$. Obviously, the next point on the number line must be $\frac{31}{2}$.

For the *Think* problem, a few children may discover that the second product through the seventh are actually multiples of the first product. However, expect most children to write the missing products by inspecting the pattern.

Assignments (page 333)

Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Follow-up

For practice in finding equivalent improper fractions, give the children a worksheet similar to the one shown.

Give the missing numerators and denominators.

1. $\frac{3}{2} = \frac{\square}{4}$	7. $\frac{11}{3} = \frac{\square}{9}$
2. $\frac{5}{4} = \frac{10}{\square}$	8. $\frac{0}{4} = \frac{\square}{2}$
3. $\frac{5}{3} = \frac{\square}{6}$	9. $\frac{6}{2} = \frac{\square}{4}$
4. $\frac{10}{4} = \frac{5}{\square}$	10. $\frac{18}{6} = \frac{\square}{2}$
5. $\frac{4}{2} = \frac{8}{\square}$	11. $\frac{14}{6} = \frac{7}{\square}$
6. $\frac{3}{3} = \frac{\square}{9}$	12. $\frac{15}{6} = \frac{5}{\square}$

Workbook, page 108

PAGES 334–335

Objective

Given two fractional numbers written with like denominators, the child will be able to add them.

Preparation

Materials

colored strips

You might use the preparation time to provide the children with further practice in expressing fractions from a study of the strips. For example, ask children to show with their strips the fraction $\frac{3}{5}$, if the yellow strip is the unit; or to show $\frac{5}{8}$, if the brown strip is the unit; or $\frac{2}{7}$, if the black strip is the unit.

Investigation

Have children use their own strips to show the addition operation illustrated in the text. Make sure they realize that the light green, yellow, and brown strips are labelled $\frac{3}{10}$, $\frac{5}{10}$, and $\frac{8}{10}$ because the orange strip is the unit. Explain that they may use any strip as the unit strip while they find 5 more addition equations. Remind them to record both the strips they use and the fractions that they add. (Note that we speak of adding "*fractions*." Actually, only *fractional numbers* may be added; however, for convenience, the term *fraction* may be used in such contexts, and it should be understood to mean the addition of the fractional numbers represented by these fractions.)

Sample addition equations follow.

Unit	*Strips*	*Equations*
black	purple + white = yellow	$\frac{4}{7}+\frac{1}{7}=\frac{5}{7}$
dark green	red + red = purple	$\frac{2}{6}+\frac{2}{6}=\frac{4}{6}$
blue	purple + red = dark green	$\frac{4}{9}+\frac{2}{9}=\frac{6}{9}$
brown	yellow + yellow = orange	$\frac{5}{8}+\frac{5}{8}=\frac{10}{8}$

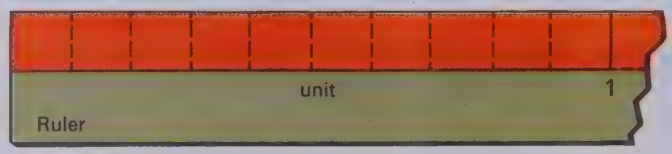
Can fractional numbers be added?

Investigating the Ideas



$\frac{3}{10} + \frac{5}{10} =$ ▢

Addition Equation



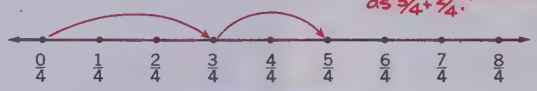
? Can you use your strips to write 5 more addition equations with fractions?

See Investigation.

Discussing the Ideas

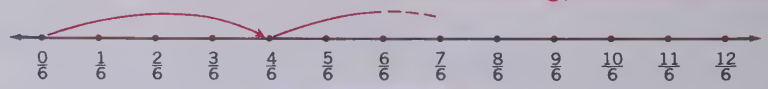
1. What unit and which strips would you use to show that $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$? Explain. See Discussion.

2. Explain how the number-line picture helps you find the sum of $\frac{3}{4}$ and $\frac{2}{4}$. It shows where you will land after one jump of $\frac{3}{4}$ and one jump of $\frac{2}{4}$, which is the same as $\frac{3}{4}+\frac{3}{4}$.



$\frac{3}{4} + \frac{2}{4} = n$ $\frac{5}{4}$

3. Explain how you would decide what the second jump would be in order to find the sum of $\frac{4}{6}$ and $\frac{5}{6}$. From $\frac{4}{6}$, count over 5 more sixths.



$\frac{4}{6} + \frac{5}{6} = n$ $\frac{9}{6}$

334

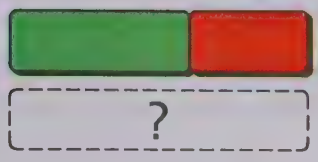
Discussion

It is essential that children understand the significance of the unit strip: the strip chosen as the unit determines which fractional numbers are represented by the other strips. For example, the purple strip plus the red strip match the equation $\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$ when the brown strip is the unit, but these strips match the equation $\frac{4}{10} + \frac{2}{10} = \frac{6}{10}$ when the orange strip is the unit. Develop this point as you discuss exercise 1: the light green strip plus the red strip match $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ when the black strip is the unit, but they match $\frac{3}{9} + \frac{2}{9} = \frac{5}{9}$ when the blue strip is the unit.

In exercises 2 and 3, the illustration with the number line clearly shows what the unit (or denominator) is in each case. Have a child explain how the final equation is related to the number-line picture. If necessary, provide similar examples with your demonstration number line, but for the time being, avoid mixed numerals and addition of fractional numbers represented by fractions having different denominators.

Using the Ideas

1. A If the [redacted] is the unit, how long is the light green strip? the red strip? $\frac{3}{8}$; $\frac{2}{8}$

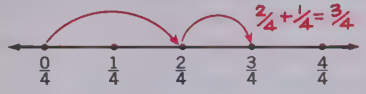


B What is the length of the single strip that has the same length as these two strips together? $\frac{5}{8}$

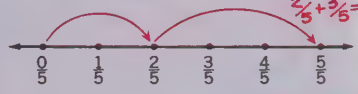
C Write an addition equation for this. $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

2. Write an addition equation for each number-line picture.

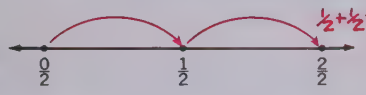
A $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$



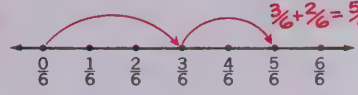
B $\frac{2}{5}+\frac{3}{5}=\frac{5}{5}$



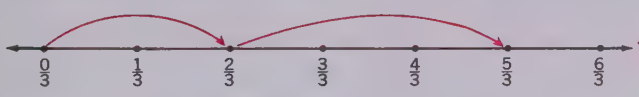
C $\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$



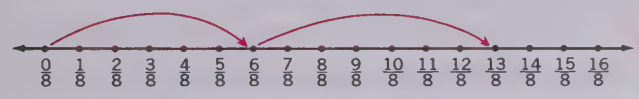
D $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$



E $\frac{2}{3}+\frac{3}{3}=\frac{5}{3}$



F $\frac{6}{8}+\frac{7}{8}=\frac{13}{8}$



3. Find the sums.

A $\frac{3}{8} + \frac{4}{8}$ $\frac{7}{8}$ C $\frac{6}{8} + \frac{1}{8}$ $\frac{7}{8}$ E $\frac{5}{7} + \frac{4}{7}$ $\frac{9}{7}$ G $\frac{4}{10} + \frac{5}{10}$ $\frac{9}{10}$ I $\frac{5}{10} + \frac{5}{10}$ $\frac{10}{10}$

B $\frac{2}{3} + \frac{4}{3}$ $\frac{6}{3}$ D $\frac{4}{6} + \frac{4}{6}$ $\frac{8}{6}$ F $\frac{5}{6} + \frac{1}{6}$ $\frac{6}{6}$ H $\frac{9}{10} + \frac{1}{10}$ $\frac{10}{10}$ J $\frac{9}{10} + \frac{11}{10}$ $\frac{20}{10}$

★**4.** A If the red strip has length $\frac{1}{3}$ and the light green strip has length $\frac{1}{2}$, what is the unit? dark green (6/6)



$\frac{1}{3} + \frac{1}{2} = ?$

B What is the length of the single strip that is as long as these two strips together? Copy and complete the equation. $\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$

$\frac{5}{6}$



Follow-up

List several equations on the chalkboard and ask children to show the operation with their strips. They may record the strips they use by writing an equation with the color names of the strips or by drawing the strips. (Note in the samples below that all denominators are ten or less than ten.)

$\frac{1}{3} + \frac{1}{3} = n$ $\quad$ $\frac{6}{10} + \frac{2}{10} = n$

$\frac{4}{5} + \frac{2}{5} = n$ $\quad$ $\frac{2}{4} + \frac{2}{4} = n$

$\frac{2}{8} + \frac{2}{8} = n$ $\quad$ $\frac{2}{7} + \frac{4}{7} = n$

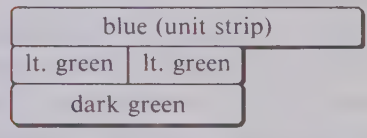
$\frac{3}{9} + \frac{4}{9} = n$ $\quad$ $\frac{1}{10} + \frac{8}{10} = n$

Sample equation worked out:

Unit: Blue →

Light green + Light green = Dark green

$\frac{1}{3}$ $\quad$ + $\frac{1}{3}$ $\quad$ = $\frac{2}{3}$.



Resources for Active Learning

Mathex: Numeration No. 7, pupil pages 42, 43, Encyclopaedia Britannica Publications Ltd.

Workbook, page 109

Using the Exercises

Assign the exercises on page 335 as independent work. When the children have finished, check the answers and have volunteers discuss them. You might ask children to show the additions of exercise 3 on the number line or with the strips. In starred exercise 4, children must realize that the dark green strip (6-strip) is the unit before they can successfully add. Make sure they do *not* think $\frac{1}{3} + \frac{1}{2} = \frac{1}{5}$, even though the red strip with the light green strip matches the yellow strip (5-strip). Help them to think first of the light green strip as $\frac{3}{6}$ and the red strip as $\frac{2}{6}$, to assure correct understanding.

Assignments (page 335)

Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

PAGES 336–337

Objectives

Given a whole number, the child will be able to write it as a fraction.

Given a fractional number that is also a whole number, the child will be able to write it in the usual whole-number notation.

Preparation

Materials

colored strips (optional)

You might use the strips to review writing improper fractions. For example, ask: If the brown strip is the unit, what fraction matches two purple strips? four red strips? eight white strips? ($\frac{2}{2}$; $\frac{4}{4}$; $\frac{8}{8}$) If the blue strip is the unit, what fraction matches three light green strips? ($\frac{3}{3}$)

Investigation

It would be helpful for the children to draw a number line of their own similar to the one in the text. They may then use it to think of different fractional segments. By counting these segments, they can find various fractional names for the whole numbers given. (See chart at the foot of this column.) Once they find the first few names for each whole number, they may recognize some patterns.

$\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots$ follows an easily recognizable pattern.

Or, they may think across:

$\frac{1}{1} \rightarrow \frac{2}{1} \rightarrow \frac{3}{1}$; $\frac{2}{2} \rightarrow \frac{4}{2} \rightarrow \frac{6}{2}$; $\frac{3}{3} \rightarrow \frac{6}{3} \rightarrow \frac{9}{3}$

Or, they may think down:

$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$
$\frac{2}{2}$	$\frac{2 \times 2}{1 \times 2}$	$\frac{3 \times 2}{1 \times 2}$
$\frac{3}{3}$	$\frac{2 \times 3}{1 \times 3}$	$\frac{3 \times 3}{1 \times 3}$

You might refer those who are having difficulty to previous pages in the text, such as page 332.

0														1														2														3
$\frac{0}{1}$														$\frac{1}{1}$														$\frac{2}{1}$														$\frac{3}{1}$
$\frac{0}{2}$					$\frac{1}{2}$									$\frac{2}{2}$					$\frac{3}{2}$									$\frac{4}{2}$					$\frac{5}{2}$									$\frac{6}{2}$
$\frac{0}{3}$			$\frac{1}{3}$			$\frac{2}{3}$			$\frac{3}{3}$			$\frac{4}{3}$			$\frac{5}{3}$			$\frac{6}{3}$			$\frac{7}{3}$			$\frac{8}{3}$			$\frac{9}{3}$															
$\frac{0}{4}$		$\frac{1}{4}$		$\frac{2}{4}$		$\frac{3}{4}$		$\frac{4}{4}$		$\frac{5}{4}$		$\frac{6}{4}$		$\frac{7}{4}$		$\frac{8}{4}$		$\frac{9}{4}$		$\frac{10}{4}$		$\frac{11}{4}$		$\frac{12}{4}$																		
$\frac{0}{5}$	$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		$\frac{5}{5}$		$\frac{6}{5}$		$\frac{7}{5}$		$\frac{8}{5}$		$\frac{9}{5}$		$\frac{10}{5}$		$\frac{11}{5}$		$\frac{12}{5}$		$\frac{13}{5}$		$\frac{14}{5}$		$\frac{15}{5}$													

Can whole numbers be matched with fractions?

Investigating the Ideas

This number line shows some fractional names for the whole numbers.

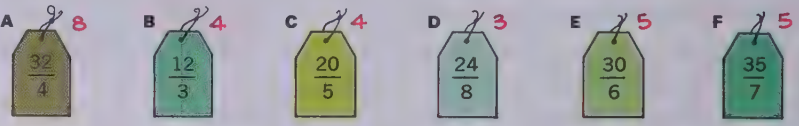


? Can you make a set of five different name tags for some other whole number?

See Investigation.

Discussing the Ideas

1. Explain an easy way to find the names for 0.
 Use 0 in the numerator in each fraction.
2. What makes it very easy to find the names for 1?
 Numerator and denominator are the same in each fraction.
3. In each fraction for 2, the numerator is greater than the denominator. How many times as great?
 Twice as great
4. In each fraction for 3, the numerator is how many times as great as the denominator? Three times as great.
5. What whole number would go with each of these tags?



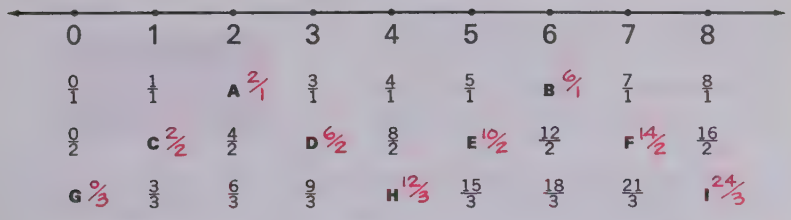
6. How could you easily find a fraction for 4?
 See Discussion.

Discussion

As you discuss each exercise, have a child exhibit on the chalkboard his fractions for that whole number. Zero as the numerator will be easily identified in exercise 1. In exercise 2, the children should realize that the numerator and denominator are equal. Write other fractional names for one such as $\frac{50}{50}$, $\frac{175}{175}$, and $\frac{1\,000\,000}{1\,000\,000}$, and ask children to identify the whole number they name. Exercise 3 emphasizes that the numerator of every fraction equivalent to $\frac{2}{1}$ is twice as great as its denominator, while exercise 4 makes the point that the numerator of every fraction equivalent to $\frac{3}{1}$ is

Using the Ideas

1. Give the missing fractions.



2. Match each whole number with a set of equivalent fractions.

A 3 (4) (1) $\{\frac{9}{1}, \frac{18}{2}, \frac{27}{3}, \frac{36}{4}, \ldots\}$

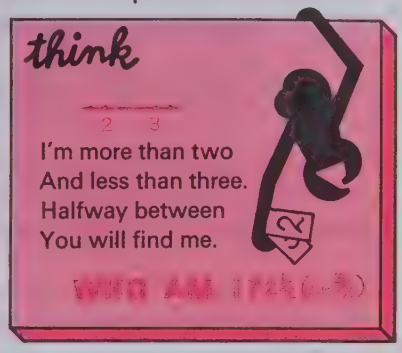
B 6 (3) (2) $\{\frac{15}{1}, \frac{30}{2}, \frac{45}{3}, \frac{60}{4}, \ldots\}$

C 1 (5) (3) $\{\frac{6}{1}, \frac{12}{2}, \frac{18}{3}, \frac{24}{4}, \ldots\}$

D 9 (1) (4) $\{\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \ldots\}$

E 0 (6) (5) $\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \ldots\}$

F 15 (2) (6) $\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \ldots\}$



3. Give the missing numerators and denominators.

A $3 = \frac{6}{\square 2}$ E $0 = \frac{\square}{6} 0$ I $5 = \frac{15}{\square 3}$ M $12 = \frac{\square}{1} 12$ Q $\frac{1}{4} = \frac{3}{\square 12}$

B $6 = \frac{6}{\square 1}$ F $15 = \frac{30}{\square 2}$ J $8 = \frac{\square}{3} 24$ N $20 = \frac{40}{\square 2}$ R $\frac{1}{2} = \frac{\square}{12} 6$

C $1 = \frac{\square}{5} 5$ G $2 = \frac{\square}{4} 8$ K $4 = \frac{12}{\square 3}$ O $11 = \frac{77}{\square 7}$ S $\frac{2}{3} = \frac{8}{\square 12}$

D $9 = \frac{\square}{1} 9$ H $7 = \frac{\square}{2} 14$ L $10 = \frac{\square}{2} 20$ P $\frac{1}{2} = \frac{\square}{10} 5$ T $\frac{3}{4} = \frac{9}{\square 12}$

4. Find the sums.

A $\frac{4}{2} + \frac{1}{2} \frac{5}{2}$ D $\frac{6}{3} + \frac{1}{3} \frac{7}{3}$ G $3 + \frac{1}{3} \frac{10}{3}$ J $\frac{12}{2} + \frac{3}{2} \frac{15}{2}$ M $2 + \frac{3}{4} \frac{11}{4}$

B $\frac{6}{2} + \frac{1}{2} \frac{7}{2}$ E $2 + \frac{1}{3} \frac{7}{3}$ H $\frac{12}{3} + \frac{2}{3} \frac{14}{3}$ K $6 + \frac{3}{2} \frac{15}{2}$ N $\frac{8}{8} + \frac{5}{8} \frac{13}{8}$

C $3 + \frac{1}{2} \frac{7}{2}$ F $\frac{9}{3} + \frac{1}{3} \frac{10}{3}$ I $4 + \frac{2}{3} \frac{14}{3}$ L $\frac{8}{4} + \frac{3}{4} \frac{11}{4}$ O $1 + \frac{5}{8} \frac{13}{8}$

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three times as great as its denominator. As you discuss exercise 5 and 6, help the children generalize this pattern so that they think, for exercise 5: "Since 32 is eight times as great as 4, or $4 \times 8 = 32$, then $\frac{32}{4}$ must equal 8." And for exercise 6, help them see that any fraction whose numerator is 4 times as great as the denominator equals 4. Also point out how the multiples of 4 appear as the numerators in this set of equivalent fractions: $\{\frac{4}{1}, \frac{8}{2}, \frac{12}{3}, \frac{16}{4}, \frac{20}{5}, \ldots\}$.

Using the Exercises

On page 337, although children could refer to the number line or strips to answer exercise 3, point out the pattern based on the check for equivalent fractions. For example, in exercise 3A, 3 can be thought of as

$$\frac{3}{1} = \frac{6}{n}$$

From the cross products, we know that $3 \times n = 6 \times 1$, so n must be 2, and $\frac{3}{1} = \frac{6}{2}$.

In exercise 4, instruct the children to think of any whole number as a fraction and express the answer as an improper fraction.

Assignments (page 337)

Minimum: 1–2. Average: 1–4.
Maximum: 1–4.

Follow-up

If the children enjoy the games with the numbered cubes (see page 331), you might have them work out rules for a game in which they use four cubes and try to roll a pair which will form a fraction that can be matched with a whole number.

Duplicator Masters, page 59

PAGES 338–339

Objectives

Given a mixed numeral, the child will be able to write it as an improper fraction, and given an improper fraction, the child will be able to write it as a mixed numeral.

Preparation

Review naming whole numbers with improper fractions by asking questions similar to the following:

2 is the same as how many fourths? How many fourths must you have to name the whole number 2? (8)

How many thirds must you have to name the whole number 4? (12)

How many fifths must you have to name the whole number 3? (15)

Alternately, you might write equations such as the following on the chalkboard and ask children to give the missing number.

$\frac{4}{1} = \frac{16}{?}$, $\frac{24}{8} = \frac{3}{?}$, $\frac{5}{1} = \frac{10}{?}$,

$\frac{8}{1} = \frac{32}{?}$, $\frac{5}{1} = \frac{25}{?}$, $\frac{9}{1} = \frac{18}{?}$,

$\frac{7}{1} = \frac{28}{?}$, $\frac{6}{1} = \frac{24}{?}$, $\frac{2}{1} = \frac{10}{?}$



What are mixed numerals?

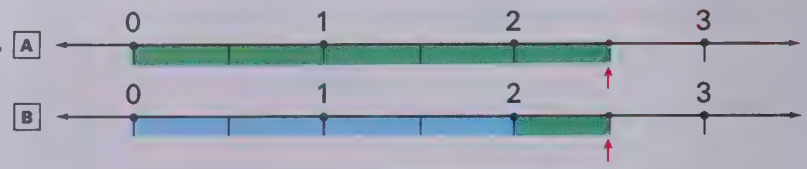
Discussing the Ideas

1. Symbols such as $2\frac{1}{2}$ and $3\frac{1}{4}$ are called mixed numerals. The two examples on the right will help you understand mixed numerals.

$2\frac{1}{2}$ means $2 + \frac{1}{2}$

$3\frac{1}{4}$ means $3 + \frac{1}{4}$

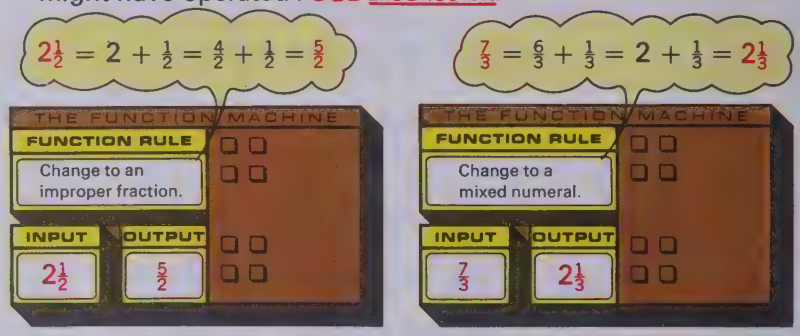
What is the meaning of $8\frac{2}{3}$? How do you think you should read the mixed numerals "$2\frac{1}{2}$," "$3\frac{1}{4}$," and "$8\frac{2}{3}$"? See Discussion.

2. 

You can name the point over the red arrow with a mixed numeral or with an improper fraction. How does the colored shading in A and B show this? You can think of the fractional number as 5 half units ($\frac{5}{2}$) or as 2 whole units plus 1 half unit ($2\frac{1}{2}$).

3. For each mixed numeral, you can find the improper fraction. For each improper fraction, you can find the mixed numeral.

A Can you explain the way these function machines might have operated? See Discussion.



B Choose other "inputs" for each machine and explain how to find the "outputs."

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Discussion

Study the text material with the children. Lead the children to read the symbol $2\frac{1}{2}$ as "two and one half." Stress the fact that $\frac{5}{2}$ and $2\frac{1}{2}$ name the same fractional number. To demonstrate this concept, use the number line or have the children use their strips to show, for example, $\frac{5}{2}$ when the purple strip is the unit. Then ask them how many purple strips may replace some of the red strips. They should realize that 5 red strips, or $\frac{5}{2}$, is the same as 2 purple and 1 red strip or $2\frac{1}{2}$. Or, using the same unit, you might have them study $3\frac{1}{4}$ by replacing a train of three purple and 1 white strip with a train of 13 white strips. In this case, make sure the children realize that the purple strip is the unit and the white strip is $\frac{1}{4}$ of it.

As shown in exercise 3, when children are to change a mixed numeral to an improper fraction, lead them to rename the whole number as an improper fraction and then add on that part of the number which is already expressed as a fraction:

$$4\frac{1}{2} = 4 + \frac{1}{2} = \frac{8}{2} + \frac{1}{2} = \frac{9}{2}$$

When children are to change an improper fraction to a mixed numeral, encourage them to ask themselves how many whole numbers are contained in this fraction and what

Using the Ideas

1. Give the correct mixed numeral for each sum.

A $2 + \frac{1}{4}$ (Answer: $2\frac{1}{4}$)

B $\frac{8}{4} + \frac{1}{4}$ (Answer: $2\frac{1}{4}$)

C $3 + \frac{1}{5}$ $3\frac{1}{5}$

D $\frac{15}{5} + \frac{1}{5}$ $3\frac{1}{5}$

E $1 + \frac{1}{8}$ $1\frac{1}{8}$

F $\frac{8}{8} + \frac{1}{8}$ $1\frac{1}{8}$

G $5 + \frac{1}{2}$ $5\frac{1}{2}$

H $\frac{10}{2} + \frac{1}{2}$ $5\frac{1}{2}$

I $1 + \frac{5}{6}$ $1\frac{5}{6}$

J $\frac{6}{6} + \frac{5}{6}$ $1\frac{5}{6}$

K $1 + \frac{3}{5}$ $1\frac{3}{5}$

L $\frac{5}{5} + \frac{3}{5}$ $1\frac{3}{5}$

M $8 + \frac{1}{4}$ $8\frac{1}{4}$

N $\frac{32}{4} + \frac{1}{4}$ $8\frac{1}{4}$

O $\frac{4}{9} + \frac{6}{9}$ $1\frac{1}{9}$

P $\frac{4}{9} + \frac{15}{9}$ $2\frac{1}{9}$

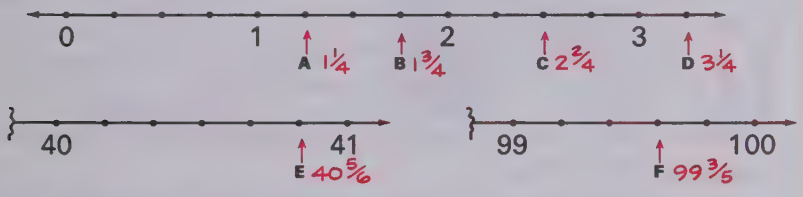
Q $\frac{6}{10} + \frac{7}{10}$ $1\frac{3}{10}$

R $\frac{10}{10} + \frac{9}{10}$ $1\frac{9}{10}$

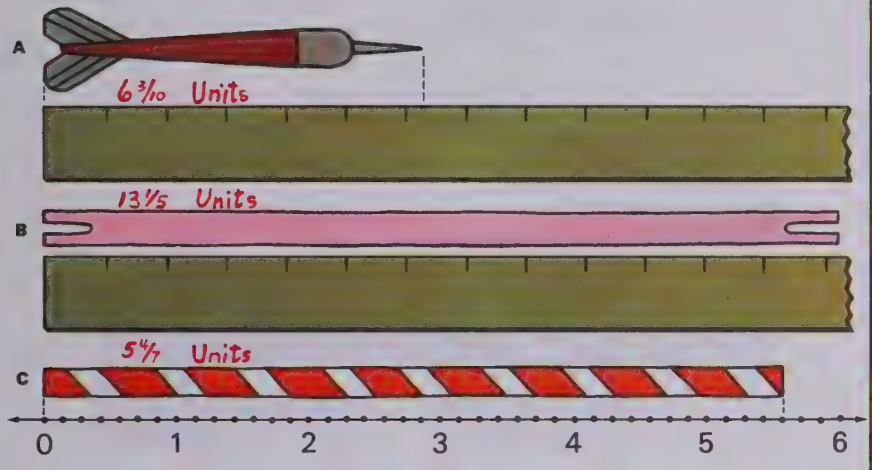
2. Give a mixed numeral for each fraction. Use exercise 1.

A $\frac{9}{4}$ $2\frac{1}{4}$ B $\frac{16}{5}$ $3\frac{1}{5}$ C $\frac{11}{6}$ $1\frac{5}{6}$ D $\frac{33}{4}$ $8\frac{1}{4}$ E $\frac{8}{5}$ $1\frac{3}{5}$ F $\frac{9}{8}$ $1\frac{1}{8}$ G $\frac{11}{2}$ $5\frac{1}{2}$

3. Give a mixed numeral for each point A through F.



★ **4.** Use mixed numerals to give the length of each object.



part of the number is left as a fraction:

$$1\frac{1}{5} = 1\frac{0}{5} + \frac{1}{5} = 2 + \frac{1}{5} = 2\frac{1}{5}$$

Using the Exercises

Assign the exercises on page 339 as independent work. Emphasize the fact that mixed numerals are simply fractional numbers written in a new way.

Note that in exercise 3 children must identify the fractional segments before they can give the mixed numeral. Although an improper fraction might correctly name the points, stress the preference here for the use of mixed numerals.

Exercise 4 is starred. This question is a challenge for the better students; however, let any child who is interested attempt it. The problem in parts A and B is that the child must decide how to subdivide the units to find a close approximation to the lengths of the objects. The closest answers are $4\frac{5}{6}$ units for part A and $10\frac{1}{5}$ units for part B, but accept any answers close to these, that is, $4\frac{2}{3}$, $10\frac{1}{4}$, and so on.

Assignments (page 339)

Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Follow-up

To give children practice in identifying equivalent fractions and mixed numerals, prepare a worksheet similar to the following.

How many names in the box are equal to the circled number?

$\frac{10}{4}$	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{15}{6}$
$\frac{12}{5}$	$(2\frac{1}{2})$	$\frac{7}{3}$	
$2 + \frac{1}{2}$	$\frac{8}{4}$	$(1 + \frac{3}{2})$	

$\frac{9}{5}$		$1\frac{3}{4}$	
$\frac{15}{6}$	$(\frac{7}{4})$	$\frac{21}{12}$	$\frac{12}{4}$
$(\frac{2}{2} + \frac{3}{4})$			
	$\frac{9}{8}$	$(1 + \frac{3}{4})$	$\frac{14}{8}$

$\frac{39}{15}$	$\frac{10}{4}$	$(\frac{8}{4} + \frac{6}{10})$	$(2 + \frac{6}{10})$
$\frac{72}{12}$	$(\frac{13}{5})$	$\frac{9}{3}$	
$2\frac{3}{5}$	$\frac{17}{5}$	$\frac{35}{7}$	$\frac{26}{10}$

$\frac{5}{2}$		$\frac{3}{3} + \frac{1}{3}$	
	$(1\frac{1}{3})$		
$\frac{7}{8}$	$\frac{8}{12}$		$\frac{3}{2}$
	$\frac{8}{6}$	$\frac{9}{6}$	
$1 + \frac{1}{3}$		$\frac{15}{6}$	$\frac{16}{12}$

Workbook, page 110

PAGES 340–341

Objective

Given addition problems involving fractions with unlike denominators (halves, thirds, and fourths), the child will be able to solve the problem by applying informal methods.

Preparation

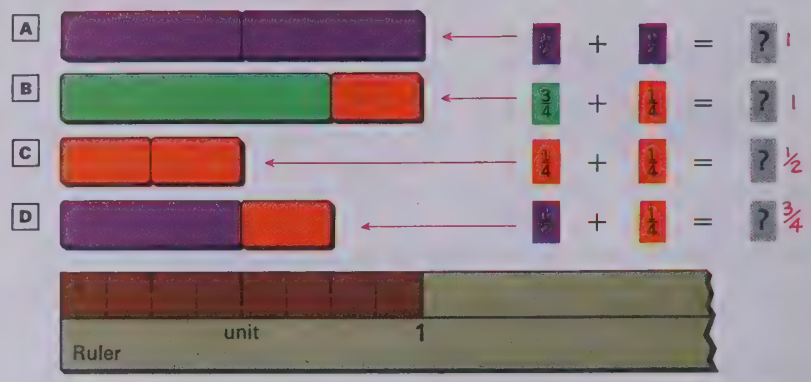
It would be helpful to review lowest-terms fractions. For example, write fractions such as $\frac{6}{8}$, $\frac{9}{12}$, $\frac{5}{15}$, and $\frac{2}{4}$ on the chalkboard. Ask children to give the lowest-terms fraction for each.

Investigation

It would be appropriate to have the children work in groups or as partners for this investigation. Although part D is illustrated, watch to see that children do *not* think $\frac{1}{2} + \frac{1}{4} = \frac{1}{6}$. Encourage any who wish to, to use the number line or the actual strips to check their answers. However, if any use the strips remind them to think in terms of the same unit as that shown in the text.

Let's find out more about addition.

Investigating the Ideas



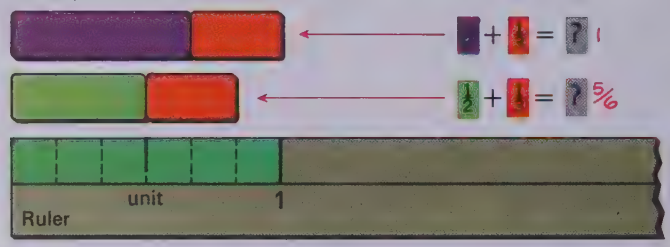
? Can you give a whole number or a lowest-terms fraction for each of the sums above? See Investigation.

Discussing the Ideas

1. Can you use the sums you found above, in the order given, to help you figure out these sums?

$1\frac{1}{2} + \frac{1}{2} =$ ▮, $2\frac{3}{4} + \frac{1}{4} =$ ▮, $3\frac{1}{4} + 3\frac{1}{4} =$ ▮, $4\frac{1}{2} + 1\frac{1}{4} =$ ▮

2. A Explain how to find these two sums.



B Can you use the sums above to help you find these sums?

$2\frac{2}{3} + 3\frac{1}{3} =$ ▮, $5\frac{1}{2} + 2\frac{1}{3} =$ ▮

Discussion

Spend ample time discussing the children's results for the investigation before proceeding to the discussion questions. Throughout the discussion, stress the preference for giving answers in whole numbers or lowest-terms fractions rather than in improper fractions or fractions of higher terms. For example, $3\frac{1}{4} + 3\frac{1}{4} = 6\frac{2}{4}$ would best be written $6\frac{1}{2}$. Similarly, we do not write $2\frac{2}{3} + 3\frac{1}{3} = 5\frac{3}{3}$ but $2\frac{2}{3} + 3\frac{1}{3} = 6$. Use other examples, but make sure your fractions have denominators that are simple and familiar to the children.

Using the Ideas



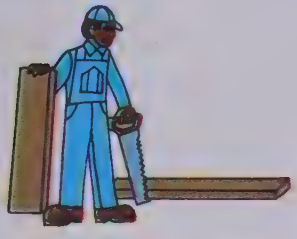
1. One Saturday, Jim had 4 lawns to mow. He mowed $2\frac{1}{2}$ lawns before lunch. How many lawns did he have left to mow in the afternoon? Solve these equations:
 - A $2\frac{1}{2} + n = 4$ $1\frac{1}{2}$
 - B $4 - 2\frac{1}{2} = n$ $1\frac{1}{2}$



2. Mrs. Brown baked 5 pies for a party. Only $3\frac{1}{3}$ pies were eaten. How many pies were left? Solve these equations:
 - A $3\frac{1}{3} + n = 5$ $1\frac{2}{3}$
 - B $5 - 3\frac{1}{3} = n$ $1\frac{2}{3}$

3. Jane lived $3\frac{1}{2}$ blocks from Sally, and Sally lived $2\frac{1}{4}$ blocks from school. If Jane walked to Sally's house and then to school, how far did she walk? Solve this equation: $3\frac{1}{2} + 2\frac{1}{4} = n$ $5\frac{3}{4}$

4. Mrs. White bought a beef roast that weighed $3\frac{1}{10}$ kilograms and a steak that weighed $2\frac{2}{5}$ kilograms. How many kilograms of meat did she buy? Solve this equation: $3\frac{1}{10} + 2\frac{2}{5} = n$ $5\frac{1}{2}$



5. A carpenter cut a board into two pieces. One piece was $2\frac{3}{10}$ metres long, and the other was $1\frac{1}{5}$ metres long. How long was the board before he cut it? Solve this equation:
 $2\frac{3}{10} + 1\frac{1}{5} = n$ $3\frac{1}{2}$



Using the Exercises

You might choose to treat the problems on page 341 orally, working through them with the class. Observe with the children that the two equations given for exercises 1 and 2 show the relationship between addition and subtraction. Help the children use reasoning to solve the problems. For example, in exercise 1, we may think, "If he mowed another $\frac{1}{2}$ lawn, he will have mowed 3 lawns, and since $3 + 1 = 4$, $2\frac{1}{2} + 1\frac{1}{2} = 4$. From this, it is easy to see that $4 - 2\frac{1}{2} = 1\frac{1}{2}$." Following discussion of these exercises, if time permits, give the children an opportunity to make up some fractional-number word problems of their own to present to the class either orally or on the chalkboard.

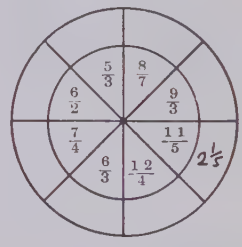
Assignments (page 341)

Minimum: 1–4, oral. Average: 1–4. Maximum: 1–5.

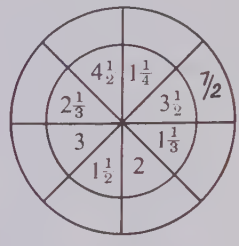
Follow-up/"Fraction Wheels"

"Fraction wheels" can help children practice finding mixed-numeral equivalents for improper fractions. To make a pair of these wheels, draw a pair of circles with diameters approximately 10 centimetres on a duplicating master. Divide each circle into eight parts, and then draw an 8-cm circle within each part, as shown. On the wedges of one wheel, write improper fractions, and in the other, write mixed numerals. Ask the children to change the improper fractions to mixed numerals and vice versa.

Change to mixed numerals.



Change to improper fractions.



Fraction Wheels

Resources for Active Learning

Mathematics in Modules, F2, Addison-Wesley.

Mathex: Graphing and Probability No. 6, "Extensions to Fractions," pp. 9–10, Encyclopaedia Britannica Publications Ltd.

Mathex: Numeration No. 7, pupil page 45, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 60
Workbook, page 111

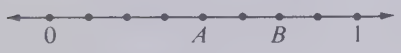
PAGES 342–343

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review any topics with which the children have had difficulty in the chapter. In particular, review how to associate exactly one number idea and one point on the number line with a given set of equivalent fractions. For example, draw on the chalkboard a number line divided into sixths. Mark certain points and ask children to name these points. Remind them that a whole set of equivalent fractions matches each of these points, and when feasible have them write out the first few members for such a set. For example:



$A \rightarrow \{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$

$B \rightarrow \{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$

1. Give the missing words.
 - A For each set of equivalent fractions, there is one __?__ on the number line. point
 - B For each fractional number, there is one set of __?__ fractions. equivalent
 - C For each set of equivalent fractions, we think of just one __?__ number. fractional
 - D To name a fractional number, we can choose any __?__ fraction from the set of equivalent fractions.
 - E The __?__ __?__ fraction is often used to name a fractional number. lowest-terms
 - F If two fractions are __?__, then they name the same fractional number. equivalent
 - G Each whole number is also a __?__ number. fractional
 - H If two fractions name the same fractional number, then they are __?__ equivalent

2. Match each set of fractions with a number-line picture.

 A $\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\}$ (2)

 B $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$ (9)

 C $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$ (1)

 D $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$ (7)

 E $\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\}$ (8)

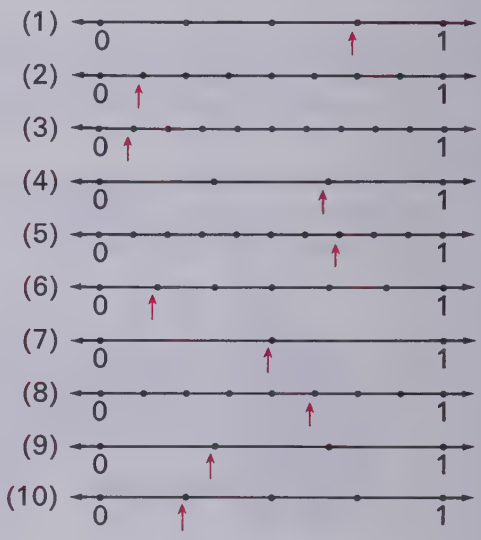
 F $\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \ldots\}$ (5)

 G $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$ (10)

 H $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$ (4)

 I $\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$ (6)

 J $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \ldots\}$ (3)



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Discussion

Have the children do the exercises independently. If you use this as a test page, you will want to check the papers yourself. However, no matter how the page is used, you should allow time for discussion when the children have completed their work.

Exercise 1 may be difficult for some of the children since children sometimes have trouble verbalizing ideas even though they understand them. However, the children should be encouraged to attempt exercise 1, and perhaps even refer to previous pages of their text in order to find the correct words for the various blanks. The children will benefit considerably from a discussion of each part of exercise 1.

The *Think* problem on page 343 is relatively simple if the children begin with Sam and trace the sharing back to Fred. Let the children who solve the problem explain it to the others. If no one solves it, guide the children through the steps as a class activity. For example, say: "Sam got half of Ned's cards. If Sam got 6, then Ned must have been given 12. If Ned got half of Tom's cards, Tom must have gotten 24 from Fred. If Tom only got half of Fred's cards, then 2×24, or 48, is the number of cards Fred started with."

3. Give the missing numerators and denominators.

A $\frac{1}{2} = \frac{5}{\square}$ 10 D $\frac{\square}{15} = \frac{1}{5}$ 3 G $\frac{3}{4} = \frac{30}{\square}$ 40 J $\frac{\square}{100} = \frac{3}{10}$ 30 M $\frac{6}{7} = \frac{\square}{14}$ 12

B $\frac{4}{\square} = \frac{1}{3}$ 12 E $\frac{1}{8} = \frac{\square}{40}$ 5 H $\frac{\square}{50} = \frac{2}{5}$ 20 K $\frac{2}{9} = \frac{\square}{36}$ 8 N $\frac{5}{\square} = \frac{50}{80}$ 8

C $\frac{1}{4} = \frac{\square}{40}$ 10 F $\frac{\square}{6} = \frac{2}{3}$ 4 I $\frac{3}{8} = \frac{\square}{24}$ 9 L $\frac{40}{\square} = \frac{4}{5}$ 50 O $\frac{3}{10} = \frac{\square}{50}$ 15

4. Answer T (true) or F (false).

A $\frac{1}{2} > \frac{1}{4}$ T D $\frac{5}{8} > \frac{1}{2}$ T G $\frac{1}{4} < \frac{1}{3}$ T

B $\frac{1}{3} < \frac{1}{4}$ F E $\frac{2}{3} < \frac{1}{2}$ F H $\frac{4}{6} > \frac{1}{3}$ T

C $\frac{1}{8} < \frac{1}{4}$ T F $\frac{3}{4} > \frac{1}{2}$ T I $\frac{1}{6} < \frac{1}{3}$ T

5. Give a mixed numeral for each sum.

A $5 + \frac{1}{3}$ $5\frac{1}{3}$ B $\frac{1}{4} + 2$ $2\frac{1}{4}$ C $6 + \frac{1}{7}$ $6\frac{1}{7}$

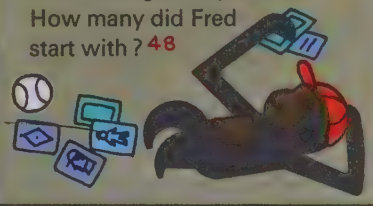
6. Give a lowest-terms fraction or a whole number for each sum.

A $\frac{1}{2} + \frac{1}{2}$ 1 C $\frac{1}{4} + \frac{1}{4}$ $\frac{1}{2}$ E $\frac{1}{4} + \frac{1}{2}$ $\frac{3}{4}$

B $\frac{1}{4} + \frac{3}{4}$ 1 D $\frac{1}{3} + \frac{2}{3}$ 1 F $\frac{1}{3} + \frac{1}{2}$ $\frac{5}{6}$

think

Fred gave Tom $\frac{1}{2}$ of his baseball cards. Tom gave Ned $\frac{1}{2}$ of the cards he got from Fred. Ned gave Sam $\frac{1}{2}$ the cards he got from Tom. Sam got only 6 cards. How many did Fred start with? 48

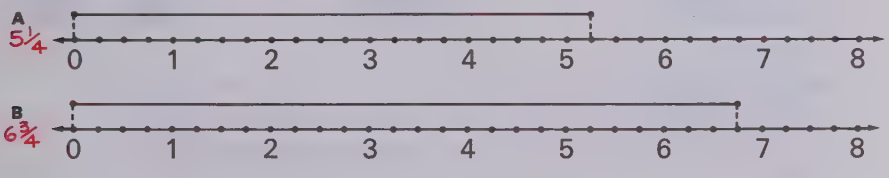


7. Give the correct sign (>, =, or <) for each ◍.

A $3 \text{ ◍ } 3\frac{1}{4}$ < C $\frac{5}{4} \text{ ◍ } \frac{6}{4}$ < E $\frac{10}{4} \text{ ◍ } \frac{5}{2}$ = G $2\frac{1}{3} \text{ ◍ } \frac{7}{3}$ =

B $2\frac{7}{8} \text{ ◍ } 3$ < D $\frac{5}{4} \text{ ◍ } \frac{5}{3}$ < F $7\frac{1}{8} \text{ ◍ } 6\frac{7}{8}$ > H $\frac{10}{4} \text{ ◍ } 2\frac{1}{2}$ =

8. Give the length of each segment.



Follow-up

The class may enjoy working some puzzles as recreation at the close of the year. Offer some intriguing problems in space perception, using toothpicks. For example, arrange 24 toothpicks to form nine squares, as illustrated. Ask the children to remove just 4 toothpicks and leave 5 squares, and to remove 8 toothpicks and leave 2 squares.

Original Square



One solution: 9 squares to 5 squares. Remove toothpicks represented by dashed lines.



One solution: 9 squares to 2 squares. Remove toothpicks represented by dashed lines.



Resources for Active Learning

Mathex: Numeration No. 7, "Puzzles and Games," pp. 43–45; "Fraction Dominoes," p. 46, Encyclopaedia Britannica Publications Ltd.

Workbook, page 112

PAGES 344–346

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Conduct a review of any algorithms with which the children have had difficulty in previous parts of the text. In particular, you might focus attention on finding the total amount when notation for money is used. You may choose to review the fact that the period, or dot, in money notation is used to separate the dollars from the cents, but avoid introducing ideas concerning the meaning of the decimal point or the relationship of this type of notation to fractional-number notation. The notation for fractional numbers using decimals will be introduced and explored in detail in Books 5 and 6.

For their work with money notation at this stage, the children should simply remember always to line up the dots or periods separating the dollars and cents. This practice will help to prevent confusion about whether they are adding numbers that represent cents or numbers that represent dollars. Note that we speak of finding the total amount or the difference in the amounts rather than of finding the sum or the difference. Addition and subtraction are operations reserved for numbers.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Measurement

1. Find the sums and differences.

A 27 + 58 85
B 126 − 59 67
C 728 + 694 1422
D 3427 + 96 3523
E 9803 − 2649 7539
F 5043 − 2174 4826
G 5043 − 3653 1390
H 528 + 367 + 493 1388
I 6203 − 2456 3747

2. Find the products.

A 38 × 6 228
B 57 × 8 456
C 54 × 12 648
D 68 × 25 1700
E 327 × 6 1962
F 438 × 5 2190
G 621 × 35 (21 735)
H 754 × 76 (57 304)
I 702 × 55 (38 610)
J 860 × 33 (28 380)

3. Find the quotients and remainders.

A 36 ÷ 4 9
B 75 ÷ 5 15
C 68 ÷ 4 17
D 93 ÷ 6 15R3
E 728 ÷ 5 (145R3)
F 436 ÷ 9 (48R4)
G 52 ÷ 13 4
H 148 ÷ 23 6R10
I 726 ÷ 91 7R89
J 343 ÷ 38 9R1

4. Find the total amounts.

A	B	C	D	E
$6.23	$13.24	$33.02	$ 31.26	$ 76.38
5.37	.67	67.00	5.48	142.50
6.43	50.83	45.28	126.50	5.78
$18.03	$64.74	$145.30	$163.24	$ 224.66

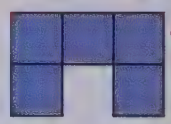
5. Find the difference of the amounts.

A	B	C	D	E
$9.28	$62.80	$37.86	$128.16	$675.24
5.67	5.43	29.59	54.37	99.98
$3.61	$57.37	$ 8.27	$73.79	$575.26

6. Give the area and perimeter for each figure.

A  a=6, p=10

B  a=5, p=10

C  a=5 p=12

7. Which is more?

A 90 minutes or 2 hours
B 3 days or 70 hours
C 50 weeks or 1 year
D 1000 days or 3 years

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Discussion

"Keeping in Touch" pages 344 to 346 may be assigned as independent work. While checking and discussing the answers for the exercises on page 344, you might have children present chalkboard demonstrations of how they worked some of the parts of exercises 4 and 5. It would be worthwhile for the children to see that the regrouping involved in work with these numbers which represent money is the same as that in the usual work with addition and subtraction. Children may enjoy checking their solutions for the short story problems on page 345, by working in groups of 3 or 4. Problems 5 and 6 might challenge many of the children. For example, it will be hard for them to realize, without much thought, that in problem 5 they must find two numbers whose sum is 15 and whose difference is 3. After a sufficient period of time, encourage those who are capable to share their solutions with the whole class.

As you discuss the chart on page 346, have the children explain how they read the chart in order to determine that the airline distance between Halifax and Charlottetown is 385 kilometres. Elicit from the children the observation that the row containing the name Halifax at the left



Short Stories



1 Found 26 shells on Monday and 47 shells on Tuesday. Found how many shells? 73

2 Vacation. Drove 447 kilometres on first day, 379 kilometres on second day, and 526 kilometres on third day. Drove how many kilometres? 1352

3 176 days in school year. Spend 45 cents for lunch each day. How much for lunches? $79.20

4 Had 77 cents. Spent 28 cents. Lost 7 cents. How much left? 42 cents

5 Played 15 games. Won 3 more than were lost. No ties. Won how many? 9



6 27 cats. Twice as many kittens as grown cats. How many grown cats? 9 How many kittens? 18



7 Auditorium. 35 rows of seats. 27 seats in each row. How many seats? 945

8 60 minutes in one hour. 24 hours in one day. 7 days in one week. How many minutes in one week? 10 080

9 Baked 93 cookies for school cookie sale. Put 5 in each bag. How many bags? How many left over? 18 bags; 3 left over

10 Movie: $1.75. Popcorn: 35¢. Paid for brother's movie and popcorn too. Paid how much? $4.20



and the column containing the name Charlottetown at the top meet at 385, and this tells them the airline distance between Halifax and Charlottetown. Similarly, other airline distances are found by looking for the intersection of a particular column and row. It might be helpful to have the children read some more distances from the chart before they do the exercises.

Exercise 6 has been starred simply because it involves large numbers. Therefore, it is quite possible that most children will understand the problem, even though they may make a mistake in organizing the numbers or in performing the two operations necessary to find the answer.

Follow-up

Your proficient problem-solvers would benefit from experiences in devising, analyzing, and solving multistep story problems. Many road maps have charts that show distance and travel time between major cities. These charts are usually constructed like the one at the top of page 346. (Similar ones may also be found in almanacs.) Once children learn to read such charts, they can use these, and many other kinds of data, to make up problems about travel, buildings, historic spots, monuments, and the like.

(See pages 344–345 for teaching suggestions for this page.)



Airline Distances

	Toronto	Charlottetown	St. John's	Halifax	Fredericton
Montreal	537	1096	2504	1201	726
Fredericton	1267	370	1764	470	
Halifax	1740	385	1505		
St. John's	3029	1436			
Charlottetown	1639				

The airline distance between Charlottetown and Halifax is 385 kilometres.



1. Use the chart to give the airline distances between the following points.
 - A Montreal and Halifax 1201
 - B Toronto and Fredericton 1267
 - C Halifax and Toronto 1740
 - D Toronto and Charlottetown 1639
 - E Charlottetown and St. John's 1436
 - F Montreal and St. John's 2504
 - G Charlottetown and Fredericton 370
 - H St. John's and Toronto 3029
2. How much farther is it from Montreal to St. John's than it is from Montreal to Halifax ? 1303 km
3. A salesman flew from Toronto to Charlottetown, then to Halifax, and from Halifax back to Toronto. How far did he travel ? 3764 km
4. If a plane takes 2 hours to fly from Montreal to Charlottetown, what is its average speed in kilometres per hour ? 548 km/h
5. An airliner made 7 round trips between Montreal and Toronto. How far did it fly ? 7518 km
6. Is it farther to go from Halifax to Montreal to Toronto than it is to go from Halifax to Toronto ? No

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How to Use the Activity Cards

Do you like to explore things for yourself? These Activity Cards will give you some exciting experiences with mathematics. Each card presents a different idea for you to explore. Often you will find that a card will give you ideas for additional activities on your own.



ACTIVITY CARD 1



In how many different ways can you measure yourself?

Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

- Pulse
- Height
- Weight
- Arm span
- Grip strength
- Length of step
- Number of calories used
- Area of bottom of foot
- Distance you can jump

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Activity Card 1

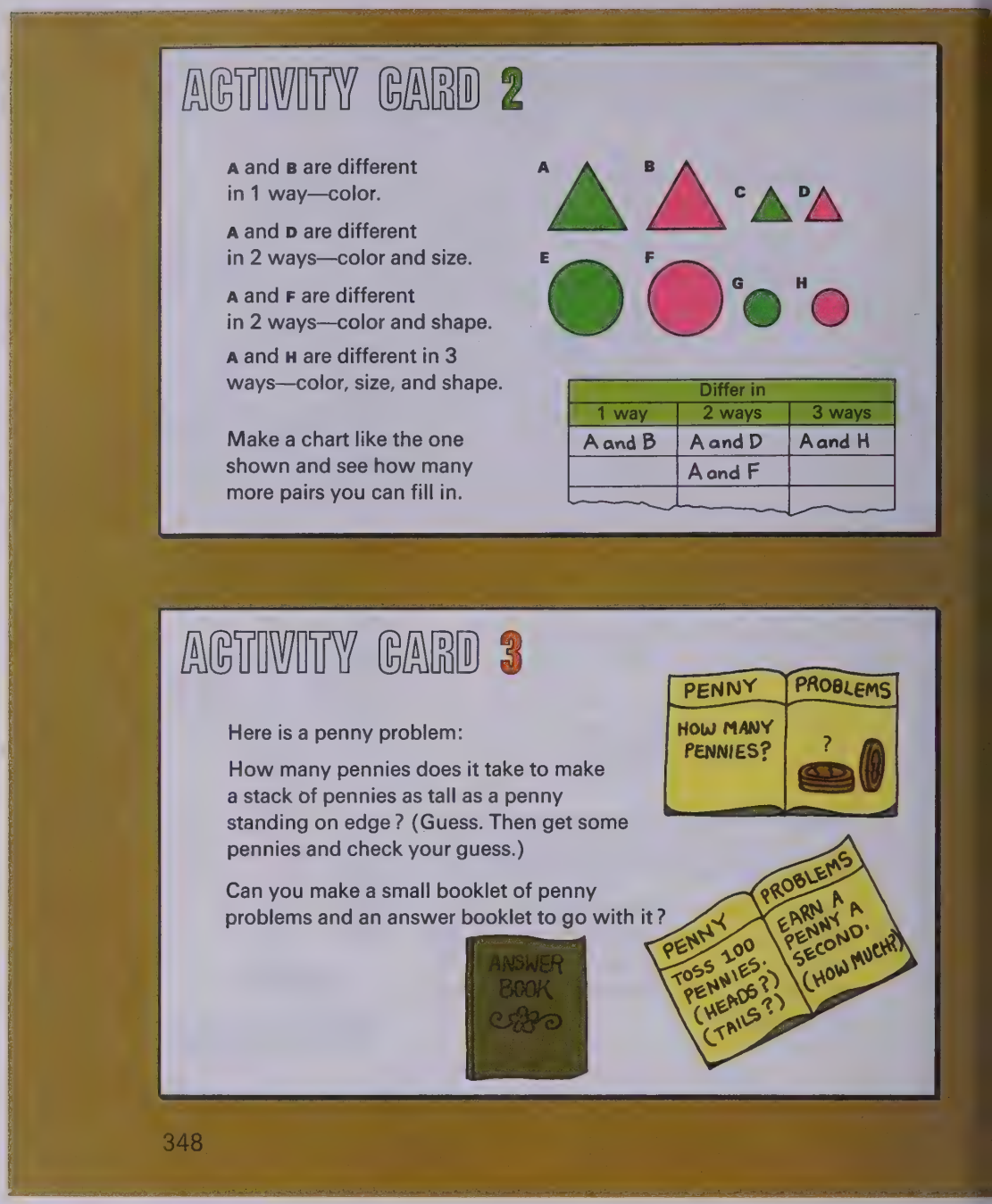
This card is designed as an open-ended experience for the children. They should be encouraged to search for unique things to measure about themselves. Some things not mentioned on the card would be breathing rate, comparisons of such things as their waist measurement and their neck measurement, or their height as compared to their arm span. Their lung capacity can be measured by blowing into a plastic bag or blowing up a large balloon. Other measurements could be shoe size, arm length, span length (span means length from the tip of the little finger to the tip of the thumb when the hand is outstretched), or cubit length (length from the tip of the elbow to the tip of the fingers). This activity could grow into a year-long study of the changes that take place in the children. If they were to keep an accurate chart recording a number of these measurements, they could fill in the chart at various times throughout the year and thus be able to observe the changes in themselves.

Activity Card 2

This card may require some extra explanation from you because the children must understand that we are thinking of the large figures as being the same size as each other, and of the small figures as being the same size as each other. This, then, gives us three different categories for these objects—size, shape, and color. Hence, the figures must differ in at least one way and may differ in as many as two or three ways. Once the children have had a chance to make a chart and to fill in as many pieces as they can, an interesting class project would be to make a chart on the chalkboard and fill in all possible pairs. An extension of this same activity can be accomplished by adding large red and green squares and small red and green squares. This, of course, would increase the number of entries on the chart considerably.

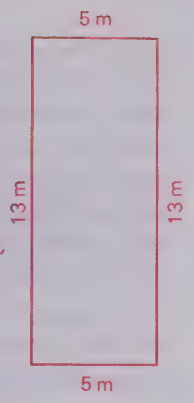
Activity Card 3

The answer to the sample penny problem is either 12 or 13. Twelve pennies stack almost as high as a penny on edge, and a stack of 13 is just a little greater than a penny on edge. Of course, this is probably a larger number of pennies than one would be likely to guess at first. Encourage the children, after they have worked with the penny problem, to generate a number of such problems of their own. At the bottom of the card are examples of some of the ideas that they might use for making up their own penny problems. Also encourage the children to make up some penny problems other than those that are suggested on the card. You should try to stimulate as much creativity as possible in making diagrams and designing the book of penny problems.



ACTIVITY CARD 4

Suppose you had 36 metres of fence wire. Here is one possible pen with **4 square corners** and a **whole number of metres on each side** that you could make with the wire.

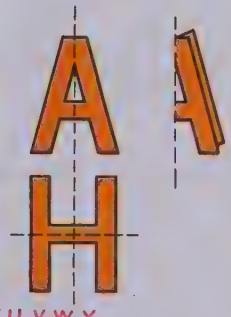


Can you draw on graph paper all the different pens of this type that you could make with the 36 metres of wire? 1×17, 2×16, 3×15, 4×14, 5×13, 6×12, 7×11, 8×10, 9×9

Which pen encloses the greatest area? 9×9
the smallest area? 1×17

ACTIVITY CARD 5

When you fold along a line of symmetry, one half exactly matches the other half.



The letter **A** has 1 line of symmetry.
The letter **H** has 2 lines of symmetry.

Which letters can you cut from old newspaper headlines and show, by folding, that they

have just 1 line of symmetry? A, B, C, D, E, K, M, T, U, V, W, Y

have exactly 2 lines of symmetry? H, I, X

O has infinitely many lines of symmetry.

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Activity Card 4

The concept involved in this activity card involves various rectangular geometric figures that have a constant perimeter but a variable area. The children will need graph paper for this activity, and they should be encouraged to find as many of the rectangles as they can. As long as the perimeter of the rectangle is held constant, the smallest area that can be produced is a rectangle 1 by 17, and the greatest area that can be produced is a square with the dimensions 9 by 9.

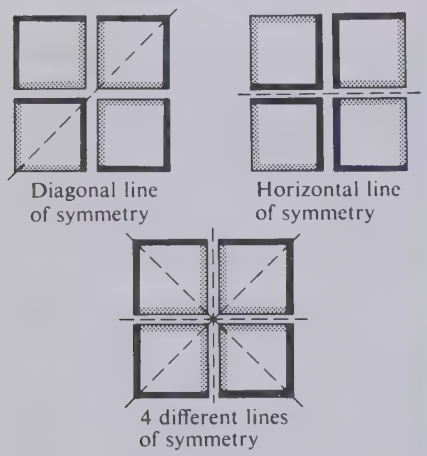
Activity Card 5

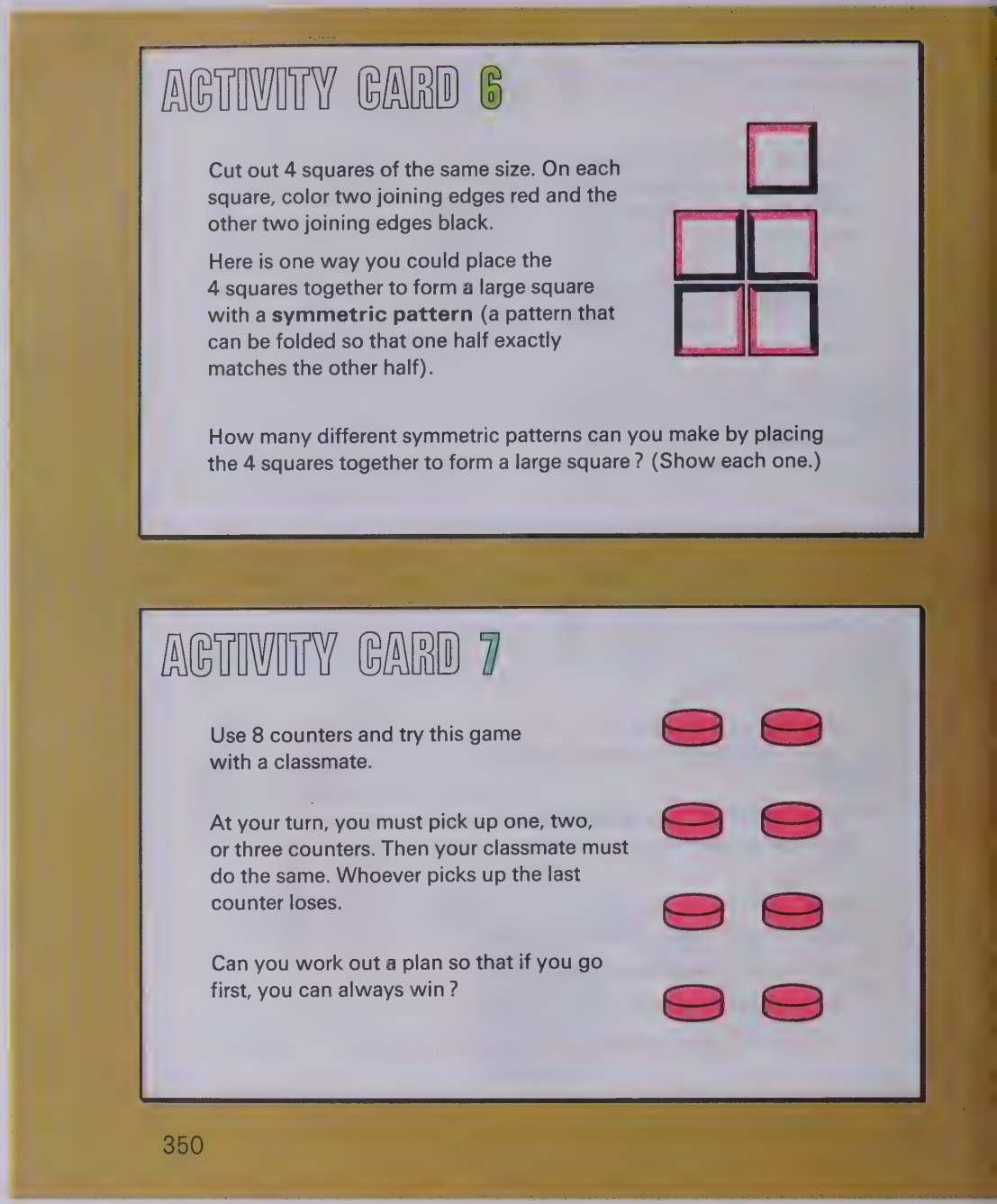
Cards 5 and 6 concern symmetric figures and lines of symmetry. In card 5, the children are working with letters of the alphabet that have one or more lines of symmetry. They will be particularly motivated by actually cutting large block letters from the headlines of newspapers. As follow-up, they can make a display of symmetric letters for the bulletin board. (It will be helpful if you or the children bring to class a number of newspapers that have unusually large headlines that they can cut out.)

Activity Card 6

The purpose of this card is to provide additional experiences in work with symmetry. Notice that the instructions on the card do not allow for rotational symmetry. On this card, the child is simply investigating symmetric figures, or, as they are called here, "symmetric patterns"—figures which have a line of symmetry. There are many patterns that children can form that will be symmetric about a given line. When the children discover one of these patterns, you should encourage them to draw a picture of their result. Sometimes the figure will be symmetric about a vertical line, as is shown on the activity card. At other times, however, the figure will be symmetric about a horizontal line or about one of the diagonals. Also, the figure can be put together so that it will have more than one line of symmetry.

Examples of Possibilities





Activity Card 7

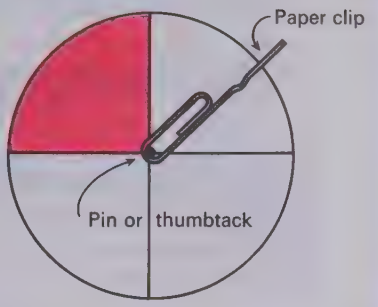
This card involves considerable reasoning and strategy in planning the various moves. Once the children have become familiar with the game and have played it for a while, you might suggest that they try it with other numbers of counters, such as 10, 11, 12, or 13. Also, rather than having them just play at random, encourage them to work out specific strategies so that if they go first they can always win. The most obvious way to work out a strategy for this game is to start from the end of the game. For example, if one thinks about the fact that if at any time there are just 4, 3, or 2 counters when it is his turn, he can clearly win, since when there are 4, he can pick up three; when there are 3, he can pick up 2; and when there are 2 left, he can pick up 1, thus forcing the other player to pick up the last counter. Therefore, the game with 8 counters involves reducing the number of counters to 4, 3, or 2 when it is his turn to play. Of course, this can be done by reducing the original number of counters (8 in this game) to 5 when it is the other person's turn to play. Therefore, the strategy, if it is his turn to play first, would be for him to take 3 counters. Then his opponent must take 1, 2, or 3 counters from the 5 counters, so the total left will be 2, 3, or 4. As more counters are used to play the game, the strategy becomes more complicated.

ACTIVITY CARD 8

Make a spinner like this one. Guess how many "reds" you will get in 10 spins. Try it.

Now guess how many "reds" you will get in 100 spins. Make a table and record your results for 100 spins.

Can you guess how many "reds" for 1000 spins?

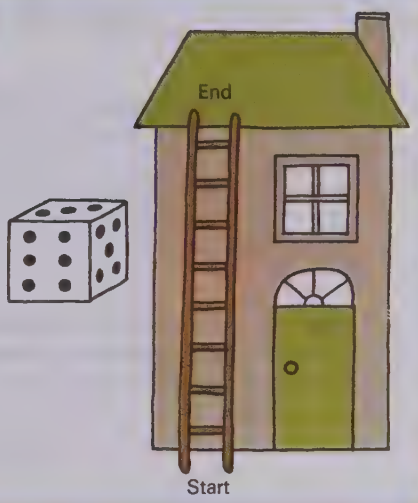


ACTIVITY CARD 9

How many tosses will it take you to climb to the roof if you follow these rules?

1. If you toss a 1 or a 6, go down one rung (if you can).
2. Go up one rung if you toss a 2, 3, 4, or 5.

Guess how many tosses. Then use a die and try it.



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Activity Card 8

This activity card involves some basic ideas of probability. Note with the children that, since one fourth of the circular region is shaded red and three fourths is white, one would expect on a large number of spins to get a reading of three fourths whites and one fourth reds. The spinner, itself, is relatively easy to make and you should encourage the children to make their own spinners out of heavy paper or cardboard. They can do this simply by drawing a circle and shading in one fourth of it, as is indicated on the card. Then by unbending one end of a paper clip and using a thumb tack in the centre of the circle, they can produce the spinner. If the children become interested in these probability ideas, you might encourage them to make other spinners, coloring different portions of the circular region with various colors and guessing how many of each color they will get in a hundred spins.

Activity Card 9

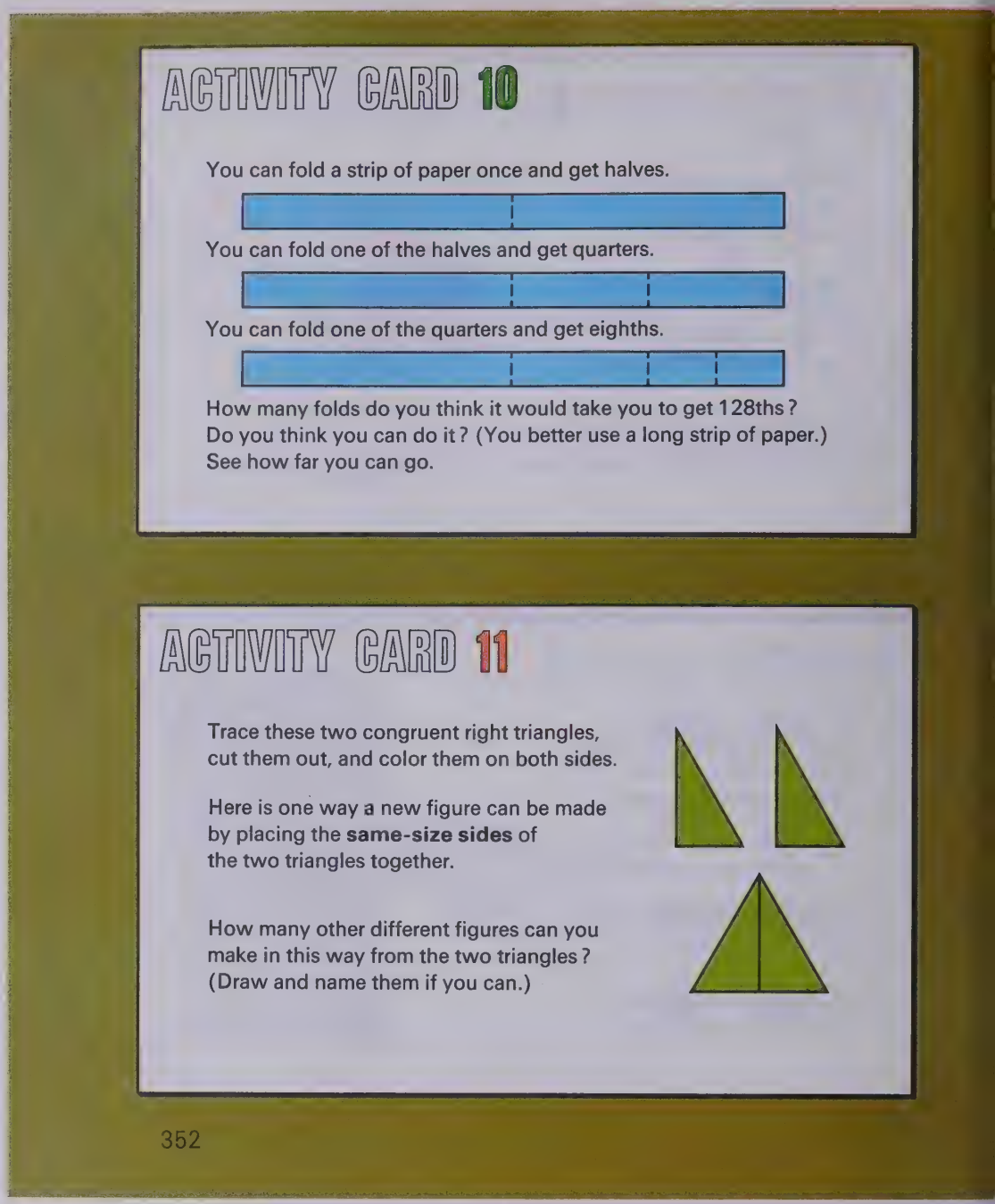
This card involves another interesting probability idea. Since the cube has 6 different faces and the rules of the game indicate that you move up on 4 of the faces, and move down on 2 of the faces, it becomes a game in which, for each 2 jumps up, one would expect to have 1 jump down. Although it often will not work out this way, over a large number of tosses, the result turns out to be 2 up and 1 down.

Activity Card 10

This card involves physical manipulations leading to an understanding of fraction concepts. A strip of paper 50 cm to 75 cm long will give the child an opportunity to investigate fractions as small as $\frac{1}{128}$ or even $\frac{1}{256}$. Simply by careful folding, the children can use a strip of paper cut from the long side of an A4 sheet and get as small as $\frac{1}{128}$. You might find it instructive to have the children label the various regions on their strip of paper according to the fractional part of the strip represented. Specifically, you could have them write the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, . . . on the strip of paper.

Activity Card 11

This activity card involves the idea of working with congruent triangles and producing other geometric figures. Do not belabor the teaching of the words "congruent" or "right triangle," but give the children an opportunity to say what they think these words mean. Some of them will possibly know the essential meaning from past experiences with geometric ideas. Six different figures can be formed by following the rules on the card: three different isosceles triangles and three different parallelograms (one of which is a rectangle).

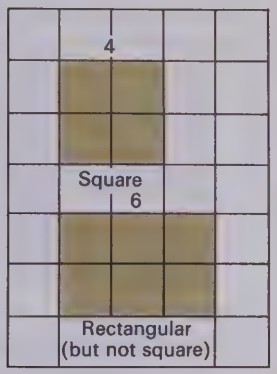


ACTIVITY CARD 12

Some numbers are square and some are rectangular but not square. Some numbers are neither square nor rectangular.

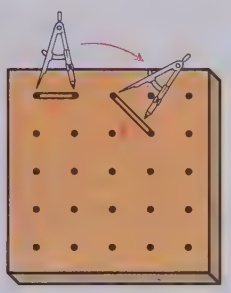
Can you color graph paper to show which numbers up to 20 are square and which are rectangular?

The rectangular numbers should have more than one row and column.



ACTIVITY CARD 13

"Line segment" *B* is longer than "line segment" *A* because the nails (dots) of *B* are farther apart than the nails of *A*. (Use your compass to check this.)



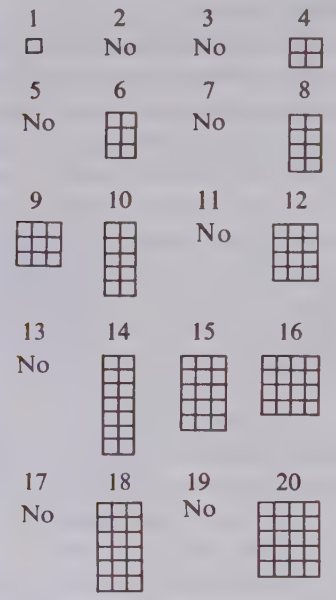
How many **different lengths** of "line segments" can you find on the geoboard and draw on dot paper? 14

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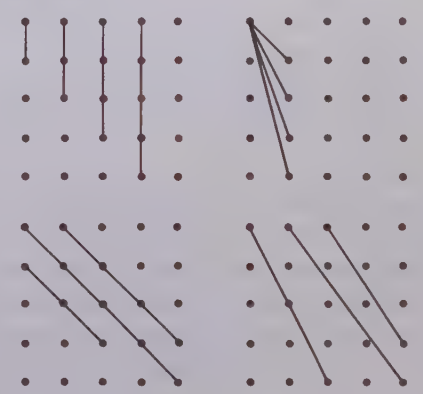
Activity Card 12

This card introduces the concept of square and rectangular numbers, which are also composite numbers if they are greater than 1. At the same time, this activity identifies the prime numbers, in that those numbers greater than 1 which are neither square nor rectangular are the primes. Note the restriction on rectangular numbers: the rectangular numbers, in order to qualify, cannot simply be a long string of squares having just one row or column. That is, they must have more than one row and more than one column to qualify as a rectangular number. You will need to provide the children with sheets of graph paper to do this activity. Do not belabor the fact that, according to our definitions here, the numbers 1, 2, and 3 are neither square nor rectangular, even though we might consider the number 1 square since we did not place the restriction of more than one row and column on the square numbers. The important point is not to allow the children to become confused or worried about whether or not the number 1 is square. Simply point out that we prefer to consider 1 square, but 2 and 3 are not rectangular. The numbers 2 and 3 are prime, but there is no need for you to call this fact to the attention of the children at this stage.



Activity Card 13

Ideally, each child should have his own geoboard to work with for the best results from this card. It is also suggested that you supply ditto sheets containing large numbers of 5×5 dot arrays so that the children can record and report the results of their findings on the geoboard. Also, the dot paper can be used instead of the geoboard if geoboards are unavailable. Card 13 is not at all difficult if the children are not expected to find every single length of line segment that is possible on the geoboard. All the children will be able to find some of the line segments simply by experimenting with rubber bands and connecting them to the various pegs or nails on the geoboard. It is also important that the children report with a drawing on dot paper the various segments they have discovered. There are 14 different lengths of segments possible on the geoboard:

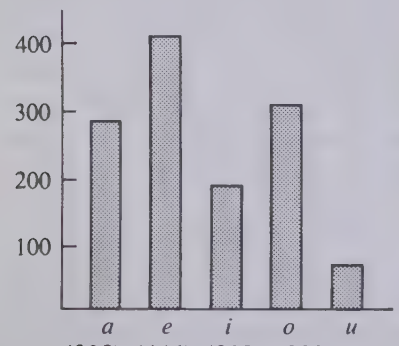


Activity Card 14

This activity has to do with symmetry and lines of symmetry. On this card, the child should be given complete freedom to cut and investigate various kinds of symmetric figures that he can produce from folding paper and then cutting off pieces of the paper. As the card suggests, however, the child should begin by producing some of the more common geometric figures.

Activity Card 15

This card gives children an opportunity to explore the frequency with which vowels are used. Because of the time required for counting the letters, you might want to suggest that children work in groups to gather the data for their graphs. If so, you could have the children work in groups of five, with each child in the group responsible for making the count for just one page. Then the children in each group could combine their data and work independently to make their bar graphs. Accuracy of the totals in an activity like this is not to be expected, but the results should yield a bar graph having the general proportions of that below.

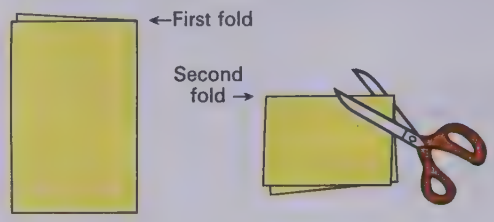


(305) (446) (208) (333) (73)

It may interest the children to know that of *all* the letters, consonants and vowels, the ten used most frequently are, in order, *e*, *t*, *o*, *a*, *n*, *i*, *r*, *s*, *h*, *d*.

ACTIVITY CARD 14

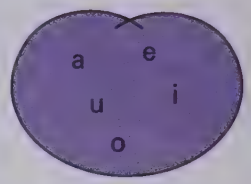
Fold a piece of paper twice and cut a piece off the corner. Then unfold the piece you cut off.



Can you cut off a piece that will unfold to be a square? a rectangle? a diamond? a four-pointed star? another interesting figure?

ACTIVITY CARD 15

Which one of these letters do you think is used most in the English language?



Use pages 146, 147, 203, 245, and 293 in this book to check your guess.

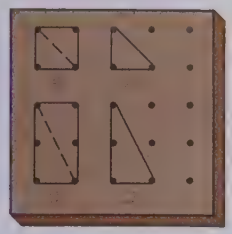
Make a bar graph to show your findings.

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ACTIVITY CARD 16

The area of *A* is 1, so the area of triangle *B* is __? __. ½

The area of *C* is 2, so the area of triangle *D* is __? __. 1



Can you find and draw on dot paper a triangle with an area of $1\frac{1}{2}$? 2? 3? 4? $4\frac{1}{2}$? 6? 8?

ACTIVITY CARD 17

Use large ruled graph paper and cut out a "5-square field" like this.

How many "5-square fields" of different shapes can you cut from your piece of graph paper?

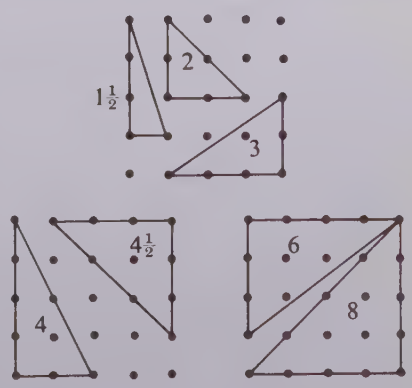


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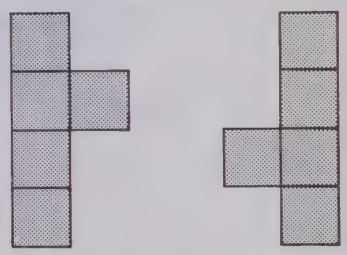
Activity Card 16

Even though it would be quite difficult for the children to find all the suggested triangles, most of the children can find some of them. You may find it necessary to give some instruction to get the children started on this card. For example, be sure they understand that the area of A is 1 so the area of B is half that, or one half, and that the area of C is 2, and hence the area of D is half that, or one. It may be helpful to have the children work in groups on this particular card so that more of the triangles can be found. Also, you should not expect the children to find all the triangles; certainly though, if any of them do, they are to be congratulated. In fact, the children should be congratulated and encouraged for any of the triangles that they find. The triangles shown here are not necessarily unique for the area of the figure given. That is, the triangle with an area of 2, for example, could well have been a "long triangle" involving a side 4 units, a side 1 unit, and then the hypotenuse connecting the other two points.

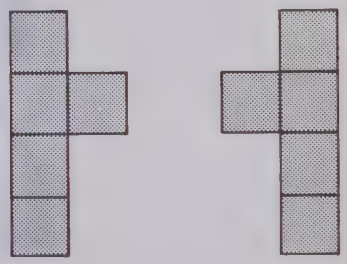


Activity Card 17

This card is designed to provide the children with an opportunity to work with the idea of a constant area that may vary in shape and, often, in perimeter. Of course, the card could easily be extended to such activities as finding fields having other numbers of squares, such as 6, 7, 8, or 9. There is an intentional ambiguity in this card: it is not completely clear when one should consider two shapes as being the same and two shapes as being different. Therefore, you will need to establish some ground rules with the children concerning what is meant by figures that are alike and figures that are different.



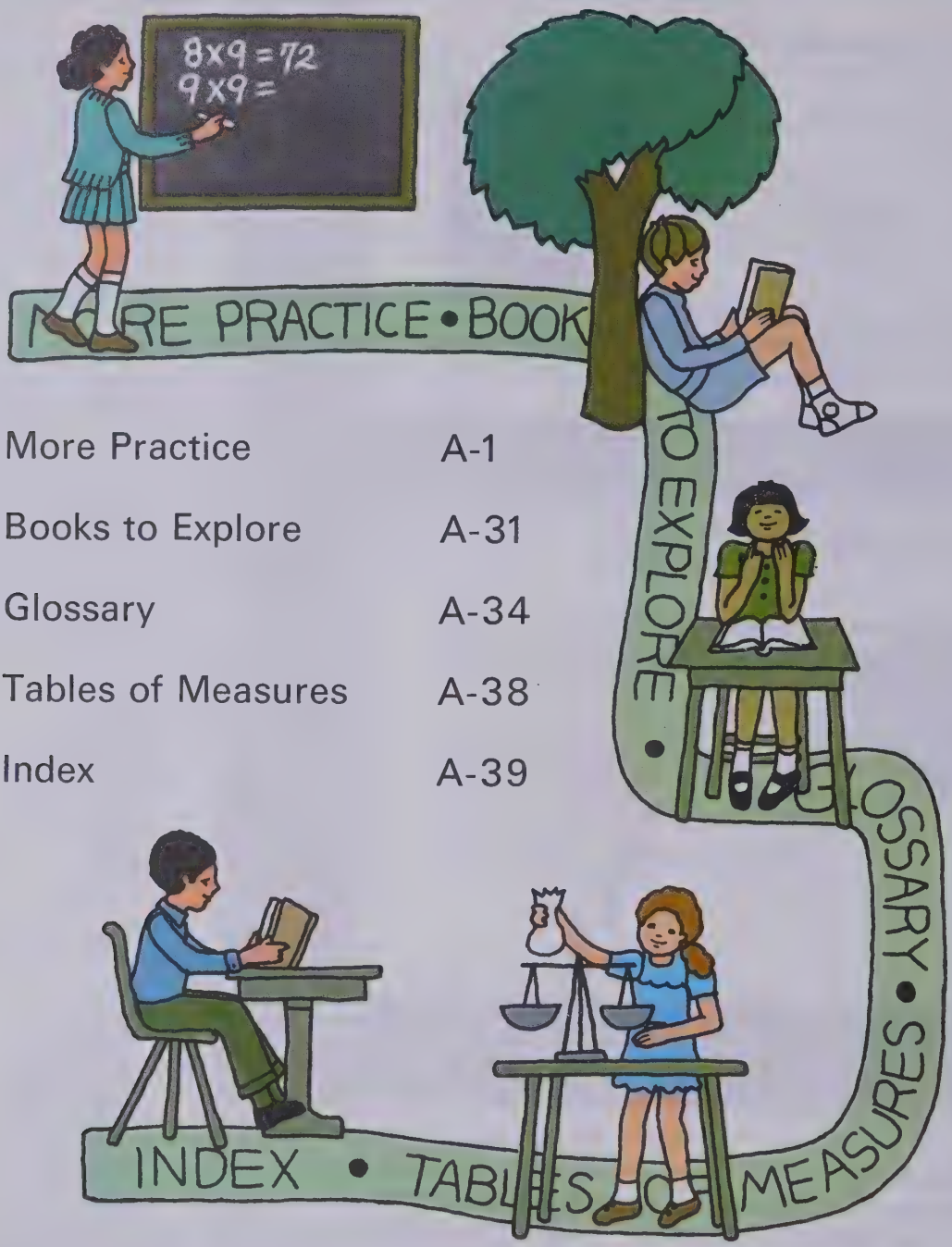
These should probably be considered the *same* since a 180° rotation matches them.



These should probably be considered *different* since one must be cut out and turned over to achieve a perfect matching.

Although the figures above suggest rules, the children should be encouraged to make their own rules about "alike" and "different." Also, you will need to establish rules such as, "One side of each must touch a full side of some other square" (the squares must not be corner to corner since the figure will not "hang together" very well this way).

Appendix



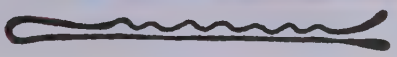
More Practice	A-1
Books to Explore	A-31
Glossary	A-34
Tables of Measures	A-38
Index	A-39

Set 1 *For use with page 13*

Give the length of each object to the nearest centimetre.

1. 13 
2. 6 
3. 12 
4. 10 
5. 14 

Give the length of each object to the nearest half centimetre.

6. 6½ 
7. 2½ 
8. 2½ 
9. 12 
10. 5 

Use your ruler to draw segments that have these lengths.

11. 5 centimetres

12. 13 centimetres

13. $10\frac{1}{2}$ centimetres

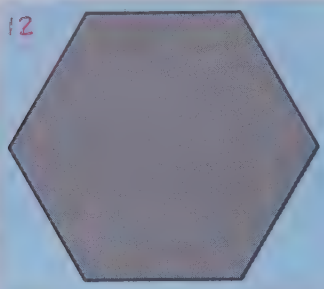
14. $5\frac{1}{2}$ centimetres

Reflected answers, Set 1: 13, 5, 6, 3, 12, 6, $6\frac{1}{2}$, 12, $5\frac{1}{2}$

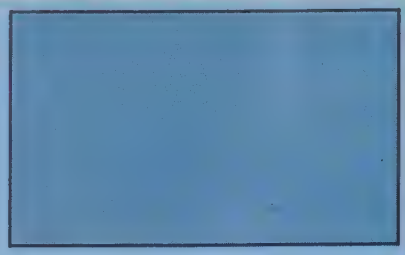
Set 2 *For use with page 17*

Use your centimetre ruler to find the perimeter of each figure.

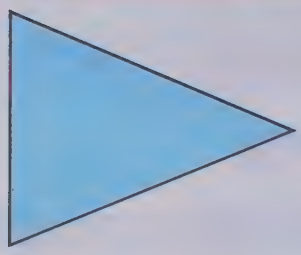
1. 12



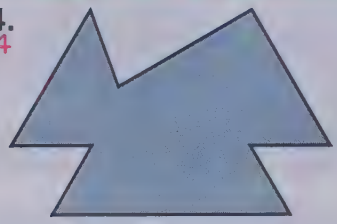
2. 16



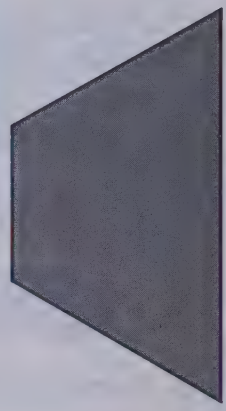
3. 11



4. 14



5. 13



Reflected answers, Set 2: 1. 15, 2. 16

Set 3 *For use with page 23*

Use your centimetre ruler to find the area of each region.

1. 10



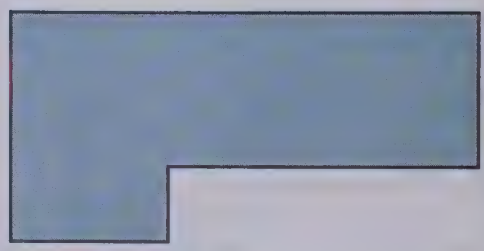
2. 9



3. 9



4. 14

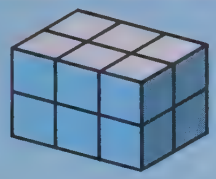


Reflected answers, Set 3: 1. 10, 2. 9

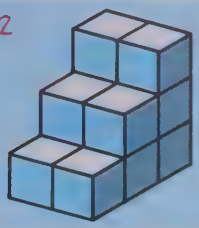
Set 4 *For use with page 27*

Find the volume of each figure.

1. 12



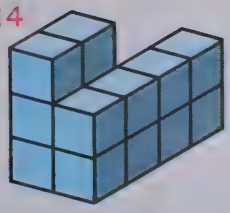
2. 12



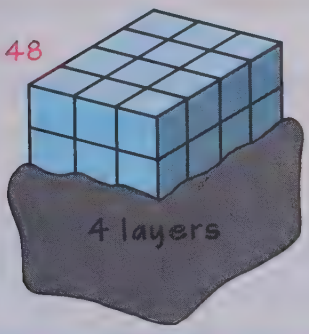
3. 10



4. 14



5. 48



6. 28



Reflected answers, Set 4: 1. 12, 2. 12, 3. 10

Set 5 *For use with page 37*

Write a numeral for each exercise.

1. 8 tens and 4 ones 84
2. 3 tens and 6 ones 36
3. 2 tens and 7 ones 27
4. 4 hundreds, 6 tens, and 5 ones 465
5. 9 hundreds, 1 ten, and 8 ones 918
6. 7 thousands, 4 hundreds, 2 tens, and 0 ones 7420

Give the missing digits.

7. 86 means 8 tens and 6 ones.
8. 43 means 4 tens and 3 ones.
9. 629 means 6 hundreds, 2 tens, and 9 ones.
10. 7585 means 7 thousands, 5 hundreds, 8 tens, and 5 ones.

Reflected answers, Set 5: 1. 84, 4. 465, 7. 8, 6, 10. 7, 5, 8, 5

Set 6 *For use with page 43*

Give the missing digits in the order indicated.

1. 436 means 4 hundreds, 3 tens, and 6 ones.
2. 208 means 2 hundreds, 0 tens, and 8 ones.
3. 500 means 5 hundreds, 0 tens, and 0 ones.
4. 1639 means 1 thousands, 6 hundreds, 3 tens, and 9 ones.
5. 6072 means 6 thousands, 0 hundreds, 7 tens, and 2 ones.
6. 36 428 means 3 ten thousands, 6 thousands, 4 hundreds, 2 tens, and 8 ones.
7. 789 201 means 7 hundred thousands, 8 ten thousands, 9 thousands, 2 hundreds, 0 tens, and 1 ones.

Write the numeral for each exercise.

8. Six hundreds, eight tens, and four ones 684
9. Seven tens, nine hundreds, and zero ones 970
10. Two thousands, five hundreds, eight tens, and two ones 2582
11. Four ten thousands, zero thousands, two hundreds, one ten, and six ones 40 216
12. Three hundred thousands, five ten thousands, eight thousands, zero hundreds, six tens, and zero ones 358 060
13. Eight hundred seventy 870
14. Four hundred thirty-two 432
15. Six thousand twenty-one 6021

Solve the equations.

16. $139 = 100 + 30 + n$ 9
17. $9567 = 9000 + n + 60 + 7$ 500
18. $7124 = n + 100 + 20 + 4$ 7000
19. $64\,729 = n + 4000 + 700 + 20 + 9$ 60 000
20. $283\,657 = n + 80\,000 + 3000 + 600 + 50 + 7$ 200 000

Reflected answers, Set 6: 1. 4, 3, 6 2. 2, 0, 8 3. 5, 0, 0 4. 1, 6, 3, 9 5. 6, 0, 7, 2 6. 3, 6, 4, 2, 8 7. 7, 8, 9, 2, 0, 1 8. 684 9. 970 10. 2582 11. 40 216 12. 358 060 13. 870 14. 432 15. 6021 16. 9 17. 500 18. 7000 19. 60 000 20. 200 000

Set 7 *For use with page 45*

Give the correct sign (< or >) for each ◍.

1. 640 ◍ 64
2. 306 ◍ 360
3. 4180 ◍ 4108
4. 7020 ◍ 7200
5. 4672 ◍ 4762
6. 98 677 ◍ 97 677
7. 25 340 ◍ 25 430
8. 49 674 ◍ 49 654
9. 319 487 ◍ 319 847
10. 560 390 ◍ 506 390
11. 22 814 ◍ 22 841
12. 27 055 ◍ 27 550
13. 48 745 ◍ 47 845
14. 61 064 ◍ 71 064
15. 74 383 ◍ 73 833
16. 69 804 ◍ 68 940
17. 56 236 ◍ 56 336
18. 42 477 ◍ 44 477
19. 25 840 ◍ 25 804
20. 62 126 ◍ 61 216

Reflected answers, Set 7: 1. > 2. < 3. > 4. < 11. < 12. < 13. > 14. < 15. > 16. > 17. < 18. < 19. > 20. >

Set 8 *For use with page 49*

For each numeral, give the number of thousands.

1. 9804 9
2. 5252 5
3. 69 414 69
4. 97 893 97
5. 28 467 28
6. 38 773 38
7. 636 532 636
8. 791 313 791
9. 381 668 381
10. 4 437 519 4437
11. 6 193 575 6193
12. 50 992 482 50 992
13. 48 242 902 242
14. 886 106 426 886 106
15. 489 870 018 489 870
16. 427 322 045 427 322

Write the number the heavy black digit stands for.

17. 77 846 7000
18. 38 944 900
19. 81 018 80 000
20. 45 872 800
21. 45 966 5000
22. 136 972 30 000
23. 626 490 6000
24. 108 774 100 000
25. 682 565 80 000
26. 122 592 500
27. 645 777 600 000
28. 473 210 10
29. 481 206 400 000
30. 121 534 20 000
31. 906 516 900 000
32. 628 361 8000

Reflected answers, Set 8: 1. 9 2. 5 3. 69 4. 97 5. 28 6. 38 7. 636 8. 791 9. 381 10. 4437 11. 6193 12. 50 992 13. 242 14. 886 106 15. 489 870 16. 427 322 17. 7000 18. 900 19. 80 000 20. 800 21. 5000 22. 30 000 23. 6000 24. 100 000 25. 80 000 26. 500 27. 600 000 28. 10 29. 400 000 30. 20 000 31. 900 000 32. 8000

Set 9 *For use with page 53*

Write the numeral for each part.

1. 50 more than one million 1 000 050
2. 300 more than six million 6 000 300
3. 10 thousand more than 22 967 687 22 977 687
4. 1 more than 3 132 679 3 132 680
5. 20 more than 17 778 088 17 778 108
6. 3 million more than 25 651 225 28 651 225
7. 1 thousand more than 6 000 575 6 001 575
8. 400 thousand more than 790 771 321 791 171 321
9. 60 million more than 327 966 151 387 966 151
10. 700 million more than 221 914 428 921 914 428

Reflected answers, Set 9: 1. 1 000 020, 2. 6 000 300, 3. 22 977 687

Set 10 *For use with page 61*

1. 5 empty bottles.
 6 full bottles.
 How many bottles? 11
2. 12 boys.
 5 wear glasses.
 How many do not wear glasses? 7
3. 8 puppies.
 Gave away 3.
 How many left? 5
4. 14 sandwiches.
 Ate 10 sandwiches.
 How many left? 4
5. 15 seeds.
 Planted 7.
 How many left to plant? 8
6. Had 3 goldfish.
 Bought 6 guppies.
 How many fish now? 9
7. 16 marbles.
 Lost 9.
 How many left? 7
8. 4 marbles needed to make a dozen in the box. How many already in the box? 8
9. Had 14¢. Spent 4¢.
 Lost 5¢.
 How much left? 5¢

Reflected answers, Set 10: 1. 11, 4. 4, 7. 7

Set 11 *For use with page 63*

Find the sums and differences.

1. 7 +7 = 14	**2.** 14 −6 = 8	**3.** 9 +2 = 11	**4.** 15 −9 = 6	**5.** 12 +0 = 12	**6.** 4 +8 = 12	**7.** 11 −7 = 4
8. 16 −7 = 9	**9.** 8 +8 = 16	**10.** 12 −7 = 5	**11.** 17 −9 = 8	**12.** 4 +7 = 11	**13.** 3 +8 = 11	**14.** 7 +9 = 16

Solve for ***n***.

15. $11 = n + 3$ 8	**18.** $5 + n = 13$ 8	**21.** $16 - n = 9$ 7	**24.** $n + 4 = 11$ 7
16. $7 + n = 12$ 5	**19.** $10 - n = 6$ 4	**22.** $13 - 5 = n$ 8	**25.** $9 - n = 0$ 9
17. $n + 6 = 6$ 0	**20.** $n - 9 = 6$ 15	**23.** $n + 5 = 14$ 9	**26.** $4 + 8 = n$ 12

Reflected answers, Set 11: 1. 14, 2. 8, 3. 11, 4. 6, 5. 12, 6. 12, 7. 4, 8. 9, 9. 16, 10. 5, 11. 8, 12. 11, 13. 11, 14. 16, 15. 8, 16. 5, 17. 0, 18. 8, 19. 4, 20. 15, 21. 7, 22. 8, 23. 9, 24. 7, 25. 9, 26. 12

Set 12 *For use with page 69*

Find the sums.

1. 36 +45 = 81	**2.** 53 +49 = 102	**3.** 67 +56 = 123	**4.** 84 +59 = 143	**5.** 65 +86 = 151	**6.** 78 +57 = 135
7. 48 39 +55 = 142	**8.** 36 77 +65 = 178	**9.** 27 47 +53 = 127	**10.** 78 33 +59 = 170	**11.** 47 64 +88 = 199	**12.** 34 76 +95 = 205
13. 73 67 +96 = 236	**14.** 43 58 +84 = 185	**15.** 99 84 +93 = 276	**16.** 78 68 +87 = 233	**17.** 87 46 +97 = 230	**18.** 54 69 +75 = 198

Reflected answers, Set 12: 1. 81, 2. 102, 3. 123, 4. 143, 5. 151, 6. 135, 7. 142, 8. 178, 9. 127, 10. 170, 11. 199, 12. 205, 13. 236, 14. 185, 15. 276, 16. 233, 17. 230, 18. 198

Set 13 *For use with page 73*

Find the sums.

1. 426 + 564 = 990	**2.** 378 + 483 = 861	**3.** 507 + 563 = 1070	**4.** 614 + 496 = 1110	**5.** 834 + 768 = 1602	**6.** 976 + 457 = 1433
7. 573 + 327 = 900	**8.** 538 + 423 = 961	**9.** 675 + 398 = 1073	**10.** 417 + 895 = 1312	**11.** 629 + 586 = 1215	**12.** 879 + 654 = 1533
13. 4375 + 2765 = 7140	**14.** 4832 + 6549 = 11 381	**15.** 5804 + 3679 = 9483	**16.** 3768 + 5954 = 9722	**17.** 6548 + 7693 = 14 241	
18. 34 785 + 56 427 = 91 212	**19.** 53 949 + 49 038 = 102 987	**20.** 64 838 + 58 147 = 122 985	**21.** 85 729 + 67 256 = 152 985	**22.** 76 617 + 76 365 = 152 982	
23. 587 + 46 + 394 = 1027	**24.** 786 + 497 + 385 = 1668	**25.** 639 + 417 + 63 = 1119	**26.** 628 + 479 + 853 = 1960	**27.** 397 + 654 + 836 = 1887	**28.** 865 + 437 + 942 = 2244
29. 5783 + 4969 + 3807 = 14 559	**30.** 8796 + 4357 + 9438 = 22 591	**31.** 3468 + 6859 + 4307 = 14 634	**32.** 8799 + 8365 + 4879 = 22 043	**33.** 2218 + 3961 + 9726 = 15 905	

Solve the equations.

34. 327 + 27 + 8 = *n* 362	**41.** *n* = 47 + 363 + 62 + 5 477
35. 75 + 826 + 97 = *n* 998	**42.** *n* = 621 + 7 + 36 + 24 688
36. 486 + 43 + 8 + 7 = *n* 544	**43.** *n* = 157 + 70 + 32 + 8 267
37. *n* = 59 + 39 + 3 + 41 142	**44.** 528 + 30 + 93 + 4 = *n* 655
38. *n* = 166 + 85 + 32 283	**45.** 12 + 409 + 53 + 3 = *n* 477
39. 103 + 99 + 6 + 4 = *n* 212	**46.** *n* = 220 + 49 + 8 + 75 352
40. 591 + 54 + 37 + 2 = *n* 684	**47.** *n* = 28 + 802 + 66 + 7 903

ζ1. 1881, 38. ζζζ4, 3ϛ. 3ϛ0, 3ε. 88ε, ϛ4. ϛ11, ϛ8. 888
ε. 1ε0ζ, e. 1ϛ33, ζ3. 1ζϛ0, ϛϛ. 1888, ζε. 1ϛ11, ϛe. 1ϛe0,
Reflected answers, Set 13: 1. 88ε, ζ. 1ε8, 3. 1ζϛ0, 4. 0ϛ11,

Set 14 *For use with page 83*

Find the differences.

1. 543 − 57 = 486	**2.** 671 − 63 = 608	**3.** 676 − 87 = 589	**4.** 847 − 50 = 797	**5.** 648 − 39 = 609	**6.** 764 − 58 = 706
7. 462 − 187 = 275	**8.** 884 − 546 = 338	**9.** 763 − 268 = 495	**10.** 846 − 457 = 389	**11.** 572 − 208 = 364	**12.** 416 − 397 = 19
13. 735 − 708 = 27	**14.** 643 − 576 = 67	**15.** 733 − 353 = 380	**16.** 454 − 328 = 126	**17.** 365 − 176 = 189	**18.** 522 − 436 = 86
19. 8742 − 967 = 7775	**20.** 2606 − 812 = 1794	**21.** 2388 − 394 = 1994	**22.** 9438 − 6189 = 3249	**23.** 9563 − 3894 = 5669	**24.** 7436 − 2879 = 4557

Reflected answers, Set 14: 1. 389, 5. 809, 3. 289, 4. 197, 2. 809, 9. 207, 8. 89, 13. 27, 14. 189, 15. 380, 16. 126, 17. 189, 18. 86

Set 15 *For use with page 87*

Solve each short story problem.

1. Sir Wilfrid Laurier. Born 1841. Became Prime Minister 1896. How old was he ? 55

2. First U.S. Satellite launched 1958. How many years ago ?
Answer depends on date solved.

3. Alexander Graham Bell. Born 1847. Died 1922. How long ago did he live ?
Answer depends on date solved.

4. Elizabeth I. Reigned 1558-1603. Victoria. Reigned 1837-1901. How much longer did Victoria reign ?
19 years

5. Prince Edward Island became a province in 1873. How many years ago ?
Answer depends on date solved.

6. First human heart transplant done by Christian Barnard in 1967. How many years ago ?
Answer depends on date solved.

Reflected answers, Set 15: 1. 55 years old, 4. 19 years

Set 16 *For use with page 89*

Find the differences.

1. 611 − 584 = 27	**2.** 840 − 323 = 517	**3.** 727 − 458 = 269	**4.** 450 − 151 = 299	**5.** 364 − 265 = 99	**6.** 436 − 328 = 108
7. 7286 − 6824 = 462	**8.** 4431 − 3872 = 559	**9.** 7048 − 5991 = 1057	**10.** 8283 − 7774 = 509	**11.** 5344 − 3675 = 1669	**12.** 6842 − 5469 = 1373

Reflected answers, Set 16: 1. 27, 2. 517, 3. 269, 4. 299, 5. 99, 6. 108

Set 17 *For use with page 93*

Find the total amounts.

1. \$3.14 + 6.95 = \$10.09	**2.** \$4.36 + 5.42 = \$9.78	**3.** \$6.86 + 7.54 = \$14.40	**4.** \$8.53 + 3.47 = \$12.00	**5.** \$14.38 + 9.76 = \$24.14
6. \$3.64 + 8.76 + 9.57 = \$21.97	**7.** \$ 2.77 + 39.81 + 2.86 = \$45.44	**8.** \$9.21 + 1.48 + 7.98 = \$18.67	**9.** \$8.82 + 6.33 + 5.87 = \$21.02	**10.** \$12.52 + 3.47 + 6.54 = \$22.53

Find the differences in the amounts.

11. \$6.97 − .83 = \$6.14	**12.** \$8.65 − 3.86 = \$4.79	**13.** \$9.42 − .98 = \$8.44	**14.** \$10.43 − 8.75 = \$ 1.68	**15.** \$15.27 − 6.38 = \$8.89
16. \$8.65 − 2.39 = \$6.26	**17.** \$19.63 − 13.56 = \$ 6.07	**18.** \$38.21 − 29.39 = \$ 8.82	**19.** \$16.54 − 7.82 = \$8.72	**20.** \$7.86 − 3.96 = \$3.90
21. \$18.31 − 5.49 = \$12.82	**22.** \$7.63 − 2.49 = \$5.14	**23.** \$23.21 − 7.54 = \$15.67	**24.** \$8.79 − 3.97 = \$4.82	**25.** \$53.23 − 17.45 = \$35.78

Reflected answers, Set 17: 1. \$10.09, 2. [illegible], 3. \$14.40, 4. \$12.00, 5. \$24.14, 6. \$21.97, 7. \$45.44, 8. \$18.67, 9. \$21.02, 10. \$22.53, 11. \$6.14, 12. \$4.79, 13. \$8.44, 14. \$1.68, 15. \$8.89, 16. \$6.26, 17. \$6.07, 18. \$8.82, 19. \$8.72, 20. \$3.90, 21. \$12.82, 22. \$5.14, 23. \$15.67, 24. \$4.82, 25. \$35.78

Set 18 *For use with page 119*

Solve the equations.

1. $5 + 5 + 5 + 5 = n$ 20
2. $4 \times 5 = n$ 20
3. $20 - 5 = n$ 15
4. $15 - 5 = n$ 10
5. $10 - 5 = n$ 5
6. $5 - 5 = n$ 0
7. $20 \div 5 = n$ 4
8. $6 + 6 + 6 + 6 + 6 = n$ 30
9. $5 \times 6 = n$ 30
10. $30 - 6 = n$ 24
11. $24 - 6 = n$ 18
12. $18 - 6 = n$ 12
13. $12 - 6 = n$ 6
14. $6 - 6 = n$ 0
15. $30 \div 6 = n$ 5

Reflected answers, Set 18: 1. 20, 5. 20, 8. 30, 9. 30

Set 19 *For use with page 129*

Find the products.

1. 2×4 8	16. 1×6 6	31. 3×9 27	46. 9×2 18	61. 6×7 42	76. 5×1 5				
2. 1×9 9	17. 8×3 24	32. 7×5 35	47. 0×0 0	62. 4×6 24	77. 1×8 8				
3. 3×3 9	18. 7×1 7	33. 9×6 54	48. 0×8 0	63. 2×5 10	78. 2×6 12				
4. 7×2 14	19. 6×9 54	34. 8×1 8	49. 3×7 21	64. 4×2 8	79. 6×5 30				
5. 5×5 25	20. 2×2 4	35. 5×9 45	50. 1×7 7	65. 8×4 32	80. 3×1 3				
6. 0×3 0	21. 2×8 16	36. 4×8 32	51. 6×2 12	66. 7×3 21	81. 1×3 3				
7. 4×5 20	22. 4×3 12	37. 2×1 2	52. 7×0 0	67. 9×5 45	82. 6×1 6				
8. 1×4 4	23. 2×7 14	38. 7×8 56	53. 9×3 27	68. 9×9 81	83. 5×2 10				
9. 9×7 63	24. 3×6 18	39. 8×9 72	54. 4×1 4	69. 2×3 6	84. 8×8 64				
10. 7×4 28	25. 3×8 24	40. 8×2 16	55. 5×0 0	70. 9×8 72	85. 8×5 40				
11. 6×3 18	26. 9×1 9	41. 3×5 15	56. 8×7 56	71. 4×0 0	86. 9×4 36				
12. 5×8 40	27. 1×2 2	42. 5×6 30	57. 1×5 5	72. 4×9 36	87. 7×9 63				
13. 7×7 49	28. 2×9 18	43. 7×6 42	58. 3×2 6	73. 6×6 36	88. 3×4 12				
14. 6×8 48	29. 4×4 16	44. 5×4 20	59. 9×0 0	74. 6×4 24	89. 5×3 15				
15. 5×7 35	30. 4×7 28	45. 9×6 54	60. 1×1 1	75. 0×1 0	90. 6×0 0				

Reflected answers, Set 19: 1. 8, 2. 9, 3. 9, 4. 14, 5. 25, 6. 0, 7. 20, 8. 4, 9. 63, 10. 28, 11. 18, 12. 40, 13. 49, 14. 48, 15. 35

Set 20 *For use with page 132*

Solve each short story problem.

1. Chorus frog: 5 centimetres long. Bullfrog: 3 times as long. How long? 15 cm
2. Field mouse: 15 centimetres. Beaver: 5 times as long. How long? 75 cm
3. 7 kilometres to Albany. 9 times as far to Mumford. How far to Mumford? 63 km
4. Joe: 4 blocks from school. Jack: 4 times as far. How far? 16 blocks
5. Dan: 9 years old. Tom: twice as old. How old? 18
6. Dolphin: 6 metres long. Killer whale: 3 times as long. How long? 18 m

Reflected answers, Set 20: 1. 15 cm, 4. 16 blocks

Set 21 *For use with page 135*

Solve the equations.

1. $7 \times n = 49$ 7
2. $49 \div 7 = n$ 7
3. $3 \times n = 18$ 6
4. $18 \div 3 = n$ 6
5. $8 \times n = 48$ 6
6. $48 \div 8 = n$ 6
7. $n \times 5 = 45$ 9
8. $45 \div 5 = n$ 9
9. $n \times 4 = 20$ 5
10. $20 \div n = 4$ 5
11. $n \times 8 = 64$ 8
12. $64 \div 8 = n$ 8
13. $3 \times n = 24$ 8
14. $24 \div n = 3$ 8
15. $7 \times n = 42$ 6
16. $42 \div 7 = n$ 6
17. $n \times 5 = 15$ 3
18. $15 \div 5 = n$ 3
19. $n \times 9 = 63$ 7
20. $63 \div 9 = n$ 7
21. $n \times 2 = 14$ 7
22. $14 \div 2 = n$ 7
23. $n \times 7 = 35$ 5
24. $35 \div 7 = n$ 5
25. $n \times 5 = 40$ 8
26. $40 \div 5 = n$ 8
27. $n \times 7 = 21$ 3
28. $21 \div n = 7$ 3
29. $n \times 5 = 10$ 2
30. $10 \div 5 = n$ 2
31. $n \times 3 = 12$ 4
32. $12 \div 3 = n$ 4
33. $n \times 4 = 28$ 7
34. $28 \div 4 = n$ 7
35. $n \times 8 = 72$ 9
36. $72 \div 8 = n$ 9

Reflected answers, Set 21: 1. 7, 2. 7, 3. 6, 4. 6, 5. 6, 6. 6, 7. 9, 8. 9, 9. 5, 10. 5, 11. 8, 12. 8, 13. 8, 14. 8, 15. 6, 16. 6, 17. 3, 18. 3, 19. 7, 20. 7, 21. 7, 22. 7, 23. 5, 24. 5, 25. 8, 26. 8, 27. 3, 28. 3, 29. 2, 30. 2, 31. 4, 32. 4, 33. 7, 34. 7, 35. 9, 36. 9

Set 22 *For use with page 137*

1. 5 ÷ 1 5	**18.** 10 ÷ 5 2	**35.** 30 ÷ 5 6	**52.** 5 ÷ 5 1	**69.** 18 ÷ 6 3
2. 21 ÷ 7 3	**19.** 28 ÷ 4 7	**36.** 24 ÷ 3 8	**53.** 49 ÷ 7 7	**70.** 27 ÷ 9 3
3. 2 ÷ 1 2	**20.** 6 ÷ 3 2	**37.** 8 ÷ 4 2	**54.** 36 ÷ 9 4	**71.** 0 ÷ 5 0
4. 40 ÷ 8 5	**21.** 16 ÷ 2 8	**38.** 54 ÷ 6 9	**55.** 56 ÷ 7 8	**72.** 4 ÷ 1 4
5. 18 ÷ 3 6	**22.** 24 ÷ 6 4	**39.** 6 ÷ 2 3	**56.** 2 ÷ 2 1	**73.** 7 ÷ 7 1
6. 72 ÷ 8 9	**23.** 35 ÷ 7 5	**40.** 6 ÷ 6 1	**57.** 32 ÷ 4 8	**74.** 14 ÷ 2 7
7. 42 ÷ 7 6	**24.** 54 ÷ 9 6	**41.** 3 ÷ 1 3	**58.** 15 ÷ 3 5	**75.** 25 ÷ 5 5
8. 12 ÷ 4 3	**25.** 30 ÷ 6 5	**42.** 15 ÷ 5 3	**59.** 64 ÷ 8 8	**76.** 18 ÷ 2 9
9. 9 ÷ 1 9	**26.** 35 ÷ 5 7	**43.** 24 ÷ 8 3	**60.** 24 ÷ 4 6	**77.** 10 ÷ 2 5
10. 12 ÷ 3 4	**27.** 4 ÷ 2 2	**44.** 81 ÷ 9 9	**61.** 72 ÷ 9 8	**78.** 12 ÷ 6 2
11. 8 ÷ 1 8	**28.** 40 ÷ 5 8	**45.** 4 ÷ 4 1	**62.** 0 ÷ 9 0	**79.** 28 ÷ 7 4
12. 48 ÷ 8 6	**29.** 0 ÷ 1 0	**46.** 20 ÷ 5 4	**63.** 1 ÷ 1 1	**80.** 9 ÷ 9 1
13. 45 ÷ 5 9	**30.** 7 ÷ 1 7	**47.** 16 ÷ 8 2	**64.** 45 ÷ 9 5	**81.** 63 ÷ 7 9
14. 32 ÷ 8 4	**31.** 48 ÷ 6 8	**48.** 3 ÷ 3 1	**65.** 8 ÷ 2 4	**82.** 42 ÷ 6 7
15. 8 ÷ 8 1	**32.** 21 ÷ 3 7	**49.** 20 ÷ 4 5	**66.** 9 ÷ 3 3	**83.** 56 ÷ 8 7
16. 16 ÷ 4 4	**33.** 27 ÷ 3 9	**50.** 36 ÷ 4 9	**67.** 63 ÷ 9 7	**84.** 36 ÷ 6 6
17. 14 ÷ 7 2	**34.** 12 ÷ 2 6	**51.** 6 ÷ 1 6	**68.** 18 ÷ 9 2	**85.** 20 ÷ 4 5

Reflected answers, Set 22: 1 · 2 · 5 · 3 · 3 · 5 · 18 · 5 · 18 · 1 · 50 · 5 ·
0 · 10 · 3 · 36 · 3 · 24 · 4 · 24 · 2 · 23 · 1 · 25 · 5 · 7 · 1 · 25 · 5 · 3 · 8 · 3 · 9 ·

Set 23 *For use with page 141*

Solve each story problem.

1. 7 boys. 3 sandwiches for each boy. How many sandwiches? 21

2. 40 Boy Scouts. 8 in each patrol. How many patrols? 5

3. 7 pies. 6 pieces per pie. How many pieces of pie? 42

4. 8 donuts per box. 8 boxes. How many donuts? 64

5. 9 teams. 5 players on each team. How many players? 45

6. 54 pieces of candy. 9 children. How many pieces per child? 6

Reflected answers, Set 23: 1 · 54 · 5 · 2 · 3 · 45

Set 24 *For use with page 159*

Find the products.

1. 5 × 40 200	**8.** 8 × 300 2400	**15.** 9 × 60 540	**22.** 3 × 700 2100	**29.** 6 × 70 420				
2. 8 × 60 480	**9.** 50 × 9 450	**16.** 800 × 7 5600	**23.** 6 × 900 5400	**30.** 70 × 9 630				
3. 4 × 70 280	**10.** 6 × 800 4800	**17.** 7 × 50 350	**24.** 5 × 900 4500	**31.** 70 × 8 560				
4. 8 × 90 720	**11.** 700 × 7 4900	**18.** 600 × 7 4200	**25.** 9 × 90 810	**32.** 600 × 6 3600				
5. 40 × 8 320	**12.** 5 × 50 250	**19.** 9 × 30 270	**26.** 4 × 400 1600	**33.** 4 × 300 1200				
6. 20 × 9 180	**13.** 300 × 5 1500	**20.** 70 × 7 490	**27.** 6 × 500 3000	**34.** 800 × 8 6400				
7. 80 × 5 400	**14.** 2 × 70 140	**21.** 8 × 200 1600	**28.** 900 × 3 2700	**35.** 700 × 4 2800				

Solve each short story problem.

36. 9 fields.
700 plants per field.
How many plants? 6300

37. 8 pages.
300 words per page.
How many words? 2400

Reflected answers, Set 24: 1. 500, 2. 480, 3. 2400, 4. 450, 5. 2100, 6. 3600, 7. 1200, 8. 2400, 9. 450, 10. 630

Set 25 *For use with page 161*

Solve the equations.

1. $n \times 5 = 250$ 50	**10.** $n \times 4 = 1200$ 300	**19.** $n \times 200 = 1000$ 5
2. $n \times 5 = 2500$ 500	**11.** $n \times 7 = 490$ 70	**20.** $n \times 70 = 420$ 6
3. $n \times 6 = 480$ 80	**12.** $n \times 3 = 900$ 300	**21.** $n \times 8 = 3200$ 400
4. $n \times 6 = 4800$ 800	**13.** $n \times 8 = 720$ 90	**22.** $n \times 600 = 1200$ 2
5. $n \times 3 = 150$ 50	**14.** $n \times 5 = 4000$ 800	**23.** $n \times 9 = 540$ 60
6. $n \times 3 = 1500$ 500	**15.** $n \times 6 = 3600$ 600	**24.** $n \times 400 = 2800$ 7
7. $n \times 4 = 160$ 40	**16.** $n \times 5 = 4500$ 900	**25.** $n \times 50 = 300$ 6
8. $n \times 9 = 3600$ 400	**17.** $n \times 20 = 180$ 9	**26.** $n \times 70 = 560$ 8
9. $n \times 3 = 270$ 90	**18.** $n \times 70 = 350$ 5	**27.** $n \times 300 = 2100$ 7

Reflected answers, Set 25: 1. 20, 2. 200, 3. 300, 4. 40, 5. 2, 6. 8, 7. 9

Set 26 *For use with page 163*

Find the quotients.

1. 160 ÷ 4 40 **4.** 360 ÷ 4 90 **7.** 90 ÷ 3 30 **10.** 140 ÷ 7 20 **13.** 180 ÷ 60 3

2. 270 ÷ 3 90 **5.** 200 ÷ 4 50 **8.** 180 ÷ 9 20 **11.** 360 ÷ 6 60 **14.** 270 ÷ 90 3

3. 210 ÷ 3 70 **6.** 630 ÷ 9 70 **9.** 120 ÷ 2 60 **12.** 560 ÷ 8 70 **15.** 140 ÷ 20 7

Find the quotients.

16. 5)$\overline{250}$ 50 **19.** 6)$\overline{120}$ 20 **22.** 6)$\overline{420}$ 70 **25.** 90)$\overline{720}$ 8 **28.** 30)$\overline{150}$ 5

17. 2)$\overline{180}$ 90 **20.** 7)$\overline{280}$ 40 **23.** 2)$\overline{140}$ 70 **26.** 40)$\overline{240}$ 6 **29.** 40)$\overline{320}$ 8

18. 2)$\overline{100}$ 50 **21.** 7)$\overline{630}$ 90 **24.** 90)$\overline{450}$ 5 **27.** 8)$\overline{640}$ 80 **30.** 70)$\overline{560}$ 8

Solve each short story problem.

31. 360 apples.
9 boxes. How many
apples in each box? 40

32. 490 bricks.
70 bricks in each layer.
How many layers? 7

Reflected answers, Set 26: 1. 40, 4. 90, 7. 30, 10. 20, 13. 3, 16. 50, 19. 20, 22. 70, 25. 8, 28. 5, 31. 40, 32. 7

Set 27 *For use with page 165*

Find the products.

1. 50 × 50 2500 **4.** 40 × 20 800 **7.** 50 × 30 1500 **10.** 50 × 400 20000 **13.** 20 × 200 4000

2. 60 × 50 3000 **5.** 60 × 90 5400 **8.** 40 × 80 3200 **11.** 80 × 200 16000 **14.** 70 × 500 35 000

3. 30 × 80 2400 **6.** 20 × 30 600 **9.** 90 × 90 8100 **12.** 60 × 800 48000 **15.** 40 × 600 24 000

Find the quotients.

16. 50)$\overline{4000}$ 80 **19.** 60)$\overline{2400}$ 40 **22.** 40)$\overline{2800}$ 70 **25.** 50)$\overline{4500}$ 90 **28.** 40)$\overline{1200}$ 30

17. 90)$\overline{5400}$ 60 **20.** 20)$\overline{1600}$ 80 **23.** 50)$\overline{1000}$ 20 **26.** 80)$\overline{4800}$ 60 **29.** 70)$\overline{4200}$ 60

18. 70)$\overline{3500}$ 50 **21.** 30)$\overline{600}$ 20 **24.** 80)$\overline{3200}$ 40 **27.** 30)$\overline{1200}$ 40 **30.** 80)$\overline{7200}$ 90

Reflected answers, Set 27: 1. 2500, 4. 800, 7. 1200, 10. 20 000, 13. 4000, 16. 80, 19. 40, 22. 70, 25. 90, 28. 30, 31. 40, 32. 7

Set 28 *For use with page 175*

Estimate each product.

1. 41 × 8 320	**6.** 58 × 4 240	**11.** 76 × 71 5600	**16.** 92 × 58 5400	**21.** 684 × 6 4200
2. 52 × 7 350	**7.** 33 × 7 210	**12.** 19 × 93 1800	**17.** 77 × 83 6400	**22.** 816 × 5 4000
3. 93 × 5 450	**8.** 22 × 3 60	**13.** 83 × 52 4000	**18.** 47 × 33 1500	**23.** 698 × 3 2100
4. 67 × 4 280	**9.** 38 × 5 200	**14.** 69 × 56 4200	**19.** 28 × 82 2400	**24.** 4 × 933 3600
5. 54 × 2 100	**10.** 8 × 91 720	**15.** 82 × 29 2400	**20.** 54 × 56 3000	**25.** 7 × 519 3500

Estimate each answer.

26. 12 eggs make 1 dozen.
How many eggs in 8 dozen ? 80

27. 52 weeks in 1 year.
How many weeks in 6 years ? 300

28. 16 children in 1 bus.
How many children in 14 buses ? 200

29. Pens: 89¢ each.
How much for 28 pens ? $27.00

11. 2800, 12. 1800, 13. 4200, 14. 4200, 15. 2400, 16. 5400, 17. 6400, 18. 1500, 19. 2400, 20. 3000

Reflected answers, Set 28: 1. 350, 2. 320, 3. 540, 4. 540

Set 29 *For use with page 177*

Estimate each quotient.

1. 4)$\overline{78}$ 20	**5.** 9)$\overline{736}$ 80	**9.** 6)$\overline{562}$ 90	**13.** 7)$\overline{5010}$ 700	**17.** 89)$\overline{277}$ 3
2. 3)$\overline{291}$ 100	**6.** 8)$\overline{571}$ 70	**10.** 8)$\overline{639}$ 80	**14.** 9)$\overline{2180}$ 200	**18.** 62)$\overline{490}$ 8
3. 6)$\overline{431}$ 70	**7.** 4)$\overline{296}$ 70	**11.** 3)$\overline{164}$ 50	**15.** 2)$\overline{1750}$ 900	**19.** 53)$\overline{608}$ 12
4. 5)$\overline{347}$ 70	**8.** 7)$\overline{643}$ 90	**12.** 5)$\overline{396}$ 80	**16.** 4)$\overline{3190}$ 800	**20.** 38)$\overline{157}$ 4

Estimate each answer.

21. 318 chairs. 8 rows.
How many chairs in each row ? 40

22. 274 books. 9 boxes.
How many books in each box ? 30

13. 100, 14. 500, 15. 3, 18. 8

Reflected answers, Set 29: 1. 50, 2. 100, 3. 80, 4. 70, 5. 90, 6. 80, 10. 80

Set 30 *For use with page 183*

Solve the equations.

1. $4 \times 58 = (4 \times 50) + (4 \times 8) = n$ 232
2. $3 \times 47 = (3 \times 40) + (3 \times 7) = n$ 141
3. $6 \times 36 = (6 \times 30) + (6 \times 6) = n$ 216
4. $5 \times 29 = (5 \times 20) + (5 \times 9) = n$ 145
5. $6 \times 21 = (6 \times 20) + (6 \times 1) = n$ 126
6. $8 \times 35 = (8 \times 30) + (8 \times 5) = n$ 280
7. $3 \times 82 = (3 \times 80) + (3 \times 2) = n$ 246
8. $5 \times 73 = (5 \times 70) + (5 \times 3) = n$ 365
9. $9 \times 34 = (9 \times 30) + (9 \times 4) = n$ 306
10. $4 \times 96 = (4 \times 90) + (4 \times 6) = n$ 384
11. $8 \times 75 = (8 \times 70) + (8 \times 5) = n$ 600
12. $5 \times 17 = (5 \times 10) + (5 \times 7) = n$ 85
13. $4 \times 74 = (4 \times 70) + (4 \times 4) = n$ 296
14. $9 \times 61 = (9 \times 60) + (9 \times 1) = n$ 549
15. $3 \times 57 = (3 \times 50) + (3 \times 7) = n$ 171
16. $6 \times 92 = (6 \times 90) + (6 \times 2) = n$ 552
17. $8 \times 24 = (8 \times 20) + (8 \times 4) = n$ 192
18. $7 \times 66 = (7 \times 60) + (7 \times 6) = n$ 462
19. $5 \times 87 = (5 \times 80) + (5 \times 7) = n$ 435
20. $4 \times 39 = (4 \times 30) + (4 \times 9) = n$ 156

13. 360

Reflected answers, Set 30: 1. 535, 5. 126, 3. 315, 11. 600, 15. 84

Set 31 *For use with page 185*

Find the products.

1. 43 ×2 = 86	2. 57 ×4 = 228	3. 36 ×3 = 108	4. 58 ×5 = 290	5. 49 ×4 = 196	6. 27 ×6 = 162	7. 34 ×8 = 272
8. 83 ×7 = 581	9. 76 ×9 = 684	10. 32 ×3 = 96	11. 74 ×2 = 148	12. 56 ×5 = 280	13. 77 ×7 = 539	14. 84 ×6 = 504

Solve the equations.

15. $5 \times 6 \times 7 = n$ 210
16. $5 \times 4 \times 9 = n$ 180
17. $8 \times 3 \times 4 = n$ 96
18. $2 \times 6 \times 9 = n$ 108
19. $2 \times 5 \times 7 = n$ 70
20. $9 \times 5 \times 3 = n$ 135
21. $6 \times 6 \times 6 = n$ 216
22. $4 \times 3 \times 6 = n$ 72
23. $7 \times 2 \times 6 = n$ 84
24. $3 \times 8 \times 5 = n$ 120
25. $9 \times 4 \times 6 = n$ 216
26. $5 \times 7 \times 5 = n$ 175

9. 162, 7. 210, 22. 240, 18. 108, 53. 84

Reflected answers, Set 31: 1. 86, 5. 558, 3. 108, 4. 580, 2. 162

Set 32 *For use with page 187*

Find the products.

1. 435 × 2 = 870	**2.** 276 × 4 = 1104	**3.** 4134 × 6 = 24 804	**4.** 2483 × 8 = 19 864	**5.** 787 × 3 = 2361	**6.** 3462 × 5 = 17 310
7. 563 × 2 = 1126	**8.** 8765 × 9 = 78 885	**9.** 406 × 3 = 1218	**10.** 3120 × 8 = 24 960	**11.** 865 × 4 = 3460	**12.** 9673 × 7 = 67 711

Find the products.

13. 3 × 7 × 34 714	**17.** 86 × 7 × 3 1806	**21.** 6 × 35 × 4 840
14. 6 × 9 × 15 810	**18.** 46 × 9 × 3 1242	**22.** 8 × 4 × 21 672
15. 7 × 24 × 4 672	**19.** 3 × 94 × 9 2538	**23.** 73 × 2 × 6 876
16. 4 × 42 × 5 840	**20.** 5 × 4 × 61 1220	**24.** 4 × 43 × 7 1204

Reflected answers, Set 32: 1. 870, 2. 1104, 3. 24 804, 4. 19 864, 5. 2361, 6. 17 310, 7. 1126, 8. 78 885, 9. 1218, 10. 24 960, 11. 3460, 12. 67 711, 13. 714, 14. 810, 15. 672, 16. 840, 17. 1806, 18. 1242, 19. 2538, 20. 1220, 21. 840, 22. 672, 23. 876, 24. 1204

Set 33 *For use with page 191*

Find the products.

1. 39 × 80 = 3120	**2.** 27 × 90 = 2430	**3.** 35 × 40 = 1400	**4.** 56 × 70 = 3920	**5.** 63 × 30 = 1890	**6.** 91 × 80 = 7280
7. 87 × 20 = 1740	**8.** 45 × 50 = 2250	**9.** 37 × 60 = 2220	**10.** 64 × 90 = 5760	**11.** 79 × 80 = 6320	**12.** 47 × 30 = 1410
13. 56 × 40 = 2240	**14.** 35 × 20 = 700	**15.** 93 × 70 = 6510	**16.** 52 × 50 = 2600	**17.** 74 × 60 = 4440	**18.** 23 × 90 = 2070

Reflected answers, Set 33: 1. 3120, 2. 2430, 3. 1400, 4. 3920, 5. 1890, 6. 7280, 7. 1740, 8. 2250, 9. 2220, 10. 5760, 11. 6320, 12. 1410, 13. 2240, 14. 700, 15. 6510, 16. 2600, 17. 4440, 18. 2070

Set 34 *For use with page 193*

Find the products.

1. 23 × 34 = 782	**2.** 57 × 25 = 1425	**3.** 19 × 27 = 513	**4.** 18 × 71 = 1278	**5.** 39 × 62 = 2418	**6.** 56 × 43 = 2408	**7.** 75 × 54 = 4050
8. 36 × 45 = 1620	**9.** 45 × 98 = 4410	**10.** 15 × 66 = 990	**11.** 77 × 59 = 4543	**12.** 68 × 18 = 1224	**13.** 59 × 77 = 4543	**14.** 41 × 26 = 1066
15. 425 × 32 = 13 600	**16.** 173 × 46 = 7958	**17.** 218 × 57 = 12 426	**18.** 423 × 38 = 16 074	**19.** 561 × 94 = 52 734	**20.** 789 × 62 = 48 918	**21.** 343 × 71 = 24 353
22. 785 × 75 = 58 875	**23.** 936 × 68 = 63 648	**24.** 421 × 56 = 23 576	**25.** 507 × 49 = 24 843	**26.** 683 × 34 = 23 222	**27.** 590 × 28 = 16 520	**28.** 789 × 87 = 68 643

Reflected answers, Set 34: 1. 782, 2. 1425, 3. 513, 4. 1278, 5. 2418, 6. 2408, 7. 4050, 8. 1620, 9. 4410, 10. 990, 11. 4543, 12. 1224, 13. 4543, 14. 1066, 15. 13 600, 16. 7958, 17. 12 426, 18. 16 074, 19. 52 734, 20. 48 918, 21. 24 353, 22. 58 875, 23. 63 648, 24. 23 576, 25. 24 843, 26. 23 222, 27. 16 520, 28. 68 643

Set 35 *For use with page 197*

Solve each short story problem.

1. 46 bags. 25 kilograms of potatoes in each bag.
 How many kilograms of potatoes? 1150

2. 28 books.
 88 pages in each book.
 How many pages in all? 2464

3. .14 crates.
 86 oranges in each crate.
 How many oranges in all? 1204

4. Drove 60 kilometres per hour for 12 hours.
 Drove how many km? 720

5. 14 classes.
 32 students in each class.
 How many students? 448

6. 31 days in May.
 24 hours in each day.
 How many hours in May? 744

Reflected answers, Set 35: 1. 1150, 4. 720

Set 36 *For use with page 199*

Find the products.

1. 343 × 71 = 24 353	**2.** 629 × 84 = 52 836	**3.** 146 × 242 = 35 332	**4.** 257 × 204 = 52 428	**5.** 375 × 155 = 58 125	**6.** 807 × 263 = 212 241
7. 709 × 87 = 61 683	**8.** 378 × 95 = 35 910	**9.** 466 × 376 = 175 216	**10.** 394 × 485 = 191 090	**11.** 580 × 297 = 172 260	**12.** 925 × 634 = 586 450
13. 648 × 705 = 456 840	**14.** 647 × 192 = 124 224	**15.** 852 × 380 = 323 760	**16.** 739 × 478 = 353 242	**17.** 139 × 637 = 88 543	**18.** 820 × 445 = 364 900

19. 7 × 4 × 8 × 3 = 672

20. 4 × 7 × 6 × 5 = 840

21. 5 × 4 × 6 × 9 = 1080

22. 6 × 7 × 5 × 4 = 840

23. 8 × 3 × 266 = 6384

24. 321 × 6 × 5 = 9630

25. 46 × 9 × 3 = 1242

26. 463 × 8 × 6 = 22 224

27. 9 × 595 × 7 = 37 485

28. 341 × 2 × 5 = 3410

29. 47 × 26 × 8 = 9776

30. 31 × 57 × 4 = 7068

31. 597 × 32 × 2 = 38 208

32. 66 × 71 × 84 = 393 624

33. 37 × 20 × 93 = 68 820

Reflected answers, Set 36: 1. 54 323, 2. 52 836, 3. 35 332

Set 37 *For use with page 200*

Solve each short story problem.

1. 55 minutes to deliver papers each day. 31 days in March. How many minutes delivering papers in March? 1705

2. 1000 grams → 1 kilogram. 56 kilograms. How many grams? 56 000

3. 23 motels. 84 rooms in each motel. How many rooms in all? 1932

4. 104 passengers per plane. 32 planes. How many passengers? 3328

Reflected answers, Set 37: 1. 1705, 3. 1932

Set 38 *For use with page 225*

Find the largest whole number that will make each sentence true.
Then find the quotient.

1. $n \times 7 < 31 \rightarrow 7\overline{)31}$ 4; 4R3
2. $n \times 4 < 23 \rightarrow 4\overline{)23}$ 5; 5R3
3. $n \times 8 < 50 \rightarrow 8\overline{)50}$ 6; 6R2
4. $n \times 3 < 19 \rightarrow 3\overline{)19}$ 6; 6R1
5. $n \times 6 < 51 \rightarrow 6\overline{)51}$ 8; 8R3
6. $n \times 5 < 42 \rightarrow 5\overline{)42}$ 8; 8R2
7. $n \times 7 < 61 \rightarrow 7\overline{)61}$ 8; 8R5
8. $n \times 9 < 60 \rightarrow 9\overline{)60}$ 6; 6R6
9. $n \times 6 < 22 \rightarrow 6\overline{)22}$ 3; 3R4
10. $n \times 4 < 19 \rightarrow 4\overline{)19}$ 4; 4R3

Find the quotients and the remainders.

11. $8\overline{)67}$ 8R3
12. $4\overline{)33}$ 8R1
13. $7\overline{)51}$ 7R2
14. $6\overline{)46}$ 7R4
15. $9\overline{)83}$ 9R2
16. $5\overline{)33}$ 6R3
17. $7\overline{)58}$ 8R2
18. $3\overline{)26}$ 8R2
19. $8\overline{)46}$ 5R6
20. $4\overline{)27}$ 6R3
21. $6\overline{)52}$ 8R4
22. $7\overline{)37}$ 5R2
23. $9\overline{)77}$ 8R5
24. $5\overline{)32}$ 6R2
25. $8\overline{)49}$ 6R1

Reflected answers, Set 38: 1. 4; 4R3, 2. 5; 5R3, 3. 6; 6R2, 4. 6; 6R1, 5. 8; 8R3, 6. 8; 8R2, 7. 8; 8R5, 8. 6; 6R6, 9. 3; 3R4, 10. 4; 4R3, 11. 8R3, 12. 8R1, 13. 7R2, 14. 7R4, 15. 9R2, 16. 6R3, 17. 8R2, 18. 8R2, 19. 5R6, 20. 6R3, 21. 8R4, 22. 5R2, 23. 8R5, 24. 6R2, 25. 6R1

Set 39 *For use with page 229*

Find the quotients and remainders.

1. $3\overline{)94}$ 31R1
2. $4\overline{)137}$ 34R1
3. $4\overline{)326}$ 81R2
4. $6\overline{)206}$ 34R2
5. $7\overline{)649}$ 92R5
6. $6\overline{)874}$ R3
7. $5\overline{)325}$ 65R0
8. $3\overline{)243}$ 81R0
9. $5\overline{)359}$ 71R4
10. $6\overline{)568}$ 94R4
11. $4\overline{)93}$ 23R1
12. $9\overline{)456}$ 50R6
13. $2\overline{)109}$ 54R1
14. $3\overline{)147}$ 49
15. $4\overline{)167}$ 41R3
16. $7\overline{)88}$ 12R4
17. $7\overline{)463}$ 66R1
18. $8\overline{)436}$ 54R4
19. $4\overline{)278}$ 69R2
20. $8\overline{)365}$ 45R5
21. $5\overline{)97}$ 19R2
22. $6\overline{)378}$ 63
23. $9\overline{)643}$ 71R4
24. $8\overline{)734}$ 91R6
25. $5\overline{)342}$ 68R2

Reflected answers, Set 39: 1. 31R1, 2. 34R1, 3. 81R2, 4. 34R2, 5. 92R5, 6. R3, 7. 65R0, 8. 81R0, 9. 71R4, 10. 94R4, 11. 23R1, 12. 50R6, 13. 54R1, 14. 49, 15. 41R3, 16. 12R4, 17. 66R1, 18. 54R4, 19. 69R2, 20. 45R5, 21. 19R2, 22. 63, 23. 71R4, 24. 91R6, 25. 68R2

Set 40 *For use with page 231*

Find the quotients and remainders. Check each answer.

1. $9\overline{)672}$ 74 R6	5. $8\overline{)747}$ 93 R3	9. $6\overline{)168}$ 28	13. $9\overline{)757}$ 84 R1	17. $5\overline{)246}$ 49 R1
2. $8\overline{)362}$ 45 R2	6. $5\overline{)429}$ 85 R4	10. $9\overline{)168}$ 18 R6	14. $7\overline{)461}$ 65 R6	18. $3\overline{)287}$ 95 R2
3. $5\overline{)372}$ 74 R2	7. $6\overline{)442}$ 73 R4	11. $7\overline{)177}$ 25 R2	15. $6\overline{)596}$ 99 R2	19. $8\overline{)321}$ 40 R1
4. $6\overline{)259}$ 43 R1	8. $8\overline{)534}$ 66 R6	12. $9\overline{)602}$ 66 R8	16. $4\overline{)307}$ 76 R3	20. $7\overline{)617}$ 88 R1

Solve each short story problem. Check each answer.

21. 374 bottles.
6 bottles in each carton.
How many cartons? 62

22. 212 tomato plants.
4 rows. How many plants in each row? 53

Reflected answers, Set 40: 1. 74 R6, 62; 2. 93 R3, 53; 3. 85, 74 R4; 4. 73 R2, 43 R1; 13. 84 R1, 65 R6; 14. 99 R2, 76 R3; 15. 25 R2, 66 R8; 16. 40 R1, 88 R1

Set 41 *For use with page 233*

Find the average of the numbers in each set.

1. {5, 3, 10} 6	5. {8, 7, 9, 4} 7	9. {26, 35, 29} 30	13. {9, 7, 8, 3, 6, 9} 7
2. {8, 4, 3} 5	6. {7, 9, 9, 15} 10	10. {80, 73, 30} 61	14. {12, 15, 11, 10} 12
3. {11, 9, 7} 9	7. {23, 31} 27	11. {47, 56, 23} 42	15. {78, 45, 99, 26} 62
4. {6, 4, 8} 6	8. {56, 72} 64	12. {94, 88, 37} 73	16. {87, 70, 32, 52, 54} 59

Solve each short story problem.

17. John 28 kilograms, Joe 32 kilograms, Jim 33 kilograms. What is their average weight? 31

18. Dart scores: 26, 34, 21
Average score? 27

Reflected answers, Set 41: 1. 6, 5; 2. 7, 5; 3. 30, 13. 5

Set 42 *For use with page 235*

From the set {100, 200, 300 . . .}, find the largest number that will make the sentence true.

1. $n \times 7 < 2361 \rightarrow 7\overline{)2361}$ 300
2. $n \times 4 < 1820 \rightarrow 4\overline{)1820}$ 400
3. $n \times 6 < 4933 \rightarrow 6\overline{)4933}$ 800
4. $n \times 3 < 2222 \rightarrow 3\overline{)2222}$ 700
5. $n \times 9 < 6737 \rightarrow 9\overline{)6737}$ 700
6. $n \times 5 < 4831 \rightarrow 5\overline{)4831}$ 900
7. $n \times 8 < 6558 \rightarrow 8\overline{)6558}$ 800
8. $n \times 4 < 2521 \rightarrow 4\overline{)2521}$ 600

Find each quotient and remainder.

9. 406 R2 $6\overline{)2438}$	13. 648 R5 $9\overline{)5837}$	17. 721 R1 $7\overline{)5048}$	21. 550 R1 $7\overline{)3851}$	25. 625 R1 $7\overline{)4376}$
10. 663 R3 $8\overline{)5307}$	14. 743 R2 $8\overline{)5946}$	18. 436 R5 $9\overline{)3929}$	22. 806 $6\overline{)4836}$	26. 590 R2 $6\overline{)3542}$
11. 337 $7\overline{)2359}$	15. 700 R8 $9\overline{)6308}$	19. 396 R2 $4\overline{)1586}$	23. 941 R6 $8\overline{)7534}$	27. 792 R4 $5\overline{)3964}$
12. 836 R2 $4\overline{)3346}$	16. 754 $6\overline{)4524}$	20. 1000 R3 $5\overline{)5003}$	24. 940 R3 $9\overline{)8463}$	28. 686 R2 $4\overline{)2746}$

Reflected answers, Set 42: 1. 300, 2. 400, 3. 800 R5, 4. 740 R2, 5. 700 R20, 6. 900 R31, 7. 800 R6, 8. 600 R121

Set 43 *For use with page 241*

Find each quotient and remainder.

1. 6 R20 $60\overline{)380}$	5. 75 R20 $30\overline{)2270}$	9. 87 R22 $50\overline{)4372}$	13. 49 $30\overline{)1470}$
2. 5 R10 $40\overline{)210}$	6. 32 R30 $50\overline{)1630}$	10. 96 R58 $60\overline{)5818}$	14. 94 R12 $40\overline{)3772}$
3. 6 R3 $90\overline{)543}$	7. 35 R27 $40\overline{)1427}$	11. 46 R50 $90\overline{)4190}$	15. 29 R21 $80\overline{)2341}$
4. 7 R9 $70\overline{)499}$	8. 57 R3 $80\overline{)4563}$	12. 63 R27 $70\overline{)4437}$	16. 84 R11 $50\overline{)4211}$

Reflected answers, Set 43: 1. 6 R50, 2. 12 R50, 3. 87 R55, 4. 64

Set 44 *For use with page 243*

Find the quotients and remainders.

1. 31)206 (6R20)	**6.** 63)520 (8R16)	**11.** 43)3659 (85R4)	**16.** 76)2194 (28R66)	**21.** 21)1778 (84R14)
2. 29)193 (6R19)	**7.** 77)410 (5R25)	**12.** 68)4809 (70R49)	**17.** 61)2431 (39R52)	**22.** 42)2190 (52R6)
3. 52)413 (7R49)	**8.** 54)317 (5R47)	**13.** 26)4390 (168R22)	**18.** 58)4153 (71R35)	**23.** 84)6414 (76R30)
4. 48)463 (9R31)	**9.** 61)271 (4R27)	**14.** 37)2784 (75R9)	**19.** 42)3251 (77R17)	**24.** 61)1591 (26R5)
5. 27)223 (8R7)	**10.** 89)675 (7R52)	**15.** 44)3874 (88R2)	**20.** 23)1148 (49R21)	**25.** 36)2648 (73R20)

Solve each short story problem.

26. 12 in 1 dozen.
364 eggs.
How many dozen eggs? 30

27. 24 hours⟶1 day.
534 hours.
How many days? 22

Reflected answers, Set 44: 1. 6 R20, 2. 6 R19, 3. 7 R49, 4. 9 R31, 5. 8 R7, 6. 8 R16, 7. 5 R25, 8. 5 R47, 9. 4 R27, 10. 7 R52, 11. 85 R4, 12. 70 R49, 13. 168 R22, 14. 75 R9, 15. 88 R2, 16. 28 R66, 17. 39 R52, 18. 71 R35, 19. 77 R17, 20. 49 R21, 21. 84 R14, 22. 52 R6, 23. 76 R30, 24. 26 R5, 25. 73 R20, 26. 30, 27. 22

Set 45 *For use with page 245*

Solve each short story problem.

1. 7 days⟶1 week.
280 days.
How many weeks? 40

2. 52 weeks⟶1 year.
208 weeks.
How many years? 4

3. 24 bottles per case.
192 bottles.
How many cases? 8

4. 8 tires per airplane.
464 tires.
How many airplanes? 58

Reflected answers, Set 45: 1. 40, 3. 8

Set 46 *For use with page 255*

1. List the factors of 15. 1,3,5,15
2. List the factors of 16. 1,2,4,8,16
3. List the factors of 18. 1,2,3,6,9,18
4. List the factors of 20. 1,2,4,5,10,20
5. List the factors of 24. 1,2,3,4,6,8,12,24
6. List the factors of 28. 1,2,4,7,14,28
7. List the factors of 30. 1,2,3,5,6,10,15,30
8. List the factors of 36. 1,2,3,4,6,9,12,18,36

Use your answers for 1-8 to answer the following questions.

9. List the common factors of 12 and 16. 1,2,4
10. What is the greatest common factor of 12 and 16? 4
11. List the common factors of 24 and 30. 1,2,3,6
12. What is the greatest common factor of 24 and 30? 6
13. List the common factors of 20 and 28. 1,2,4
14. What is the greatest common factor of 20 and 28? 4
15. List the common factors of 24 and 36. 1,2,3,4,6,12
16. What is the greatest common factor of 24 and 36? 12

Reflected answers, Set 46: 1. 1, 2, 3, 4, 6, 12 2. 1, 2, 4, 8, 16 3. 1, 2, 3, 4, 5, 6, 8, 9, 12, 24, 36, 72

Set 47 *For use with page 259*

Tell whether each number is prime or not prime.

1. 17 Yes
2. 52 No
3. 33 No
4. 69 No
5. 25 No
6. 27 No
7. 41 Yes
8. 37 Yes
9. 57 No
10. 51 No
11. 39 No
12. 23 Yes
13. 71 Yes
14. 43 Yes

Write an equation to show that each number is the product of prime numbers.

15. 21 3×7
16. 15 3×5
17. 28 2×2×7
18. 12 2×2×3
19. 26 2×13
20. 27 3×3×3
21. 32 2×2×2×2×2
22. 34 2×17
23. 38 2×19
24. 39 3×13
25. 40 2×2×2×5
26. 42 2×3×7

Reflected answers, Set 47: 1. prime, 2. not prime, 3. not prime, 4. prime, 5. 5×5, 6. $3 \times 3 \times 3$, 7. prime, 8. not prime, 9. 3×19, 10. $2 \times 5 \times 5$, 11. 2×17, 12. $2 \times 2 \times 3$, 13. 2×13, 14. 2×19, 15. 3×7, 16. 3×5, 17. $2 \times 2 \times 7$, 18. $2 \times 3 \times 7$, 19. $2 \times 2 \times 2 \times 2 \times 2$, 20. $5 \times 5 \times 5 \times 5 \times 5$, 21. 3×13, 22. $3 \times 3 \times 3 \times 3$, 23. $2 \times 2 \times 2 \times 5$, 24. 2×17, 25. 3×13, 26. 2×19

Set 48 *For use with page 267*

What fraction is suggested by each of these?

1. 3 out of 5 $\frac{3}{5}$
2. 9 out of 11 $\frac{9}{11}$
3. 2 out of 17 $\frac{2}{17}$
4. 7 out of 9 $\frac{7}{9}$
5. 5 out of 8 $\frac{5}{8}$
6. 13 out of 14 $\frac{13}{14}$
7. 6 out of 7 $\frac{6}{7}$
8. 12 out of 13 $\frac{12}{13}$
9. 7 out of 10 $\frac{7}{10}$

Write a fraction for each number-pair story.

10. 15 cookies in the cookie jar. Sue ate 2 of them. What fraction of the cookies did she eat? $\frac{2}{15}$

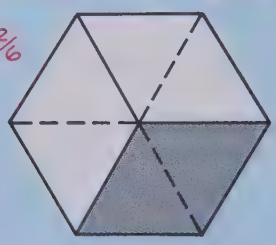
11. 7 cars in the parking lot. 3 are blue. What fraction of the cars are blue? $\frac{3}{7}$

Reflected answers, Set 48: 1. $\frac{2}{3}$, 4. $\frac{2}{7}$, 7. $\frac{6}{7}$

Set 49 *For use with page 275*

Write the pair of equivalent fractions suggested by each picture.

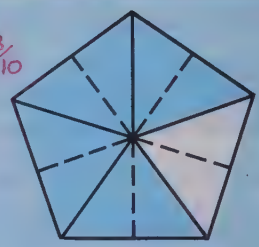
1. $\frac{1}{3}$, $\frac{2}{6}$



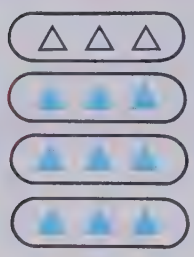
2. $\frac{1}{4}$, $\frac{2}{8}$



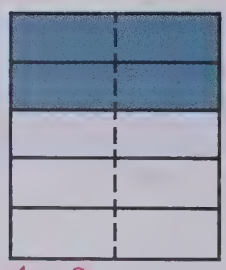
3. $\frac{4}{5}$, $\frac{8}{10}$



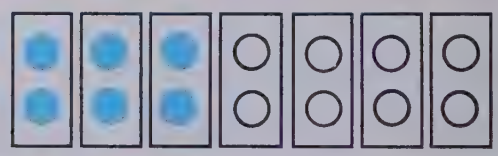
4. $\frac{9}{12}$, $\frac{3}{4}$



5. $\frac{4}{10}$, $\frac{2}{5}$



6. $\frac{6}{14}$, $\frac{3}{7}$



Reflected answers, Set 49: 1. [illegible], 5. $\frac{4}{10}$, $\frac{2}{5}$, 3. $\frac{8}{10}$

Set 50 *For use with page 277*

Find the next three fractions for each set of equivalent fractions.

1. $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \ldots\}$ $\frac{12}{20}, \frac{15}{25}, \frac{18}{30}$
2. $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \ldots\}$ $\frac{8}{12}, \frac{10}{15}, \frac{12}{18}$
3. $\{\frac{2}{9}, \frac{4}{18}, \frac{6}{27}, \ldots\}$ $\frac{8}{36}, \frac{10}{45}, \frac{12}{54}$
4. $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \ldots\}$ $\frac{8}{20}, \frac{10}{25}, \frac{12}{30}$
5. $\{\frac{3}{11}, \frac{6}{22}, \frac{9}{33}, \ldots\}$ $\frac{12}{44}, \frac{15}{55}, \frac{18}{66}$
6. $\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \ldots\}$ $\frac{12}{28}, \frac{15}{35}, \frac{18}{42}$
7. $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \ldots\}$ $\frac{4}{40}, \frac{5}{50}, \frac{6}{60}$
8. $\{\frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \ldots\}$ $\frac{28}{32}, \frac{35}{40}, \frac{42}{48}$

Reflected answers, Set 50: 1. $\frac{50}{15}, \frac{55}{12}$ 2.

Set 51 *For use with page 289*

Tell whether or not the two fractions are equivalent.

1. $\frac{2}{5}, \frac{7}{35}$ No
2. $\frac{3}{11}, \frac{8}{44}$ No
3. $\frac{5}{6}, \frac{35}{42}$ Yes
4. $\frac{5}{2}, \frac{15}{6}$ Yes
5. $\frac{4}{9}, \frac{3}{8}$ No
6. $\frac{3}{2}, \frac{24}{16}$ Yes
7. $\frac{3}{10}, \frac{12}{40}$ Yes
8. $\frac{7}{6}, \frac{48}{46}$ No
9. $\frac{5}{8}, \frac{40}{64}$ Yes
10. $\frac{3}{5}, \frac{2}{3}$ No

Find the lowest-terms fraction in each set of equivalent fractions.

11. $\{\frac{20}{24}, \frac{35}{42}, \frac{5}{6}, \frac{10}{12}, \frac{45}{54}\}$ $\frac{5}{6}$
12. $\{\frac{24}{64}, \frac{6}{16}, \frac{21}{56}, \frac{3}{8}, \frac{9}{24}\}$ $\frac{3}{8}$
13. $\{\frac{6}{27}, \frac{2}{9}, \frac{10}{45}, \frac{4}{18}, \frac{20}{90}\}$ $\frac{2}{9}$
14. $\{\frac{25}{55}, \frac{50}{110}, \frac{5}{11}, \frac{15}{33}, \frac{35}{77}\}$ $\frac{5}{11}$
15. $\{\frac{7}{10}, \frac{63}{90}, \frac{42}{60}, \frac{56}{80}, \frac{28}{40}\}$ $\frac{7}{10}$
16. $\{\frac{64}{56}, \frac{16}{14}, \frac{32}{28}, \frac{48}{42}, \frac{8}{7}\}$ $\frac{8}{7}$
17. $\{\frac{7}{4}, \frac{28}{16}, \frac{63}{36}, \frac{49}{28}, \frac{14}{8}\}$ $\frac{7}{4}$
18. $\{\frac{50}{10}, \frac{25}{5}, \frac{45}{9}, \frac{5}{1}, \frac{40}{8}\}$ $\frac{5}{1}$
19. $\{\frac{16}{10}, \frac{8}{5}, \frac{64}{40}, \frac{72}{45}, \frac{48}{30}\}$ $\frac{8}{5}$
20. $\{\frac{35}{14}, \frac{10}{4}, \frac{60}{24}, \frac{20}{8}, \frac{5}{2}\}$ $\frac{5}{2}$

Give the lowest-terms fraction for each fraction.

21. $\frac{9}{21}$ $\frac{3}{7}$
22. $\frac{15}{18}$ $\frac{5}{6}$
23. $\frac{16}{12}$ $\frac{4}{3}$
24. $\frac{15}{20}$ $\frac{3}{4}$
25. $\frac{9}{15}$ $\frac{3}{5}$
26. $\frac{18}{27}$ $\frac{2}{3}$
27. $\frac{8}{12}$ $\frac{2}{3}$
28. $\frac{24}{32}$ $\frac{3}{4}$
29. $\frac{6}{14}$ $\frac{3}{7}$
30. $\frac{10}{16}$ $\frac{5}{8}$

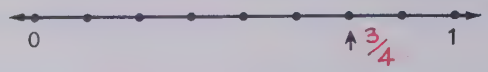
Reflected answers, Set 51: 1. no 3. yes 5. no 7. yes 9. yes

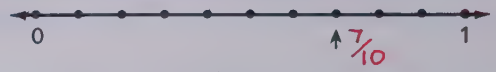
Set 52 *For use with page 323*

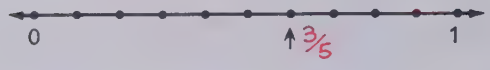
Give the lowest-terms fraction for the point over the arrow.

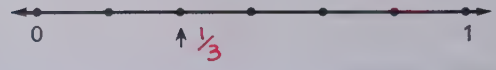
1. 

2. 

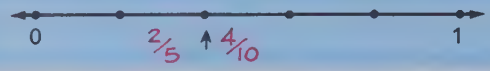
3. 

4. 

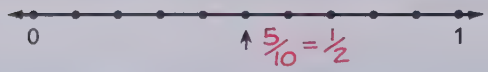
5. 

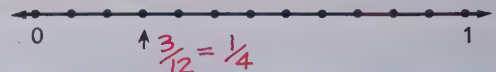
6. 

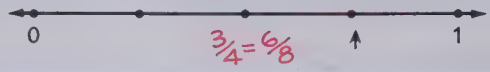
Give two fractions for the point over the arrow.

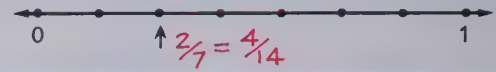
7. 

8. 

9. 

10. 

11. 

12. 

Reflected answers, Set 52: 1. $\frac{1}{4}$, 2. $\frac{4}{5}$, 3. $\frac{1}{2}$, $\frac{5}{10}$, 4. $\frac{1}{4}$, $\frac{3}{12}$, 5. $\frac{3}{4}$, $\frac{6}{8}$, 6. $\frac{1}{3}$, $\frac{2}{7}$, $\frac{4}{14}$

Set 53 *For use with page 325*

Find the missing numerators and denominators.

1. $\frac{4}{8} = \frac{1}{\square}$ 2
2. $\frac{6}{9} = \frac{\square}{3}$ 2
3. $\frac{4}{8} = \frac{\square}{2}$ 1
4. $\frac{3}{9} = \frac{1}{\square}$ 3
5. $\frac{2}{14} = \frac{\square}{7}$ 1
6. $\frac{7}{14} = \frac{\square}{2}$ 1
7. $\frac{6}{14} = \frac{3}{\square}$ 7
8. $\frac{8}{14} = \frac{\square}{7}$ 4
9. $\frac{5}{15} = \frac{\square}{3}$ 1
10. $\frac{10}{15} = \frac{\square}{3}$ 2
11. $\frac{3}{15} = \frac{1}{\square}$ 5
12. $\frac{6}{15} = \frac{\square}{5}$ 2
13. $\frac{4}{16} = \frac{\square}{4}$ 1
14. $\frac{2}{16} = \frac{1}{\square}$ 8
15. $\frac{8}{16} = \frac{\square}{2}$ 1
16. $\frac{3}{18} = \frac{\square}{6}$ 1
17. $\frac{15}{18} = \frac{\square}{6}$ 5
18. $\frac{9}{18} = \frac{\square}{2}$ 1
19. $\frac{12}{18} = \frac{\square}{3}$ 2
20. $\frac{16}{18} = \frac{\square}{9}$ 8

Reflected answers, Set 53: 1. 2, 2. 3, 3. 4, 4. 3, 5. 7, 6. 1, 7. 7, 8. 4, 9. 3, 10. 3, 11. 5, 12. 5, 13. 4, 14. 8, 15. 2, 16. 6, 17. 6, 18. 2, 19. 3, 20. 9

Set 54 *For use with page 331*

Write the symbol ($<$, $=$, $>$) for each ◯.

1. $\frac{1}{2}$ ◯ $\frac{2}{4}$ $=$	5. $\frac{1}{4}$ ◯ $\frac{1}{2}$ $<$	9. $\frac{1}{3}$ ◯ $\frac{1}{6}$ $>$	13. $\frac{1}{3}$ ◯ $\frac{2}{6}$ $=$	17. $\frac{1}{4}$ ◯ $\frac{1}{3}$ $<$
2. $\frac{5}{8}$ ◯ $\frac{3}{8}$	6. $\frac{4}{12}$ ◯ $\frac{3}{9}$	10. $\frac{2}{3}$ ◯ $\frac{5}{6}$	14. $\frac{4}{8}$ ◯ $\frac{5}{5}$	18. $\frac{1}{12}$ ◯ $\frac{1}{2}$
3. $\frac{1}{3}$ ◯ $\frac{4}{6}$	7. $\frac{1}{5}$ ◯ $\frac{1}{2}$	11. $\frac{1}{4}$ ◯ $\frac{1}{8}$	15. $\frac{1}{4}$ ◯ $\frac{2}{8}$	19. $\frac{5}{6}$ ◯ $\frac{1}{1}$
4. $\frac{3}{4}$ ◯ $\frac{3}{8}$	8. $\frac{1}{10}$ ◯ $\frac{1}{5}$	12. $\frac{3}{4}$ ◯ $\frac{15}{20}$	16. $\frac{4}{4}$ ◯ $\frac{5}{5}$	20. $\frac{7}{8}$ ◯ $\frac{15}{16}$

Reflected answers, Set 54: 1. = 2. < 3. > 4. = 5. <

Set 55 *For use with page 337*

Give the missing numerators and denominators.

1. $4 = \frac{\square}{2}$ 8	5. $16 = \frac{\square}{2}$ 32	9. $6 = \frac{54}{\square}$ 9	13. $14 = \frac{42}{\square}$ 3
2. $2 = \frac{10}{\square}$ 5	6. $12 = \frac{36}{\square}$ 3	10. $4 = \frac{\square}{4}$ 16	14. $\frac{1}{6} = \frac{\square}{12}$ 2
3. $7 = \frac{\square}{6}$ 42	7. $5 = \frac{\square}{6}$ 30	11. $8 = \frac{32}{\square}$ 4	15. $\frac{3}{5} = \frac{6}{\square}$ 10
4. $3 = \frac{24}{\square}$ 8	8. $9 = \frac{27}{\square}$ 3	12. $10 = \frac{40}{\square}$ 4	16. $\frac{2}{3} = \frac{\square}{12}$ 8

Find the sums.

17. $\frac{8}{2} + \frac{1}{2}$ 9/2	22. $6 + \frac{4}{5}$ 6 4/5	27. $\frac{18}{6} + \frac{1}{6}$ 19/6	32. $5 + \frac{2}{3}$ 5 2/3
18. $\frac{4}{4} + \frac{1}{4}$ 1 1/4 or 5/4	23. $4 + \frac{1}{6}$ 4 1/6	28. $\frac{12}{3} + \frac{1}{3}$ 13/3	33. $7 + \frac{4}{9}$ 7 4/9
19. $\frac{6}{3} + \frac{2}{3}$ 8/3	24. $3 + \frac{2}{3}$ 3 2/3	29. $7 + \frac{3}{8}$ 7 3/8	34. $\frac{9}{9} + \frac{2}{9}$ 11/9
20. $\frac{12}{4} + \frac{1}{4}$ 13/4	25. $\frac{15}{5} + \frac{1}{5}$ 16/5	30. $\frac{8}{8} + \frac{5}{8}$ 13/8	35. $\frac{24}{6} + \frac{1}{6}$ 25/6
21. $\frac{12}{6} + \frac{5}{6}$ 17/6	26. $9 + \frac{1}{8}$ 9 1/8	31. $\frac{12}{6} + \frac{5}{6}$ 17/6	36. $\frac{20}{5} + \frac{3}{5}$ 23/5

Reflected answers, Set 55: 1. 8 2. 35 3. 42 4. 8 5. 32 6. 3 7. 30 8. 3 9. 9 10. 16 11. 4 12. 4 13. 42 14. 12 15. 10 16. 8 17. 9/2 18. 5/4 19. 8/3 20. 13/4 21. 17/6 22. 6 4/5 23. 4 1/6 24. 3 2/3 25. 16/5 26. 9 1/8 27. 19/6 28. 13/3 29. 7 3/8 30. 13/8 31. 17/6 32. 5 2/3 33. 7 4/9 34. 11/9 35. 25/6 36. 23/5

Set 56 *For use with page 339*

Give the mixed numeral for each sum.

1. $4 + \frac{1}{4}$ $4\frac{1}{4}$
2. $\frac{3}{3} + \frac{1}{3}$ $1\frac{1}{3}$
3. $\frac{12}{4} + \frac{3}{4}$ $3\frac{3}{4}$
4. $7 + \frac{3}{8}$ $7\frac{3}{8}$
5. $6 + \frac{5}{8}$ $6\frac{5}{8}$
6. $\frac{8}{4} + \frac{1}{4}$ $2\frac{1}{4}$
7. $\frac{15}{3} + \frac{2}{3}$ $5\frac{2}{3}$
8. $13 + \frac{2}{5}$ $13\frac{2}{5}$
9. $\frac{9}{9} + \frac{4}{9}$ $1\frac{4}{9}$
10. $\frac{10}{5} + \frac{3}{5}$ $2\frac{3}{5}$
11. $5 + \frac{1}{12}$ $5\frac{1}{12}$
12. $\frac{12}{12} + \frac{5}{12}$ $1\frac{5}{12}$
13. $\frac{18}{6} + \frac{1}{6}$ $3\frac{1}{6}$
14. $\frac{9}{3} + \frac{2}{3}$ $3\frac{2}{3}$
15. $\frac{14}{7} + \frac{2}{7}$ $2\frac{2}{7}$
16. $\frac{21}{3} + \frac{2}{3}$ $7\frac{1}{3}$
17. $\frac{14}{2} + \frac{1}{2}$ $7\frac{1}{2}$
18. $3 + \frac{7}{8}$ $3\frac{7}{8}$
19. $5 + \frac{9}{10}$ $5\frac{9}{10}$
20. $\frac{5}{5} + \frac{1}{5}$ $1\frac{1}{5}$

Give a mixed numeral for each fraction.

21. $\frac{7}{2}$ $3\frac{1}{2}$
22. $\frac{11}{4}$ $2\frac{3}{4}$
23. $\frac{13}{6}$ $2\frac{1}{6}$
24. $\frac{15}{7}$ $2\frac{1}{7}$
25. $\frac{21}{5}$ $4\frac{1}{5}$
26. $\frac{36}{7}$ $5\frac{1}{7}$
27. $\frac{10}{3}$ $3\frac{1}{3}$
28. $\frac{16}{5}$ $3\frac{1}{5}$
29. $\frac{17}{4}$ $4\frac{1}{4}$
30. $\frac{21}{8}$ $2\frac{5}{8}$
31. $\frac{6}{4}$ $1\frac{1}{2}$
32. $\frac{8}{3}$ $2\frac{2}{3}$
33. $\frac{9}{5}$ $1\frac{4}{5}$
34. $\frac{29}{6}$ $4\frac{5}{6}$
35. $\frac{18}{5}$ $3\frac{3}{5}$
36. $\frac{43}{8}$ $5\frac{3}{8}$
37. $\frac{52}{7}$ $7\frac{3}{7}$
38. $\frac{31}{9}$ $3\frac{4}{9}$

Reflected answers, Set 56: 21. $3\frac{5}{7}$, 22. $2\frac{1}{4}$, 23. $2\frac{1}{6}$, 24. $2\frac{1}{7}$, 25. $4\frac{1}{5}$, 26. $5\frac{1}{7}$, 27. $3\frac{1}{3}$, 28. $3\frac{1}{5}$, 29. $4\frac{1}{4}$, 30. $2\frac{5}{8}$, 31. $1\frac{1}{2}$, 32. $2\frac{2}{3}$, 33. $1\frac{4}{5}$, 34. $4\frac{5}{6}$, 35. $3\frac{3}{5}$, 36. $5\frac{3}{8}$, 37. $7\frac{3}{7}$, 38. $3\frac{4}{9}$

Set 57 *For use with page 341*

Find each sum.

1. $\frac{1}{2} + \frac{1}{2}$ 1
2. $2\frac{1}{5} + 1\frac{2}{5} = 3\frac{3}{5}$
3. $\frac{2}{5} + \frac{1}{5}$ $\frac{3}{5}$
4. $5\frac{2}{6} + 3\frac{1}{6} = 8\frac{1}{2}$
5. $\frac{5}{2} + \frac{4}{2}$ $\frac{9}{2}$ or $4\frac{1}{2}$
6. $4\frac{2}{3} + 6\frac{1}{3} = 11$
7. $3 + \frac{1}{3}$ $3\frac{1}{3}$
8. $3\frac{1}{12} + 5\frac{3}{12}$ $8\frac{1}{3}$

Solve each short story problem.

9. $2\frac{1}{2}$ dozen cookies baked.
 $1\frac{1}{4}$ dozen more to bake.
 How many cookies in all? $3\frac{3}{4}$ dozen

10. Recipe: $\frac{1}{2}$ litre milk,
 $\frac{1}{4}$ litre water.
 How much liquid in all? $\frac{3}{4}$ litre

Reflected answers, Set 57: 1. 1, 2. $3\frac{2}{3}$, 3. $\frac{2}{5}$, 4. $8\frac{1}{2}$ or $\frac{5}{9}$, 5. $4\frac{1}{2}$, 6. 11, 7. $3\frac{1}{3}$

Books to Explore

Adler, Irving. ***The Giant Book of Mathematics.***
New York, Golden Press, 1960.
(Available from Whitman Golden Ltd., Cambridge, Ontario)

Have you ever wondered how a tree grows or why a volcano is shaped as it is or what makes a card trick work? This colorful book answers these and many other questions, through exploring the world of mathematics. You'll find all kinds of exciting ideas about numbers and what they mean in our daily lives. Here are just a few of the interesting topics:

What an Egyptian carpenter used for a ruler 14
How to locate north with a stick and a watch 60
The speed of a falling satellite 65
Your chance of getting 2 heads when you toss 2 coins 73

Hogben, Lancelot. ***The Wonderful World of Mathematics.***
New York, Doubleday, 1968.
(Available from Doubleday Publishers, Toronto)

This book shows that the story of how man became civilized is also the story of how mathematics became a science. You will enjoy going back to the time of the cave man and finding out how man learned to measure and to count, to build and to navigate, to design and to calculate with computers.

Some topics covered in this book are:

The geometry of land measuring 17
How the Greeks measured the earth 36
Hindu numerals 45
Arabian lattice multiplication 50
Invention of the telescope 61

Jonas, Arthur. ***New Ways in Math.***
Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962.
(Available from Prentice-Hall of Canada Ltd., Scarborough, Ontario)

Cartoons tell the story of mathematics, including sets, probability, and algebra, and make this book fun to read. The chapter "Men in Math" on page 62 describes contemporary mathematicians, like Einstein and Von Neumann, as well as those from the past, like Pythagoras and Archimedes.

Other chapters you'll probably find exciting include:

The magic of two 24
When 1001 = 9 31
What is your hunch? 45
Doughnuts and pretzels 50
Math without numbers 57

You may also enjoy reading *More New Ways in Math* (1964), by Mr. Jonas.

Here are some other books you may enjoy:

Adler, Irving and Ruth. ***Numerals: New Dresses for Old Numbers.***
New York, The John Day Company, 1964.
(Longman Canada Ltd., Don Mills, Ont.)
Several new ways of counting are clearly explained in this little book. The authors tell how to change numerals to other bases, then add and multiply; they also explain place value.

Andrews, F. Emerson. ***Numbers, Please.***
Boston, Little, Brown and Company, 1961.
(Little, Brown & Co. (Canada) Ltd., Toronto)
This book about numbers from base 2 to 12 tells how counting began, how to use an abacus, and what decimals mean. You'll learn useful shortcuts in figuring, too.

Bendick, Jeanne. ***The First Book of Time.***
New York, Watts, 1963.
(Grolier Limited, Toronto)
Excellent pictures help trace the history of time and how we measure it. All kinds of clocks are described—from sun dials and water clocks to the atomic clock and clocks in your body.

Bendick, Jeanne and Levin, Marcia. ***Mathematics Illustrated Dictionary.***
New York, McGraw-Hill, 1965.
(McGraw-Hill Ryerson, Scarborough, Ontario)
A handy tool for students. If you need to know about ancient or contemporary mathematicians, mathematical terms and definitions, or any facts and figures, use this dictionary.

Feravolo, Rocco. ***Wonders of Mathematics.***
New York, Dodd, Mead Book Company, 1963.
(Dodd, Mead & Co. (Canada) Ltd., Toronto)
This book uses many activities to develop several bases and number systems.

Kettlekamp, Larry. ***Spirals.***
Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1964.
(Prentice-Hall of Canada Ltd., Scarborough, Ontario)
An enjoyable look at spirals—some are in nature, some man-made.

Leeming, Joseph. ***Fun with Puzzles.***
Philadelphia, J. B. Lippincott Company, 1946.
Also available in paperback from Scholastic Book Service, 1966.
(McClelland & Stewart Ltd., Toronto)
This book contains more than 200 match, coin, paper-and-pencil, cutout, and word puzzles. The answers are all in the back of the book.

Massoglia, Elinor. ***Fun-Time Paper Folding.***
Chicago, Children's Press, 1959.
(Scholars Choice Limited, Stratford, Ont.)
Many kinds of objects and shapes to make. No cutting and pasting as each shape can be folded from a single piece of paper.

Shulman, Alix. ***Bosley on the Number Line.***
New York, McKay Company, 1970.
(Musson Book Company, Don Mills, Ontario)
Here is an adventure story with a mathematical plot, all of which adds up to a fun book.

Simon, Leonard and Bendick, Jeanne. ***The Day the Numbers Disappeared.***
New York, McGraw-Hill Book Company, 1963.
(McGraw-Hill Ryerson, Scarborough, Ontario)
By taking numbers away from his class, Mr. Dibbs shows why we need numbers in everyday life. The Egyptian, Greek, and Roman number marks are traced and compared.

Sitomer, Mindel and Sitomer, Harry. ***What Is Symmetry?***
New York, Thomas Y. Crowell, 1970.
(Fitzhenry & Whiteside Ltd., Don Mills, Ont.)
Take a tour with a colorful alligator to see the point symmetry in a clover leaf, the plane symmetry in a mirror reflection, or the symmetry in the arc of a comet's orbit.

Steadman, Ralph. ***The Little Red Computer.***
New York, McGraw-Hill Book Company, 1969.
(McGraw-Hill Ryerson, Scarborough, Ontario)
The little red computer dropped out of a robot-instructed school because he could never make sense of numbers. Finally, he gets a chance in a space adventure.

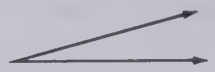
"Wise Owl Books."
New York, Holt, Rinehart and Winston, 1965.
(Holt, Rinehart and Winston of Canada Ltd., Toronto)
A set of 20 books for enjoyable and informative reading. These five have to do with mathematics:
I've Got Your Number, John, by **Olive Berg;**
Millions of People, by **Thomas Dripdale and John Dunworth;**
Number Patterns Make Sense, by **Howard Fehr;**
Dr. Frick and His Fractions, by **Henry W. Ford;**
Optical Illusions, by **Jack and Robert Strimban.**

Glossary

addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the one number which is the result of combining the two numbers is called the sum of the addends.

angle Two rays from a single point.



approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

borrow A commonly used term for the regrouping process involved in certain types of subtraction.

Example:

$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{13}\cancel{3} \\ -\,1\;7 \end{array} \rightarrow \begin{array}{r} 3\;0 \\ -\,1\;0 \\ \hline 2\;0 \end{array} \qquad \begin{array}{r} 1\;3 \\ -\,7 \\ \hline 6 \end{array} \rightarrow \begin{array}{r} 4\;3 \\ -\,1\;7 \\ \hline 2\;6 \end{array}$$

carry A commonly used term for the regrouping that is involved in addition.

Example:

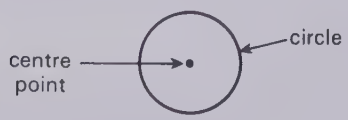
$$\begin{array}{r} 57 \\ +26 \\ \hline 83 \end{array} \rightarrow \begin{array}{r} 50 + 7 \\ 20 + 6 \\ \hline 70 + 13 = 83 \end{array}$$

centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

circumscribed circle A circle is circumscribed about a polygon when each vertex of the polygon is a point of the circle. In the figure, the circle is circumscribed about the triangle.

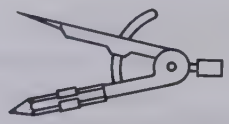


circle The set of all points in a plane which are a specified distance from a given point called the centre or centre point.



common factor When a number is a factor of two different numbers, it is said to be a common factor of the two numbers.

compass A device for drawing models of a circle.



composite number Any whole number greater than 1 that is not prime.

cone Generally thought of as a right circular cone, which is illustrated below.



congruent figures Figures that have the same size and shape.



congruent triangles

co-ordinate Number pair used in graphing.

co-ordinate axes Two number lines intersecting at right angles at 0.

count To name numbers in regular succession.

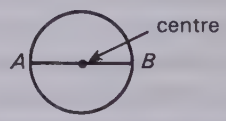
cube A rectangular prism (box) such that all faces are squares.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment AB.



diameter A chord that passes through the centre point of the circle.



difference The number resulting from the subtraction operation.

digits The basic Hindu-Arabic symbols used to write numerals. In the base-ten system, these are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

dividend In the problem $33 \div 7$, 33 is called the dividend.

$$\begin{array}{r} 4 \\ \text{Example: } 7\overline{)33} \leftarrow \text{dividend} \\ \underline{28} \\ 5 \end{array}$$

division An operation related to multiplication as illustrated:

$$3 \times 4 = 12 \begin{cases} \nearrow 12 \div 3 = 4 \\ \searrow 12 \div 4 = 3 \end{cases}$$

divisor In the problem $33 \div 7$, 7 is called the divisor.

$$\begin{array}{rr} \text{Example:} & 4 \\ \text{divisor} \rightarrow & 7\overline{)33} \\ & \underline{28} \\ & 5 \end{array}$$

edge An edge of a space figure is one of the segments making up any one of the faces of the space figure.

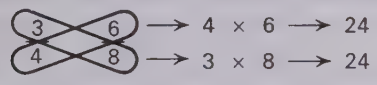
empty set A set that has no objects in it.

equality (equals; or $=$) A mathematical relation of being exactly the same.

equation A mathematical sentence involving the use of the equality symbol.

Examples: $5 + 4 = 9$; $7 + \square = 8$; $n + 3 = 7$.

equivalent fractions Two fractions are equivalent when it can be shown that they each can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:



equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, · · ·).

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

factor See multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

fraction A symbol for a fractional number, usually written $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$, and so on.

fractional number The one number we think about for each set of equivalent fractions.

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

greater than ($>$) One of the two basic inequality relations.

Examples: $8 > 5$, $28 > 25$, $80 > 50$.

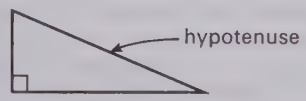
greatest common factor The largest, or greatest, number that is a factor of each of two numbers.

grouping principle (associative principle) When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

Examples: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 2) = (3 \times 4) \times 2$

hexagon A six-sided polygon.

hypotenuse The side opposite the right angle in a right triangle.



improper fraction A fraction in which the numerator is greater than or equal to the denominator.

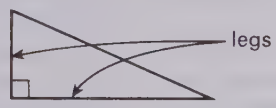
Examples: $\frac{8}{5}, \frac{9}{6}, \frac{12}{3}, \frac{7}{7}$

inequality ($<, \neq, >$) In arithmetic, a relation indicating that the two numbers are not the same.

inscribed circle A circle is said to be inscribed in a polygon if the circle lies within the polygon and each side of the polygon is tangent to the circle.



legs of a right triangle The two sides of a right triangle other than the hypotenuse.



length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

less than ($<$) One of the two basic inequality relations.

Examples: $5 < 8$, $25 < 28$, $50 < 80$.

line A line is a set of points that "goes on and on" in both directions. There is only one line through any two points.

line segment See segment.

lower terms A first fraction is in lower terms than a second fraction if the first fraction is equivalent to the second fraction and if the denominator of the first fraction is less than the denominator of the second fraction.

Example: $\frac{6}{8}$ is in lower terms than $\frac{9}{12}$.

lowest terms A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor greater than 1.

measure (1) A number indicating the relation between a given object and a suitable unit. (2) The process of finding the number described in (1).

midpoint A point that divides a line segment into two parts of the same size.



minus (−) Used to indicate the subtraction operation, as in 7 − 3 = 4 (read, "7 minus 3 equals 4").

mixed numerals Symbols such as $2\frac{1}{2}$ and $3\frac{1}{4}$.

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number. Example: 24 is a multiple of 6 since 4 × 6 = 24.

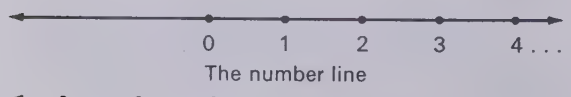
multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the one number which is a result of combining the two numbers is called the product of the two numbers.

multiplication-addition principle (distributive principle) This principle is sometimes described in terms of "breaking apart" a number before multiplying.
Example: 6 × (20 + 4) = (6 × 20) + (6 × 4)

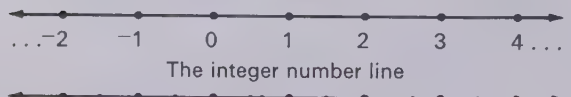
negative number If a number adds to a whole number to give 0, it is a negative number.

For example: 5 + $^{-}$**5** = 0
19 + $^{-}$**19** = 0

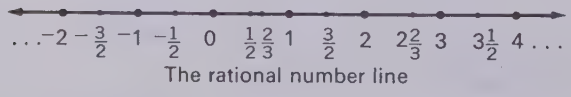
number line A line with a subset of its points matched with a subset of the real numbers. We say that the rational number line has "holes" in it because some points are not matched with rational numbers. The real number line is said to be "complete" because each point is matched with some real number.



The number line



The integer number line



The rational number line

number pair Any pair of numbers. In this book, usually a pair of whole numbers.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not even.

one principle (for multiplication) Any number multiplied by 1 is that same number.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set and each element of the second set is matched with exactly one element of the first set.

order principle (commutative principle) When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product).

Examples: 4 + 5 = 5 + 4
2 × 3 = 3 × 2

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with its opposite sides parallel.

parentheses A pair of curved symbols, (), used to indicate grouping or order of performing operations.

Examples: (5 × 4) − 2 = 18
5 × (4 − 2) = 10

pentagon A five-sided polygon.

perfect number A number that is half the sum of its factors.

Examples: 6, 28, and 496.

perimeter The sum of the lengths of the sides of a given polygon.

period In arithmetic, each set of three digits is called a period. These periods are called (right to left) units' period, thousands' period, millions' period, and so on.

Example:	**3 4 2**	**6 7 4**	**2 0 8**
	millions' period	thousands' period	units' period

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. In the numeral 3257 the 5 stands for 50; in the numeral 36 289 the 6 stands for 6000.

plus (+) Used to indicate the addition operation, as in 4 + 3 = 7 (read, "4 plus 3 equals 7").

polygon A closed geometric figure made up of line segments.

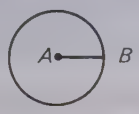
prime number A number greater than 1 whose only factors are itself and 1.

product The result of the multiplication operation. In 6 × 7 = 42, 42 is the product of 6 and 7.

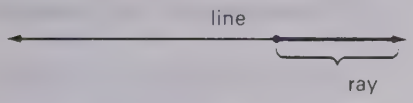
quadrilateral A four-sided polygon.

quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radius (1) Any segment from the centre point to a point on the circle. (2) The distance from the centre point to any point on the circle.



ray The heavy part of the line shows a ray.



rectangle A quadrilateral that has four right angles.

regrouping A method of handling place value symbols in adding or subtracting numbers.

remainder:

Example:
$$\begin{array}{r} 6 \\ 7\overline{)47} \\ 42 \\ \hline 5 \end{array} \leftarrow \text{remainder}$$

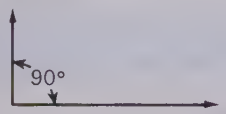
repeated addition Finding the sum of a set of numbers, each of which is the same.

Example: $5 + 5 + 5 + 5$

repeated subtraction Starting with a number and repeatedly subtracting the same given number from each difference that is obtained.

rhombus A parallelogram with 4 sides of the same size.

right angle An angle that has the measure of 90 degrees.



a right angle

right triangle A triangle that has one right angle.

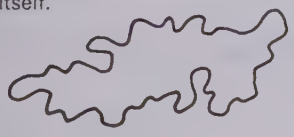
Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

segment Two points on a line and all the points on that line that are between the two points.

sequence A collection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group or collection of objects.

simple closed curve Can be thought of as a loop of string that is on a flat surface and does not cross itself.



skip count To count by multiples of a given number. Example: Counting by fives — 0, 5, 10, 15, 20, . . .

solution The number or numbers which result from solving an equation or a given problem.

solve To find the number or numbers which, when substituted for the variable or placeholder, make a given equation true.

square A quadrilateral that has four right angles and four sides that are the same length.

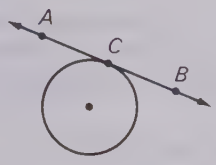
subtraction An operation related to addition as illustrated:

$$7 + 8 = 15 \begin{cases} \nearrow 15 - 8 = 7 \\ \searrow 15 - 7 = 8 \end{cases}$$

sum The result obtained by adding any set of numbers.

symmetric figure A figure that can be folded in half so the two halves match.

tangent A line is tangent to a circle if the two figures are in one plane and have exactly one point in common.

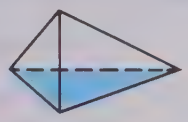


Line *AB* is tangent to the circle at point *C*.

times ($\times$) Used to indicate the multiplication operation, as in $3 \times 4 = 12$ (read, "3 times 4 equals 12").

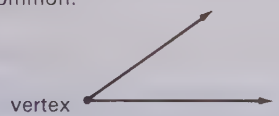
triangle A three-sided polygon.

triangular pyramid A 4-sided space figure that has triangular regions for all faces.



unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.



volume The measure, obtained using an appropriate unit (usually a cube), of the interior region of a space figure.

whole number Any number in the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, \ldots\}$.

zero principle (for addition) Any number added to zero is that same number.

Tables of Measures

LENGTH	
10 millimetres (mm) = 1 centimetre (cm)	1000 millimetres = 1 metre
10 centimetres = 1 decimetre (dm)	100 centimetres = 1 metre
10 decimetres = 1 metre (m)	10 decimetres = 1 metre
1000 metres = 1 kilometre (km)	1 / 1000 kilometres = 1 metre

TIME	
60 seconds (s) = 1 minute (min)	52 weeks = 1 year (yr)
60 minutes = 1 hour (h)	12 months (mo) = 1 year
24 hours = 1 day	365 days = 1 year
7 days = 1 week (wk)	366 days = 1 leap year

CAPACITY
10 millilitres (ml) = 1 centilitre (cl)
10 centilitres = 1 decilitre (dl)
10 decilitres = 1 litre (*l*)
1000 litres = 1 kilolitre (kl)

WEIGHT
1000 grams (g) = 1 kilogram (kg)
1000 kilograms = 1 tonne

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A Text for Teachers

Investigating Mathematics Learning

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Introducing the Metric System

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INVESTIGATING MATHEMATICS LEARNING

I. Some Thoughts About Learning

Almost everyone has some observations on teaching and learning. A recent quote that has been making the rounds is: "If we tried to teach children to speak, they would never learn." However, in *The Process of Education* (Harvard University Press, 1960), Jerome Bruner observes, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development." But Linus (of *Peanuts* cartoon fame), considerably less optimistic, laments: "How can I learn 'New Math' with an 'Old Math' mind?"

In a more critical vein, John Holt in *How Children Fail* (Pitman Publishing Co., 1964) asserts: "In our classes, we begin with words, carry on with words, and often fail to get beyond words." He also says, "All too often the mathematics classroom becomes a temple of worship for the right answers, and the way to get ahead is to lay plenty of them on the altar." We know, of course, that many teachers for many years have been doing an excellent job helping elementary school children learn mathematics. Yet, it is worthwhile for us to reevaluate our approaches and, if possible, find even better ways to create situations where children learn more effectively.

The implications of the research of Piaget and others in how children learn mathematics and the observations of countless classroom teachers concerning the directions we should take are well summarized by a familiar Chinese proverb:

I hear and I forget.
I see and I remember.
I do and I understand.

The message of this proverb is that hearing and seeing are not enough: to learn with understanding, the child should experience *active involvement* with mathematical ideas. In order for the child to become actively involved, it has been found that the use of *physical materials* which contain the seeds of the mathematical ideas are valuable and often necessary. Coupled with the idea of active involvement with physical materials is the idea that teachers should encourage *student responsibility* and create conditions in which the student is not always encouraged to rely solely on the teacher but rather to take initiative for figuring out some things for himself.

Z. P. Dienes summarized a multitude of suggestions from researchers and teachers when he said: "It is suggested that we shift the emphasis from teaching to learning, from our experience to the children's, in fact, from our world to their world."

Teachers vary considerably in their views of how best to help children become actively involved with mathematics. While one teacher desires to convert his classroom immediately into a mathematics laboratory, another teacher may prefer a very modest beginning with a limited amount of active student involvement with physical materials inserted into his usual classroom approach. In this text we suggest a number of approaches for modest beginnings and indicate ways in which these approaches might be expanded to provide for a total laboratory approach and a more extensive individualized program.

To introduce one possible approach, let us simulate a teaching strategy by outlining one way to organize a specific lesson. Thus, suppose a teacher wanted to devise a lesson which would help children understand the idea of congruent segments in geometry. First the teacher provides each child with a geoboard and a sheet containing several 5-by-5 arrays of dots. Then he reviews, very briefly, the idea of a segment and the endpoints of a segment. Next, after helping the children see that they can use a rubber-band around two nails to represent a segment on the geoboard, he passes out the investigation suggested in Figure 1.

INVESTIGATION

How many different segments with endpoints just as far apart as these can you find on your geoboard? Record your results on dot paper.

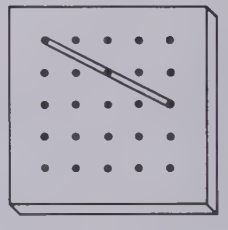


Figure 1

The teacher may choose to have chairs or desks rearranged so that children can communicate with each other as they become involved in the investigation. The teacher will check to be sure that everyone understands the investigation question; then he should encourage the children to find their own way to answer the question and record their findings. (To gain a fuller appreciation of an investigation situation, play the role of the child and complete the investigation yourself.)

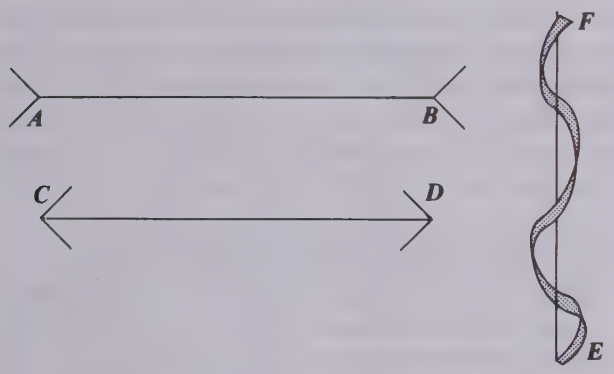
Brief discussions among children or between teacher and children may occur during investigations, but the main discussion is most effective after the investigation has been completed. At this time, the teacher might ask such questions as: "How many different

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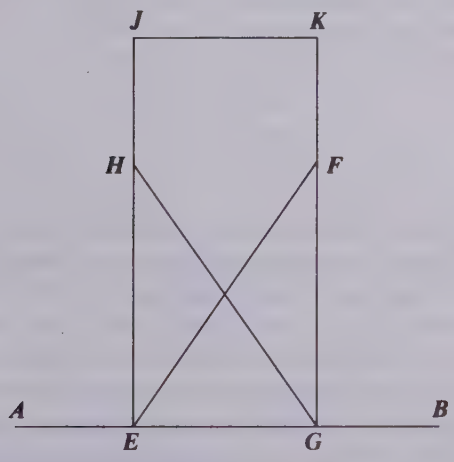
segments did you find?" "How can you be sure that you have found all such segments?" "How could you convince someone that each of your segments has endpoints just as far apart as all the others?" Such questions could then be followed up with a definition of congruent segments: When the endpoints of one segment are just as far apart as the endpoints of another segment, we say the segments are *congruent*. Then ask, "Can you think of some other ways to tell when two segments are congruent?" This question might lead into a discussion of how tracing paper, compasses, or marks on the edge of a piece of paper can be used to determine whether or not two segments are congruent.

After the children have discussed the ideas, the teacher may provide them with some problems which *utilize* these ideas. The child would probably be encouraged to use the ideas for testing congruence of segments that were developed in the discussion. The following are examples of possible exercises.

1. Find 2 segments below that are congruent to each other.



2. Name each pair of congruent segments in this picture.



One way to individualize a lesson is through an *extension* of the exercises. Extending the exercises can provide for remediation, reinforcement, or enrichment. As an extension to individualize this lesson, the teacher might give certain students the follow-up investigation below. (For a fuller appreciation of this lesson, complete the exercises and the investigation yourself.)

INVESTIGATION

Segment AB is not congruent to segment CD.
How many different segments (no two congruent) can you find on your geoboard? Record your results on dot paper.

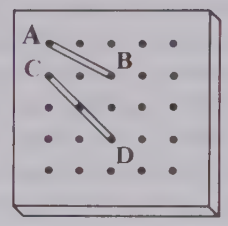


Figure 2

This abbreviated "lesson" provides a preview of one possible technique for encouraging children to become actively involved with physical materials in situations where they take more responsibility in the learning of mathematical ideas.

In the next section of this text, the parts of lessons such as the one described above will be analyzed and discussed. An outline for planning such lessons will be given, and various suggestions for carrying out each part of such a lesson will be proposed.

Since the investigation phase of the lesson provides the encouragement for active involvement by the child and since the kind of investigation used depends upon the type of learning involved, Section III in this text will focus on specific types of learning in elementary school mathematics. For example, the "lesson" described above helped children learn the *concept* of congruent segments; other lessons might be concerned with developing a *skill*, forming a *generalization*, learning a *fact*, or developing an *attitude*. Each type of learning will be analyzed and related to activity-oriented lessons that provide modest beginnings toward an active approach to mathematics learning.

Edith Biggs and James MacLean, in their book *Freedom to Learn* (Addison-Wesley, 1969), state: "A few schools scattered throughout the world are responding with some speed to a message which has been repeated with increasing urgency for some three hundred years. It is a simple message: Schools should be organized, not for teachers to teach, but for children to learn." In the same book, there appears an extensive list of "homemade" materials and devices that can be easily acquired for use in the mathematics classroom. Many materials, from newsprint and drinking straws, to string, popsicle sticks, beans, and homemade geoboards, can be made available to children at minimal cost. Rather than dismiss the possibility of actively involving children with materials in the classroom because no funds are available, a teacher should study this list carefully; he may be amazed by how much can be done with minimum expense.

Teachers sometimes feel that to involve children with physical materials and allow them to communicate with other children in the classroom is to invite

chaos. On the contrary, it has been found that, when children really become involved in using materials to investigate a situation, there may be a bit more low-keyed noise about the room but the usual discipline problems are almost nonexistent. It is helpful if there are tables available in the classroom so that children can work in small groups. If tables are not available, desks could be moved to assist in small-group work. On occasion, an investigation might call for children to leave their desks and to engage in other activity in the room. A simple set of "ground rules" should suffice to make the situation quite manageable.

It is interesting to consider the number of elementary school teachers who prefer to say that they are "helping children learn mathematics" rather than that they are "teaching mathematics." What one says, of course, does not always describe accurately what one does. It does seem important, however, in the light of recent studies and observations about how children learn mathematics, to focus on the child and try to create an environment in which the child has a greater opportunity to make decisions and to become really interested in his study. It is hoped that the following sections of this teachers' text will provide some ideas which may help you improve your ability to "help children learn mathematics."

EXERCISE SET 1

1. What was your reaction to the investigations in this section? **A** Did you become involved in the activity? **B** Were you interested? **C** Did you watch the clock? **D** Did you talk to anyone else while completing the investigation? (If so, was it helpful?) **E** Did the investigation situation help you better understand the idea involved? **F** What other feelings did you have?
2. Which quotation in this section seemed most significant to you? Why?
3. **A** Do you think most teachers teach the same way they were taught as elementary school children? **B** What do they do differently? **C** What are some ways you think our teaching of elementary school mathematics might be improved?
4. Look through the *Investigating School Mathematics* text at your grade level. How do the comments in this first section of the text relate to the approach taken in the child's text?

II. A Plan for a Learning Experience

First consider the practical matter of how the teacher proceeds in the daily task of helping children learn mathematics. A structured outline (inherently flexible) around which daily learning experiences may be planned can be a valuable organizational aid for the teacher and can give him a fresh insight into the role of new approaches to instruction.

Here is the outline that was used in planning the "lesson" in Section I. It has proven to be quite useful, especially for those teachers who have desired to make a beginning toward providing children more opportunities for active involvement with mathematical materials and ideas.

Preparation and Investigation
Discussion
Utilization and Extension

Since this outline offers a variety of possibilities for a teacher to reevaluate his approach to classroom instruction, the following sections provide an examination of its individual elements.

Preparation and Investigation

The investigation phase (often called simply "the investigation") is central to the learning experience. In this phase, the children are encouraged to become actively involved, individually or in groups, in the investigation of a situation that contains the seed for the central idea of the lesson. The investigation should be the "main event" in terms of pupil activity and involvement. The teacher should think of the investigation as a child-centred activity. Completion of the investigation in Figure 3 will help clarify the ideas of investigation.

INVESTIGATION

Can you find an investigation in a text from the *Investigating School Mathematics* series that

(a) uses centimetre strips?
(b) utilizes paper folding?
(c) has a question like "How many can you find?"
(d) involves the geoboard?
(e) encourages children to use graph paper?
(f) asks the children to record their findings?
(g) directs the children to use reference material?

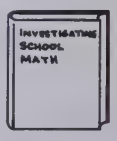


Figure 3

Homemade or commercially produced manipulative materials often provide the stimulus for the situation to be investigated. At other times, even more simple teacher-devised activities provide this stimulus. For example, the suggested investigation in Figure 4 might have been made by a teacher to initiate an investigation in a lesson designed to help children form the generalization, "You can rearrange three addends any way you please, and the sum will always be the same."

Sometimes by asking appropriate questions about a situation of interest to the children the teacher may involve them in an exploration of a central idea to be developed.

Regardless of how an investigation is initiated, a teacher should remember that the investigation situation is specifically designed to encourage children to

INVESTIGATION

Make three slips of paper like these.



Then turn them over and mix them up. Pick any two slips and add the numbers on them. Then add the number on the other slip.

If you do this five times, will you get the same number each time?

Figure 4

take responsibility for the thinking and exploring. Too much "teacher help" can hinder the achievement of these aims.

In an investigation, it is not uncommon to see children deeply involved and assuming full responsibility for completing the task at hand. The teacher, who plays a key role in initiating the investigation, may appear not to be needed as he moves about the room. Occasionally, a brief discussion between teacher and child occurs, but most of the larger-group discussion occurs after the investigation. The investigation itself should embody an attitude toward learning that could be easily stifled by too many words from the teacher. Perhaps, in an investigation, a new adage should replace the old: the teacher, rather than the children, should be "seen but not heard."

The investigation is predicated on the assumption that the best way to minimize the need for words is to substitute an appropriate question for a wordy explanation at a time when a child's interest in a mathematical situation is beginning to ripen.

For example, suppose a certain group of children understand the concept of a triangle and are ready to consider characteristics that distinguish one type of triangle from another. An appropriate question to initiate an investigation might be the one shown in Figure 5. (Try this investigation yourself.)

INVESTIGATION

How many triangles with different shapes can you make that have no nails inside?

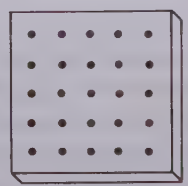


Figure 5

This question is both activity-stimulating and activity-sustaining. It helps involve the child in a search which he will continue with little further motivation. Notice also that the answer is not as important as the experiences the child will have as he responds to the question. Further, the question is sufficiently clear that the child immediately becomes involved with the challenge of the investigation rather than dissipating energy in efforts to understand the question. Another characteristic of this type of question is that it provides for individual differences: when the child is asked "How many can you find?" he can feel successful even if he finds only one. Of course, not all investigations can or should be introduced by this type of question, but it is important for the teacher to recognize that as the children respond to these questions, they will achieve in widely differing ways. In an investigation, the teacher should give recognition for all levels of achievement.

It should be noted that the amounts of time used for the investigations may vary considerably. One investigation may involve a very brief "happening" which sparks a simple idea within the child. Another investigation may utilize a large part of the period of time available for the mathematics lesson and might involve the child in a sustained exploration of a game or a set of manipulative materials.

To set the stage for an investigation of any duration, a preliminary *preparation* phase is sometimes needed. This phase provides for a brief review of key ideas needed for the investigation and for any motivational activity helpful in introducing it. This phase should be kept fairly short and care should be taken to see that this preliminary work does not preempt the central idea or activities involved in the investigation or the work that follows it.

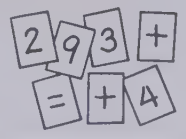
In summary, the investigation phase is the child-involvement phase. It often requires materials, and is usually motivated by a carefully selected question which focusses the student's attention on the central idea of the lesson. Proper consideration of this phase in your lesson planning can be highly rewarding.

EXERCISE SET 2

1. Find some investigations in the *Investigating School Mathematics* text that contain features not mentioned in Figure 4.
2. Choose a lesson from an *Investigating School Mathematics* text and write a description of the role you think the teacher would play in using the investigation phase of the lesson.
3. Choose an idea to be taught and prepare an investigation situation which has the potential of involving the child in working with this idea.
4. Two investigations follow. Give the central idea of a possible lesson based on the use of each one.

A

INVESTIGATION

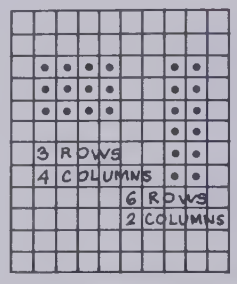


Cut out 7 slips of paper. Put one of these numerals or one of these signs on each one.

How many different equations with 3 addends can you write with your slips of paper? Record each equation you find.

B

INVESTIGATION



The graph paper shows two different ways to arrange 12 counters in a rectangular array.

How many different ways can you arrange 24 counters in a rectangular array? Record your findings by drawing pictures on graph paper.

5. Here is an interesting investigation you may like to try. Through it, you will be introduced to a basic idea of mathematics. Be sure to record your findings and be ready to discuss them further in the next section.

Copy and continue this array of numbers until you reach 52.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
	...		

Then circle all the prime numbers in the array.

Notice that the numbers in the right-hand column can be written as $4 \times$ (a whole number). For example: $8 = 4 \times 2$, $12 = 4 \times 3$, and $20 = 4 \times 5$.

Can you make a statement about prime numbers that is suggested by this activity?

Discussion

Following the investigation, a *discussion* phase allows teacher and children to further share ideas in a discussion of what they found in the investigation. The teacher has an excellent opportunity in this phase to ask questions and to supply examples to help children further develop their understanding of the ideas germinated by the investigation.

Another valuable aspect of the discussion phase is that it provides additional opportunities for children to communicate with other children as a means of shaping their ideas. In a good discussion, it is not unusual for children, having reached an impasse in *their* thinking and communication about an idea, to ask the teacher if he can clarify the point. This is when the teacher as a resource person emerges. At other times, when ideas new to the teacher arise, the teacher participates in the discussion, not as a resource person, but as a fellow-learner. Both of these situations can contribute to a comfortable, meaningful discussion, but its potential benefits may never be realized if the teacher monopolizes the discussion to the extent that the children are denied the opportunity to draw their own inferences and make their own decisions. Since it is the child who is involved in the investigation, the child's ideas about the findings should be of primary importance, and the child should supply as many details leading to the understanding of the idea as possible.

By listening to the child and asking appropriate questions, the teacher can build on the child's initial ideas and help him develop a deeper understanding in preparation for further work. This understanding cannot be developed, however, by always asking questions which require simply that a child remember a fact correctly or perform a practical skill. Nor is it sufficient to ask questions to which a child can respond with a guess of "Yes" or "No." Rather, the questions that should be asked often are those that require a deeper thinking on the part of the child.

For examples of the more effective type of question, consider again the investigation described in Figure 7. This investigation, designed to set the stage for the development of the concepts of isosceles triangle, right triangle, scalene triangle, and equilateral triangle, might be followed by a discussion in which the teacher would ask questions such as the following:

1. Can you choose a pair of triangles you found and describe ways in which one is different from another?
2. In what ways are some triangles you found alike? (Note: Children may respond, "Some have a square corner," "Some have two sides the same," "Some have no sides the same," "Some are large," and so on.)
3. How would you describe a triangle that is different from any of the triangles you formed on the geoboard?

As the teacher asks thought-provoking questions and listens to the children's responses, he will be able to find ways to clarify the basic idea of the lesson and to prepare the children for the independent work which is to follow. It is in the latter stages of the discussion that the teacher may want to explain more carefully, show additional examples, and, in general, lead the child to a deeper mastery of the ideas involved.

EXERCISE SET 3

1. Can you find a question in the "Discussing the Ideas" section of an *Investigating School Mathematics* text which **A** asks the children to recall something previously learned? **B** asks the children to restate or explain an idea in their own words? **C** asks the children to interpret a diagram, picture, or explanation? **D** asks the children to analyze a given situation? **E** asks the children to evaluate a given situation?
2. What do you think about the effectiveness of the investigation described in Figure 5 as a means of meeting the goals indicated?
3. Write five questions you might ask while conducting a discussion in a mathematics lesson of your choice.
4. The following discussion exercises refer to the investigation presented in exercise 5 of Exercise Set 2. **A** What statement did you make about prime numbers? **B** Can you find a prime number that does not appear in the first or the third column? Can you find more than one? **C** $4 \times n$ is an algebraic expression. What algebraic expression can you devise to describe the prime numbers in the third column? in the first column? **D** Of the prime numbers less than 100, which type of prime occurs more often? **E** 113 is a prime number. Which type of prime is it?
5. Investigation questions may be open ("In how many ways can you measure a ball?") or closed ("Can you find the circumference and diameter of this ball?"). Discuss the merits of open and closed questions.

Utilization and Extension

The *utilization* phase allows each child to work on his own and to use the ideas developed in the investigation and discussion phases.

Often children need to practice recalling facts that have been developed or introduced in the lesson. Appropriate exercises requiring written answers are often valuable in providing this practice.

In another lesson, a child may have learned an algorithm or a skill. In order to refine this skill, he may need considerable practice using it. Appropriately designed written exercises which children complete independently can be quite helpful in polishing these skills.

In another lesson, a new idea may have been presented. In order to become more familiar with this idea and to understand how it relates to other ideas, the child may need thought-provoking problems which involve the idea. The *utilization* phase presents an opportunity for the child to solve problems which involve ideas that have been presented previously or to look at an idea that is different but closely related to one he has already encountered.

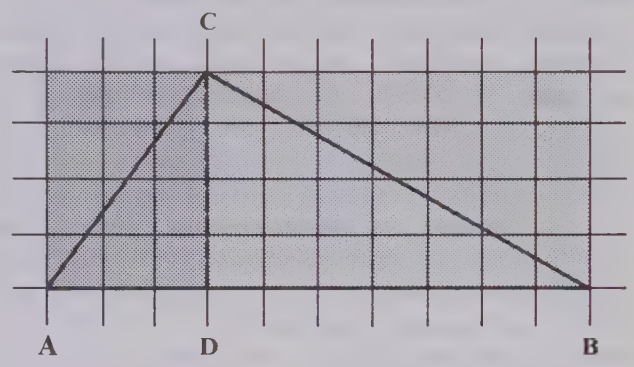
Creative activities for independent work can do much to extend the learnings developed in the investigation and discussion phases. The utilization exercises in examples A and B below are sequenced in such a way that the child has an opportunity to discover a new procedure or new ideas as a result of his work.

EXAMPLE A

Find the differences.

75	75	75	75	75	75	75
-32	-33	-34	-35	-36	-37	-38
43	42	41				

EXAMPLE B



What is the area of the region shaded dark gray?
What is the area of the region shaded light gray?
What is the area of the two regions together?
What is the area of triangle ADC?
What is the area of triangle BDC?
What is the area of triangle ABC?
The area of triangle ABC is what part of the entire shaded region?

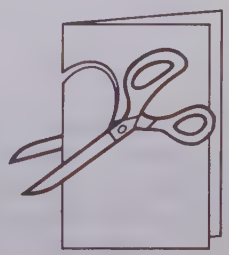
The teacher should appreciate the great potential value of discovery-sequenced exercises such as these, and should look for opportunities to make his own exercise sets using such sequences. Another set of utilization exercises might encourage the child to independently delve more deeply into the idea initiated in the investigation. Further activities with mathematical materials often provide opportunities for the child to use and extend the idea of the investigation. Example C provides an opportunity for the child to reinforce his concept of symmetry.

EXAMPLE C

Do this to make symmetrical figures.



Fold a piece of paper.



Make a cut that starts and ends on the fold.



Unfold the piece you cut out. It will be symmetrical.

Make cuts so that the unfolded shape will be:

A a rectangle	**D** a square	**G** a rocket
B a leaf	**E** a house	**H** a hexagon
C a triangle	**F** a pumpkin	**I** a butterfly

Regarding the utilization phase, it should be noted that on occasion it may be more valuable to have pairs or small groups of children work the exercises together.

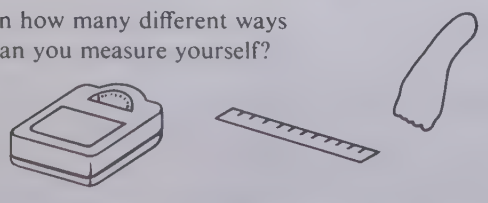
Finally, the *extension* phase provides for use of remedial, maintenance, or enrichment activities to further individualize the learning opportunities. This individualization offers numerous advantages. The slower children can avoid the frustration of having to proceed to new ideas before the previously presented ideas are understood, and the more capable children are spared the tedium of completing long lists of drill problems involving ideas they already understand.

The teacher might look for creative ways to meet individual differences in the ability to learn mathematics. For example, the slower child might profit from additional drill on certain facts and skills. Drill tapes or audio cassettes made by the teacher might provide a novel way to present the necessary practice. Duplicator masters and commercial workbooks are also available to provide extra work for those who need it. For other situations, an appropriate programmed instructional unit might serve the needs of the slower child. Single-concept film loops, which the child can play again and again, often are useful in helping him grasp an important concept. Appropriately conceived tutorial situations, in which classmates who understand the ideas work with children who do not, can be quite effective. Simple investigations utilizing physical objects which clarify more abstract ideas can also provide remedial work for certain children.

The teacher must also be concerned with those children who understand the basic ideas of the lesson and who can quickly work all the utilization exercises provided. These children can often become quite interested in activity cards which contain "open-ended" questions, such as the card shown below. (You are encouraged to try the suggested activity yourself.)

ACTIVITY CARD 10

In how many different ways can you measure yourself?



Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

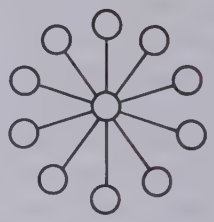
Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump

Activities such as these give the child an opportunity to make his own decisions about which ideas he uses from the lesson and how he uses them.

Puzzles or riddles can also provide a useful extension of ideas for your children. Consider, for example, those shown in Figure 6.

Think

Draw a figure like this one on your paper. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so that the sum along any line is 21.



Think

I can be found halfway between Twenty-seven and seventeen.

WHO AM I?



Figure 6

Conceptually fertile games can also provide valuable experiences to supplement the basic lesson. For example, the game "Sleuth" (3M Company) is fun for children and gives them valuable experience in classification and drawing logical inferences.

The methods suggested for extending the ideas for slower children are often suitable for use in certain situations with more capable children. Similarly, the more exciting modes of extension suggested for faster children can often be quite stimulating and valuable if used appropriately for the slower children.

It is to be hoped that the teacher will share a sense of excitement in providing extra stimulation to broaden the mathematical perspective of the children. Perhaps, he will also see that much of the extension activity can truly be fun for children while at the same time inspiring new interest and involvement in mathematical ideas. In using this suggested lesson outline, if the teacher chooses to maximize the investigation phase while deemphasizing the others, it might justly be said he is using the laboratory approach. On the other hand, should he maximize the discussion phase, he may find increased options for a guided discovery approach to mathematics learning. Also, it is possible that maximization of the utilization phase accompanied by appropriate student materials would allow the teacher to embark on a course of individually prescribed instruction.

EXERCISE SET 4

1. Find an example of an exercise set in which a learning sequence occurs in an *Investigating School Mathematics* text.

2. Choose a mathematics topic and write a set of exercises which might lead the student to discovery of a central idea.
3. Can you find a lesson in an *Investigating School Mathematics* text in which the "Using the Ideas" section provides for varying degrees of student ability.
4. Choose a learning experience appropriate for your children and list some possible specific activities for use in the extension phase of this learning experience.
5. Describe your views concerning the role of drill for slow, average, and bright children.
6. Select and play a game that could be used to extend a lesson with children.
7. In Exercise Set 2, you investigated an idea of mathematics. In Exercise Set 3, you had an opportunity to discuss this idea. The exercises below enable you to use the idea you learned, and suggest an extension of the idea.

Complete each exercise.

A List five prime numbers of the "$4n + 1$" type that are greater than 50.

B List five prime numbers of the "$4n - 1$" type that are greater than 50.

C 997 is the largest prime number less than 1000. Is it a "$4n + 1$" or a "$4n - 1$" prime?

D Suppose you used a continuation of the array of numbers shown below and circled all the prime numbers. What does this suggest about another way to classify the primes?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

. . .

III. A Focus on Specific Types of Learning

In considering the more specific aspects of mathematics learning it is helpful to categorize the general types of things children learn. A simplified categorization is given below.

Concepts
Skills
Generalizations
Facts
Attitudes

It is important to recognize that each of these types of learnings has unique characteristics. Because of this, the approaches and children's activities chosen to promote these learnings may often be quite different. In the sections that follow, we will consider each of these types of learning and suggest possible approaches and activities.

Concepts

Suppose that a child is having difficulty and comes to the teacher for assistance. When the teacher asks what the difficulty is, the child points to the multiplication 9×6 and says, "I can't do this because we haven't had it yet." This reflects a common attitude among children who have been in school for a few years. Somehow they learn to feel that they are incapable of figuring out anything new in mathematics. Literally, they can do nothing that they "haven't had yet."

If this child had confidence in his ability to "figure something out" and had a clear understanding of the *concept* of multiplication, he could have found the product by perhaps adding sixes, using sets, or making jumps on the number line. Another child who knew no division "facts" but who had a clear concept of division (as illustrated below) could use his knowledge of multiplication to find any of the basic quotients desired.

$$\begin{matrix} \mathbf{P} & \mathbf{F} & \mathbf{F} \\ 72 \div 8 = n & \leftarrow & \text{You find this quotient,} \end{matrix}$$

$$\begin{matrix} & & & \mathbf{F} & \mathbf{F} & \mathbf{P} \\ \text{when you find this factor.} & \rightarrow & n \times 8 = 72 \end{matrix}$$

A *concept*, then, may be thought of as an idea which, when properly understood, will help the child to solve problems he "hasn't had yet," to figure something out for himself. As another example, consider the concept of prime number. Once a child understands that a prime number is a whole number with exactly two factors, he has the power, providing he understands how to find the factors of a number, to seek out and list those numbers that are prime. Of course, the task of deciding whether or not a given number is prime may be quite laborious, but understanding the concept does give the child the power to succeed.

To look more carefully at what concepts are and how they are taught, consider a model in which concept learning is relatively easy, namely, that of a set of attribute pieces. Suppose there are pieces of four different shapes (triangles, squares, circles, and rectangles), of three different colors (red, blue, and yellow), and of two different sizes (large and small), as pictured in Figure 7. (In the figures the colors red, blue, and yellow are denoted by the initials R, B, and Y.)

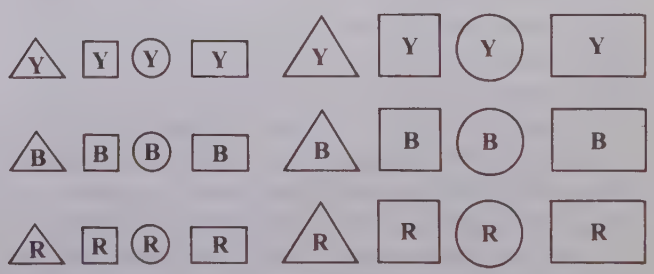


Figure 7

Now consider the Concept Card in Figure 8.

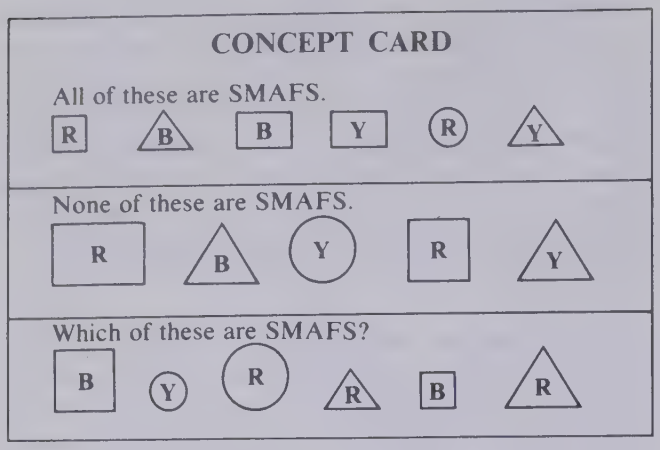


Figure 8

If you study the preceding Concept Card carefully, you will develop the simple concept of a SMAF. Notice that the key means used to teach this concept is by examples, along with *non-examples*. Both examples and non-examples play important roles in teaching many concepts in mathematics. The concept of a triangle may be taught to young children by using the Concept Card in Figure 9.

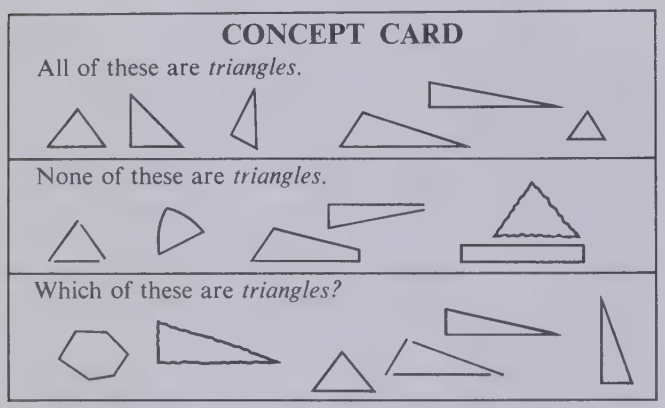


Figure 9

Clearly, the child would need further experiences in order to develop fully the concept of triangle, but the beginnings are embodied in the Concept Card shown in Figure 9.

One of the important ideas to remember when considering concepts is that concepts, unlike some other things that children learn, are developed over a period of time. Simple concepts may be developed very quickly, but other, more complicated concepts must be germinated when the child is very young and broadened through a spiralling return to the concept at various stages throughout the child's development. Many concepts are not fully developed until the child becomes an adult and encounters the idea in a variety of situations. For example, the concept of a fraction or fractional number may be introduced in grade 1 or grade 2, but a full understanding of this concept may not come until many years later. The child may acquire only an embryonic idea of a concept the first time it is presented, so it is important for the teacher to recognize the true nature of concepts and be willing to return often to the idea and carefully nurture its growth within the child. If he does not expect complete mastery after the initial presentation, he will spare himself considerable frustration when he recognizes later that the child needs further development of the basic idea.

Another key feature of concept learning suggested by the experiments of Piaget and supported and extended by the theories developed by Z. P. Dienes concerns the role of physical manipulative materials in young children's concept learning. In general, the implication of these authors' works is that it is through child involvement with physical environment that a firm basis for the development of more abstract concepts is laid. In fact, it is suggested that concept learning is facilitated by exposing children to as many different physical situations which embody the concept as possible.

It should be recognized that there are different levels of concept development and different types of concepts within these levels. For example, in the very earliest stages of mathematical learning, most concept learning involves the *concept of physical objects* such as balls, blocks, and circular or triangular objects. Very soon, the concept of certain *relations between objects* is developed: above, below, taller, shorter, larger, wider, longer, behind, and so on. A subsequent stage involves the concept of a *set of objects* such as a set of golf clubs, a set of dishes, a box of crayons, a set of blocks, a collection of stamps, or the children in a classroom. A slightly higher level of concept learning involves *relations between sets of objects:* equivalent, equal, has more than, has less than, and so on. It is at this stage that the important concept of *number* arises. For, in a sense, the concept of number involves a consideration of a set of equivalent sets. At a higher level of abstraction, the concept of certain *relations between numbers* (is less than, is greater than, is equal to, and so on) is developed. Ascending the ladder of abstraction, another level of development might involve the concept of *sets of numbers,* such as odds, evens, primes, composites, and perfects.

Clearly, the realm of concepts is vast, and the elementary teacher need not concern himself directly with many of the types of higher-level concepts. He must recognize, however, that the beginning stages in the development of many important concepts occur in the elementary school and that, through utilization of a variety of manipulative materials and appropriate strategies, he can do much to help the children learn concepts appropriate for their level.

EXERCISE SET 5

1. Use the attribute pieces shown in Figure 13. Invent a concept, name it, and make an appropriate concept card for it.

2. Choose at least two *Investigating School Mathematics* concepts from the list given below and develop concept cards which illustrate the use of examples and non-examples to teach the concepts you have chosen.
 - **A** quadrilateral
 - **B** simple closed curve
 - **C** odd number
 - **D** greater than (the relation)
 - **E** right triangle
 - **F** is congruent to (the relation)
 - **G** lowest-terms fraction
 - **H** parallelogram
 - **I** diagonal of a polygon
 - **J** parallel lines
 - **K** one half
 - **L** isosceles triangle
 - **M** equivalent sets
 - **N** symmetrical figure
3. Answer the questions on the sets of Creature cards from the set of attribute materials published by the Webster Division of McGraw-Hill Book Company (if available).
4. Choose an unusual concept of your own invention and make a concept card from which a person might discover your concept.
5. The investigation in Figure 1 was used to teach the concept of congruent segments. Make a card to teach this concept using examples and non-examples.
6. Complete "Learning a Concept" on pages I-18 and I-19; then answer the following questions.
 - **A** What are some examples of the concept you learned?
 - **B** Give some characteristics of the concept you learned.
 - **C** What were your feelings about the lesson? How could the lesson be improved to make the learning of the concept easier?

Skills

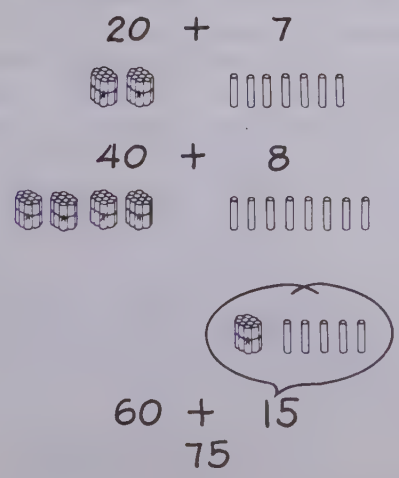
Broadly speaking, there are several types of skills that children develop in the elementary school. Hopefully, many children will develop a skill in estimating distance, weight, capacity, and time. Some teachers may wish to help children develop skill in drawing geometric figures. Some teachers set goals for upper-grade children which include developing skills in reasoning and even in "proof" of simple ideas. In elementary mathematics the most fundamental skill, by far, is that of computation with whole and rational numbers. It is these specific computational skills involving addition, subtraction, multiplication, and division and the processes related to these operations with which we are particularly concerned in the discussion that follows.

Two types of skills, power skills and speed skills, are available for completing each arithmetic process. A *power skill* is any effective way to find an answer. A *speed skill* is the most efficient way to find an answer. A power skill is a process through which a given problem is attacked by means of some technique which, though possibly quite inefficient, can produce a correct solution. This power skill may involve a long, tedious process, one which may be totally unrelated to the most efficient method for arriving at the solution. On the other hand, when a speed skill is employed, the problem is attacked with the most efficient technique available, and the problem is solved relatively quickly, usually in a mechanical fashion.

For example, suppose a child wants to find the sum of 27 and 48. If he simply starts at 48 and counts on 27 more, he is using a power skill. If, however, he finds the answer by using the usual algorithm for addition, then a speed skill is being employed.

Two additional points are worth noting about the previous example. First, in order to utilize the power

POWER SKILL B
– Bundles and Grouping



POWER SKILL C
– Expanded Notation

$$\begin{array}{r} 27 \\ +48 \\ \hline \end{array} \qquad \begin{array}{r} 20 + 7 \\ 40 + 8 \\ \hline 60 + 15 \\ 75 \end{array}$$

POWER SKILL D
– Addition Algorithm with Intermediate Step

$$\begin{array}{r} 27 \\ +48 \\ \hline 15 \\ 60 \\ \hline 75 \end{array}$$

Figure 10

skill, the child needed a clear concept of addition as it relates to counting. Thus, a power skill relies on a previously learned concept. As the child uses the concept in a power-skill situation, he gains new confidence in his ability to do something he "hasn't had yet." Secondly, the teacher should observe the evolution from power to speed. In finding the sum of 27 and 48, the initial power skill involved a basic concept of addition and the counting process. In practice, the child may continue the evolutionary trek from power to speed by next utilizing power skills B, C, and D as shown in figure 10.

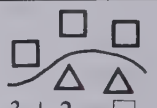
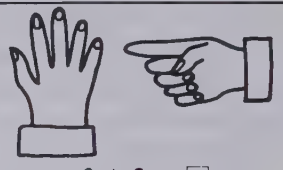
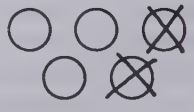
Note that each of these power skills represents a small step toward the ultimate, more efficient speed skill. When considering this process of evolution, it should also be noted that the earlier stages in a power-skill sequence often involve manipulative materials with subsequent power skills exhibiting a transition from the concrete to the more abstract. This physical beginning, which utilizes bundles and grouping, is illustrated as Power Skill B in Figure 10.

The use of power skill is available to all children. The slower child may well attempt the problem by the only means he knows, one which may often be quite laborious. For example, in finding the quotient $5863 \div 72$, the slower child might subtract 1 seventy-two at a time until he has reduced the dividend to some number less than seventy-two. The more able and creative child might tire of this method and attempt to subtract some multiple of seventy-two, such as 10 seventy-twos. Since each child is working on his own for a period of time, the development of power skill is extremely helpful in working with individual differences.

One decision that the teacher must make in relation to each child is the extent to which he should be encouraged to develop an efficient speed skill for a given algorithm. Obviously, skills are important and should be taught in elementary mathematics, yet it is the good judgment of the teacher that plays the crucial role in guiding a given child from power to speed. For certain processes, children should probably never be forced to attain a speed skill, but should be allowed to operate at the power-skill level. Other children should be directed toward the speed skill as quickly as possible in order that they may proceed to more interesting aspects of mathematics. In rare instances, a child might profit from an initial consideration of a speed skill with no previous power-skill development of a given process. The emphasis on the role of conceptual power in the performance of a skill is a key feature of the so-called "new" mathematics. It is quite probable that we cannot predict the future mathematical needs of children in our classes today, but we can help them develop the confidence, even in the area of learning skills, to utilize concepts previously learned to discover some of the basic processes for themselves.

EXERCISE SET 6

1. Write *power* or *speed* depending on the type of skill you think is being employed.

Specific Skill	Example
A Using sets to find sums	 $3 + 2 = \square$
B Using number line to find sums	 0 1 2 3 4 5 $3 + 2 = \square$
C Counting fingers to find sums	 $3 + 2 = \square$
D Memorizing that $3 + 2 = 5$	Think: $3, 2 \rightarrow 5$ $3 + 2 = 5$
E Thinking about "take away" to find differences	 $5 - 2 = \square$
F Using the inverse relation (missing addend) to find differences	Think: $? + 2 = 5$ $5 - 2 = \square$
G Memorizing that $5 - 2 = 3$	Think: $5, 2 \rightarrow 3$ $5 - 2 = 3$
H Using sets to find products	 $3 \times 6 = \square$
I Using the number line to find products	 0 5 10 15 $3 \times 5 = \square$
J Using logic (basic principles) to find products	Since $5 \times 5 = 25, 6 \times 5 = \square$ or Since $3 \times 5 = 15, 6 \times 5 = \square$

2. Four different power skills are shown for finding $91 \div 7$. These skills would lead up to finding this quotient by "ordinary short division."

$$7\overline{)91}\quad 13$$

In what order should these be presented?

A
$$\begin{array}{r} 7\overline{)91} \\ \underline{70} \\ 21 \\ \underline{21} \\ 0 \end{array}$$
(10) (3)

B Subtract 1 seven at a time.

$$\begin{array}{r} 91 \\ -\ 7 \\ \hline 84 \\ -\ 7 \\ \hline \vdots \end{array}$$

C
$$\begin{array}{r} 13 \\ 7\overline{)91} \\ \underline{70} \\ 21 \\ \underline{21} \end{array}$$

D Group 91 objects into sets of 7.

3. Complete the "Learning a Skill" lesson on pages I-19 and I-20; then do these exercises.

A Discuss the skill you learned and the way you learned in terms of power skills and speed skills.

B What part of the lesson helped you evolve a speed skill?

C What were your feelings about the lesson? How could it be improved?

Generalizations

Imagine that one of your students is engaged in an investigation in which he was asked to cut out a large quadrilateral and draw colored lines connecting the midpoints of each side of the quadrilateral. The question stimulating the investigation was, "Can you make an odd-shaped quadrilateral so that when you connect the midpoints you do not form a parallelogram?" As a result of this investigation and the subsequent discussion of his findings, the child was led to form a generalization: "The segments connecting the midpoints of any quadrilateral form a parallelogram."

In another lesson, a child might be responding to an investigation question which asked: "If you cut off the corners of a triangle and place the tips at the centre of a circle, what part of the circle can you cover? Can you find a triangle for which this is not true?"

As the child completes the investigation and engages in the discussion which follows, he forms this tentative, unproved *generalization:* "If a compass is used to draw arcs on the corners of any triangle and these corners are cut off along the arcs, then these corners will cover exactly one half of a circle drawn with the same compass opening." This tentative generalization, of course, is the forerunner of the familiar generalization that the sum of the degree measures of the three angles of any triangle is 180.

A generalization provides the economy of moving from consideration of isolated, specific cases to a general statement which holds true for a complete set of numbers or geometric figures. For example, the generalizations stated above deal with the set of all quadrilaterals and the set of all triangles. The regular occurrence of the word "any" in the generalization statements implies that the observation is true for *every* such geometric figure.

The key to teaching a generalization effectively is to provide children with appropriately chosen examples (or instances) which lead them to the generalization. An approach often used by teachers to help children learn generalizations is that of *guided discovery.* In this approach the teacher uses carefully sequenced questions and carefully chosen examples to focus the child's thought on the generalization to be discovered.

It is instructive for children in the upper elementary grades to have experiences in forming generalizations which seem obvious from a set of examples, but which, in fact, do not hold true. For example, consider the equations below.

$\boxed{1} \times \boxed{1} - \boxed{1} + 11 = 11$
$\boxed{2} \times \boxed{2} - \boxed{2} + 11 = 13$
$\square \times \square - \square + 11 = ?$

Figure 11

If 1 is written in the box and the operations are performed, the result is 11, which is a prime number. If 2 is written in the box, the result is 13, also a prime number. Upper-grade children are likely to conjecture that the sum is always a prime number. When they try 3, the sum is 17, also a prime. Similarly, the child finds that the numbers 4, 5, 6, 7, 8, 9, and 10, when written in the box produce a prime number. A child accustomed to forming generalizations from even fewer examples than this will likely conclude that this formula will always produce a prime number. It is instructive to note that when the next number, 11, is written in the box, the result is 121, which, being divisible by 11, is not a prime. This example illustrates the important idea that, even though the generalizations the child might make seem quite plausible and are most often true, it is only by means of a mathematical proof of a generalization that one can be completely sure that it is correct. These proofs, of course, are often not accessible to elementary school children. Thus, a healthy attitude might be characterized by references to generalizations which include phrases such as, "appears to be true," "is probably true," or "could most likely be proven."

Often a search for a generalization is initiated by a question such as, "Do you see any patterns?" For example, several simple generalizations might be formulated about the multiplication table in Figure 12. One child might observe that every number on the main diagonal of the table is a square number. Another student might observe that for every number on one side of this main diagonal, such as 10, there is a matching number symmetrically placed on the other side of the main diagonal.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

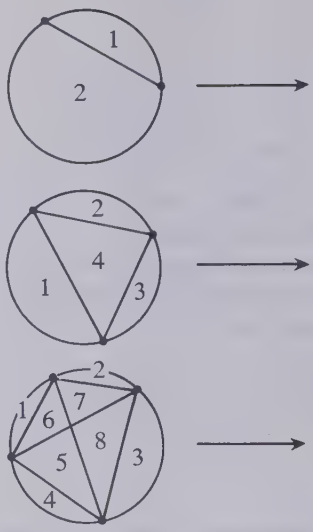
Figure 12

This last generalization is the table counterpart of the commutative principle for multiplication; that is, in the case of 12, $6 \times 2 = 2 \times 6$ or $4 \times 3 = 3 \times 4$. Another generalization that might be reached by careful consideration of the table is that the only primes in the table occur in the one-row or one-column of that table. Still another interesting generalization suggested by the table is that the sum of any number in the two-row and a number below it in the five-row will equal the number below these numbers in the seven-row. Of course, there are many other generalizations ranging from the very simple to the more complex that could be made about this multiplication table.

Perhaps the illustrations above will suggest that the mathematics available to the elementary school child is replete with possibilities for discovery of generalizations. The teacher's task is to create a learning environment in the classroom, not only in terms of physical materials and situations, but in terms of attitude toward learning and toward children, which provides opportunities for discoveries of generalizations and an atmosphere in which it is rewarding to make these discoveries. The teacher should be ever aware of the possibility that the habit of seeking generalizations may well be one of the most valuable things the child learns from his experiences in mathematics.

EXERCISE SET 7

1. Choose a text from the *Investigating School Mathematics* series and list some generalizations which the students who study this text might discover.
2. Investigate the Madison Project shoe boxes and complete the activities for at least two boxes.
3. The illustrations and the table which follow show that if you connect two points on a circle, you divide the interior of the circle into two regions; if you connect three points on a circle, you divide its interior into four regions; if you connect four points on a circle, you divide its interior into eight regions. Note that the points chosen should not be evenly spaced on the circle.



Number of points on a circle	Number of regions formed inside circle
2	2
3	4
4	8
5	
6	

 A Fill in the table to show how many regions are formed if five points on a circle are connected.
 B Form a generalization about the right-hand column of the table.
 C Test your generalization by finding out how many regions are formed inside when six points on a circle are connected.
4. Devise an investigation which might enable a student to discover this generalization: "The sum of the degree measures of the angles of a quadrilateral is 360."
5. Write some questions you would ask and show some examples you would use in guiding a child to discover one of the following generalizations.
 A The commutative principle for multiplication
 B The volume of a "box" is found by multiplying length times width times height.
 C In measuring length, the shorter the unit, the greater the measure.
 D Any angle inscribed in a semicircle is a right angle.
 E Every even number ends in 0, 2, 4, 6, or 8.
6. Complete the "Learning a Generalization" lesson on page I-20; then answer the following questions.
 A What generalization did you learn from the lesson?
 B How many specific examples did you consider before you understood the generalization?
 C In what way did you use the generalization after you discovered it?

Facts

In elementary mathematics, there are certain bits of information that are used so frequently that it is

beneficial for the child to be able to recall them quickly when they are needed. These items are ordinarily called *facts*. There are three main types of facts that are of major concern. The first type of fact is one which evolves from a concept. It might be an example of a specific concept ("Two is a prime number," "25 is a square number," "A parallelogram is a quadrilateral"), or it might be a characteristic of a specific concept, possibly even a part of the definition for the concept ("An isosceles triangle has two congruent sides," "An even number is a number divisible by two," "A pentagon has five sides"). Examples of, or characteristics of, concepts are not always considered as facts; only if such an example or characteristic is deemed important enough to be remembered for immediate recall, is it considered to be a fact and committed to memory.

A second type of fact is a fact derived from a generalization; that is, if a generalization is simple, or deemed important enough to remember for immediate recall, it might often be considered a fact. For example: "The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of the right triangle"; or "The length of the segment joining the midpoints of two sides of a triangle is one half the length of the third side." Each of these statements might be considered facts since they are sometimes useful for immediate recall. A third type of fact—one that is given a great deal of attention in the elementary school mathematics program—is the type of fact derived from a power skill. For example, the child may have utilized a sequence of power skills for finding sums such as $4 + 3$. He may have used sets of counters, centimetre strips, jumps on the number line, or reasoning from facts such as $3 + 3 = 6$. These power skills, based on certain important concepts, provided the evolutionary progression toward the final speed skill used in finding sums. In this particular case, however, the speed skill used is simply that of memorizing the sum. Whenever the speed-skill stage involves memorization, the particular learning which was classified as a skill or a process during the power-skill stage is reclassified as a fact. The basic addition and multiplication facts fall into this category, and they are given major attention in the elementary school. It is these facts to which primary attention will be given in this section.

A first important point to be made in discussing the teaching of facts is that extensive power-skill work preceding the memorization stage can pay valuable dividends. The broad base of understanding provided by the power-skill work removes the aura of magic from this aspect of mathematics and not only makes the task of memorization of the facts easier, but helps the child view it as a "reasonable thing to do." Figure 13, for example, shows some of the power skills that might be utilized in the initial development of procedures for finding products. Careful development using some or all of these power skills can give the child a basic feeling for a procedure by which products may be found.

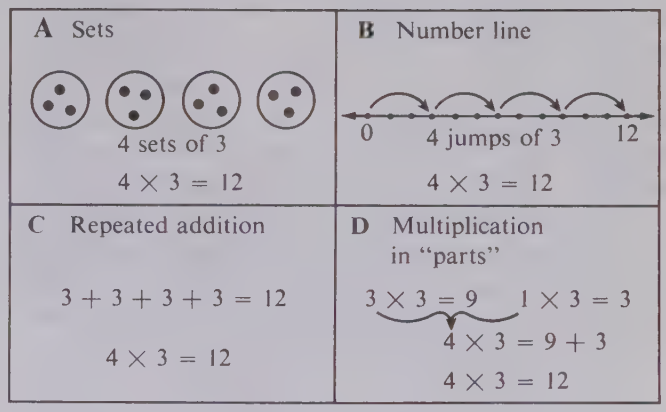


Figure 13

The teacher must use good judgment in deciding when a given child should move from this power-skill stage to memorization of the facts. The appropriate time could vary extensively depending upon the ability and experiences of the child. If the power-skill work is started early in the elementary grades, the child will have ample time to reap the benefits of this basic experience with materials and concepts before the transition to speed skill is made.

When the time has come to memorize the facts, it is important for the child to have a clear idea of the nature of this goal and the reasons it is appropriate. The teacher should even take time to help the child see the very clear difference between "figuring out the fact" and "memorizing the fact." Hopefully, he could help the child develop a feeling for situations in which the facts will be used and in which immediate recall would be quite valuable and time-saving for the child.

After the addition or multiplication facts to be memorized have been placed in perspective, the teacher should seek interesting situations and creative ways in which to practice recalling the facts. For example, the children might make their own flash cards and use a timer to see how long it takes them to give these facts. If desired, two children could work together and see which of them could go through the flash cards most quickly. Another game utilizes a pair of homemade colored dice and an empty multiplication table (Figure 14) for each child. As the game

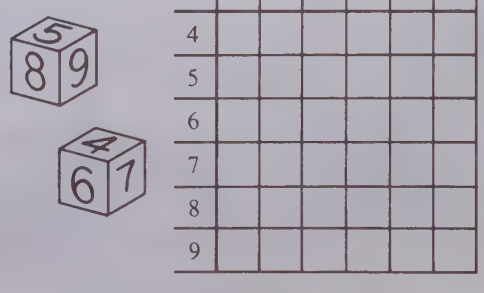


Figure 14

proceeds, a child rolls his dice and writes the product of the numbers on the dice in the appropriate space on his multiplication table. His partner then does the same thing when it is his turn. If a child arrives at an incorrect product or writes the product in the wrong space in the table, he is penalized by missing a turn. The object of the game is to see who can complete the table first. Various modifications of this game are possible, including one in which each child works independently and keeps a tally of the number of times he rolls the dice and also keeps track of the time it takes. The basic objective, of course, is to provide an interesting situation in which the child is motivated to recall multiplication facts rapidly.

Some children may need to spend considerable time in the power-skill stage before they begin to memorize. If there are children who have attempted to memorize the facts and find the job more difficult than anticipated, the teacher may want to consider allowing them to prepare a fact card on which they write the facts that they still do not know. Perhaps it would be realistic and beneficial to let some children use this fact card during the year whenever they desire, thus relieving the tension that could result from difficulties they encounter in memorizing the facts at one specific time. As the school year progresses, the teacher may want to suggest from time to time that a particular child concentrate on one of the troublesome facts and attempt to memorize it so that he can remove it from his fact card. The accomplishment of this goal, of course, would merit recognition and reward. After one fact is removed, the child might start working on removing another fact. The ultimate goal would be to remove all of the facts by the end of the year. Teachers who are interested in helping children learn mathematics in a comfortable way may find that a more realistic, less pressured approach to learning facts may enable the child to find greater enjoyment and success in his mathematical experience.

EXERCISE SET 8

1. Invent a game that could be used to help children practice recalling addition or multiplication facts.
2. Find a commercially produced game that is designed to help children practice recalling facts.
3. Complete the "Learning Some Facts" lesson on pages I-20 and I-21 of this text; then answer the following questions.
 - **A** How many of the facts did you know?
 - **B** What techniques did you use to help you memorize the remaining facts? Did you find this lesson difficult?
 - **C** Can you imagine some of the difficulties your children might have in learning facts?
 - **D** Did you find any mnemonic devices which were helpful in remembering the facts?

ATTITUDES

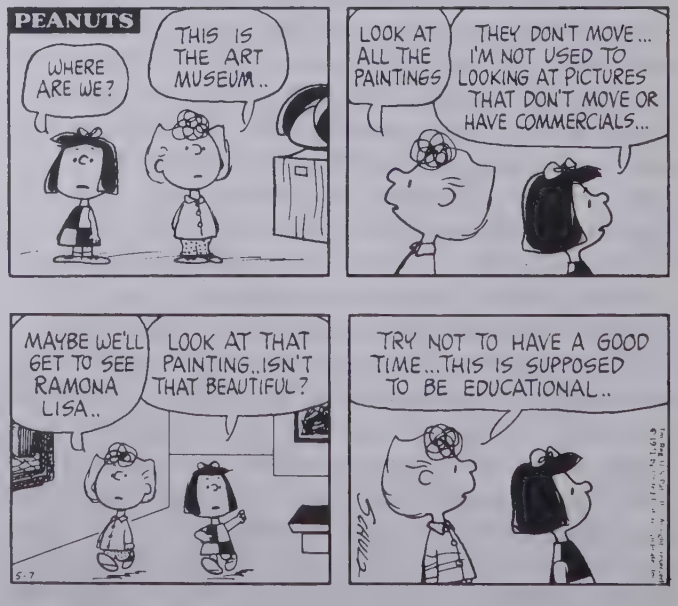
In his poem "Arithmetic," Carl Sandburg wrote:

> Arithmetic is numbers you squeeze from your head
> to your hand to your pencil to your paper
> till you get the answer.
> Arithmetic is where the answer is right
> and everything is nice
> and you can look out of the window
> and see the blue sky—
> or the answer is wrong
> and you have to start all over and try again
> and see how it comes out this time.
>
>
>
> Arithmetic is where you have to multiply—
> and you carry the multiplication table in your head
> and hope you won't lose it.*

The attitude toward mathematics, school, one's ability, and learning in general that one senses on reading this part of Sandburg's poem is surely typical of many children in classrooms today. Perhaps, it was a feeling similar to this that caused Huckleberry Finn to say:

> I had been to school 'most all the time and could spell and read and write just a little and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could get any further if I was to live forever. I don't take no stock in math, anyway.

There are many different kinds of attitudes exhibited by children who have been exposed to classroom mathematical experiences in different parts of the world. There are, of course, the more general attitudes that a child has toward his teacher, toward his school, toward his fellow students, and toward the process of education. All too often the child's attitude toward education in general is that suggested by Charles Schulz in this *Peanuts* cartoon.



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*From *Complete Poems*, copyright, 1950, by Carl Sandburg. Reprinted by permission of Harcourt Brace Jovanovich, Inc.

Two of the attitudes to be considered here, however, are the child's attitude toward mathematics and the child's attitude toward himself as he relates to mathematics. It has been said that the mathematical experiences of a child before the age of 11, and the responses he has been encouraged to make to those experiences, largely determine his potential mathematical development. If this is so, then a child's attitude toward mathematics and his feelings about how he relates to mathematics are extremely important considerations for the classroom teacher.

A moment's reflection on the number of people who are willing to say that they hate mathematics and on the multitude of others who seem to harbor a fear regarding their inability to cope with ideas of mathematics leads the teacher to realize that he does indeed teach attitudes, whether he tries to or not. Clearly, the teacher who conducts a classroom in which children's achievements are evaluated almost exclusively on the basis of how many right answers they can come up with must surely engender attitudes in children which differ greatly from those engendered by the sensitive teacher who recognizes the child's need to think his own thoughts and to become involved in an exciting exploration of ideas that interest him. Or, consider the difference between the teacher who teaches only speed skills and facts and the teacher who recognizes the central importance of concepts and generalizations, as well as the facts and skills. The child exposed to the first teacher must surely have a feeling toward mathematics, and his ability to interact with it, that is far different from that of the child who learned with the second teacher.

If what happens in the classroom is of such importance in developing attitudes within the child, then the teacher may want to reevaluate his approaches to instruction by reconsidering certain fundamental questions. What subject matter and methods most effectively instill within the child the feeling that mathematics is interesting, fun, and a source of adventure? Will these means provide an opportunity for the child to exercise his freedom of choice and to make decisions about what he does with mathematics? Aldous Huxley said: "A child is a genius until the age of ten." Could it be that our classroom approaches squelch this genius? Can we select mathematical experiences and materials that enable the children to experience success and thus maintain that sense of worthiness and prestige with peers that is of such importance? Can we structure these experiences in such a way that the child maintains within this atmosphere of freedom a sense of security and safety, thus avoiding the fear that can erode his ability to approach mathematical situations with confidence? Can we help children see the usefulness and importance of mathematics without boring them?

Clearly, the questions just raised are difficult to answer and specific techniques for developing healthy attitudes are hard to come by. But even though prescriptions for developing attitudes are scarce, many of the ideas about teaching suggested in earlier sections of this text can provide assistance for the teacher. The investigation, for example, provides the child with an opportunity to make independent decisions and to interact with mathematics and materials and encourages him to take responsibility for his own learning. As difficult as it may seem at times, a child's acceptance of responsibility for his own learning inculcates an attitude that is ultimately invaluable. Also, the manipulative materials or activities that are made available to the child in the investigation situation provide an interaction with the physical world that is often extremely valuable in making mathematics real to a student. Unless a child is ready for more abstract thinking, he cannot be induced to sense the adventure in mathematics without a physical environment to explore. Opportunities for attitude development are implicit not only in the investigation phase of a lesson but in the discussion as well. If a teacher can convince the child that his ideas are important, then the child finds himself in a situation, albeit a mathematical one, in which *he* feels important. His prestige with his peers increases and he feels successful. Exercises in the utilization phase of a lesson that begin simply and gradually increase in difficulty can also help the child feel that he can do mathematics on his own; and, of course, carefully selected extension activities can provide the child with a variety of opportunities to experience the fun of mathematics.

Not only do the phases of the learning experience provide unique opportunities of attitude development, but the particular types of learnings involved within these phases also have their effect. The teaching of concepts and generalizations provides the child with a feeling of power regarding mathematics, for when he experiences the thrill of discovering a concept or a generalization, or when he uses these to solve a problem, he is also developing a useful and wholesome attitude toward mathematics learning. He is developing a habit of reacting to a mathematical situation which will be invaluable when he later encounters mathematical situations possibly undreamed of today. Also, careful teaching of skills and facts can provide the child with that basic sense of security that comes simply from being able to do something or to remember something.



Figure 16

Regarding the child's level of confidence in his ability to cope with mathematical problems, one of the child's paramount needs is to experience success, and as mentioned previously, having entertaining experiences with mathematics might decrease the fear that can erode his confidence. To provide these experiences, the teacher might create in the classroom a "Fun with Mathematics" centre (see Figure 16) that contains mazes, puzzles, design materials, and so on. This centre represents an extra effort to encourage the child to successfully play with mathematics. Some of the materials that might be in such a centre are as follows: the soma cube, the tangram pieces, 2-cm cubes, materials for curve stitching, a kaleidoscope, pattern blocks, Cuisenaire rods, multi-base arithmetic blocks, geoboards, a wide variety of counters, attribute blocks, scales and balances, timers, calendars, measuring tapes and rulers, yarn and string, an assortment of boxes and cans, magazines and catalogues, mirrors, dice, play money, graph paper, assorted plane and solid shapes, abacus, pegboard, compass, mathematical balance, etc.

Perhaps, as you consider the attitudes more carefully and reevaluate the effects of your approaches to instruction, you will find other ways to help children develop a healthy attitude toward mathematics and an enthusiasm for the enjoyment it can offer. Each day as the teacher enters the classroom with plans for a learning experience, he might well ask himself: "What effect will *this* have on the attitudes of the students in my classes?"

EXERCISE SET 9

1. Select a text from the *Investigating School Mathematics* series and find at least five activities which could contribute to the child's development of a positive attitude toward mathematics.
2. Explain how you think some of the other types of learning might also contribute to better child attitude toward learning in general and mathematics specifically.
3. Complete the "Learning an Attitude" lesson on page I-21 of this text; then answer the following questions.
 - **A** Was the lesson fun?
 - **B** How did you feel when you had finished the lesson?
 - **C** Did the lesson change any of your ideas about mathematics?

IV. Some Learning Experiences for the Teacher

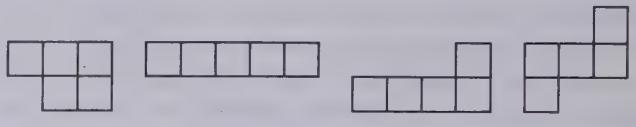
In Section II you were introduced to an outline for a learning experience which involved preparation, investigation, discussion, utilization, and extension. In Section III the types of things children learn—concepts, skills, generalizations, facts, and attitudes—were categorized. In this section, we combine these ideas and use them in presenting five learning experiences designed especially for the teacher. That is, in order to gain a first-hand view of lessons which develop these types

Lesson 1. Learning a Concept

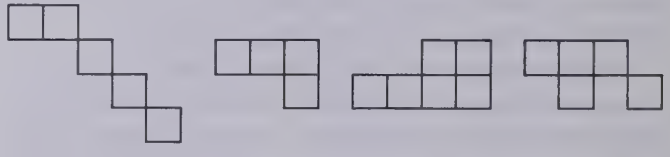
What is a pentominoe?

Investigating the Ideas

Each of these is a **pentominoe.**



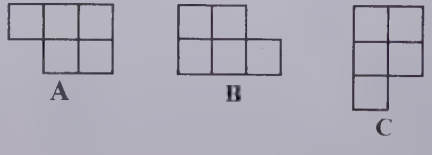
None of these is a **pentominoe.**



How many more pentominoes can you find and show on graph paper?

Discussing the Ideas

1. How many pentominoes did you find?
2. Can you give some characteristics of a pentominoe?
3. How would you "broadly classify" a pentominoe?
4. Can you define a pentominoe?
5. Are the pentominoes in Figures A, B, and C the same?



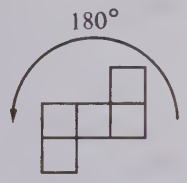
6. How could you convince someone that you have found all possible pentominoes?

of learning, the teacher will have experiences with each of these in the five lessons; and, in order to become more familiar with the suggested structure for a learning experience, each of these five lessons will involve an investigation, a discussion, a utilization, and an extension of the ideas.

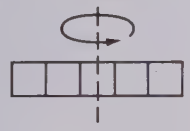
It might be valuable for the teacher, after he has become involved in each of these lessons and has completed the activities, to rethink and discuss his reactions to the various phases of the lesson structure and to the various types of learnings involved. In this way, he might gain a new insight into the way the children in his classes might react to these kinds of situations.

Using the Ideas

1. Which of the pentominoes can be folded to form a box with the "lid missing"?
2. Some pentominoes can be rotated about a point 180° and returned to their starting position. These pentominoes are said to have 180° rotational symmetry. Which pentominoes have 180° rotational symmetry?



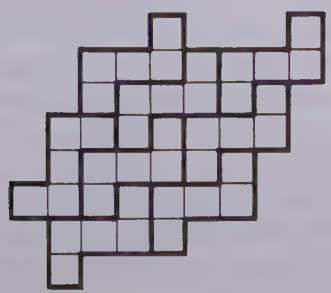
3. Some pentominoes can be flipped about a line and returned to their starting position. Such pentominoes are said to have reflectional symmetry. Which pentominoes have reflectional symmetry?



4. What do you think a hexominoe would be? How many hexominoes can you find?

Extension

Some pentominoes can be used to tessellate (fill without overlapping) the plane, as shown below. Can you find at least two more pentominoes that can be used to tessellate the plane? Show the tessellations on graph paper.

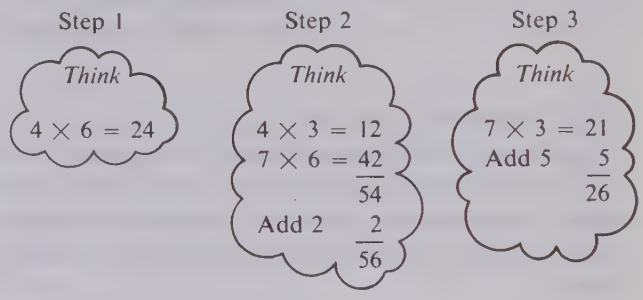


Lesson 2. Learning a Skill

Can you find the product of two 2-digit numbers "in your head"?

Investigating the Ideas

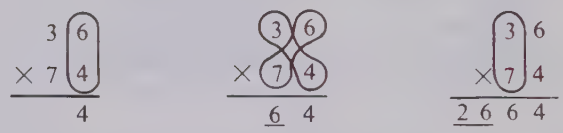
Follow these steps for writing the *answer only* for 74×36.



Write 4
Remember 2

Write 6
Remember 5

Write 26



Can you use this method to write answers only for the products below? Check your answer using the "long" method.

53	37	45	67
×48	×62	×23	×32

Discussing the Ideas

1. Explain this statement: In Step 1 you are finding the number of ones.
2. In Step 2 you are finding the number of ?.
3. The 2 you remembered is really 2 ?.
4. Explain what you are finding in Step 3.

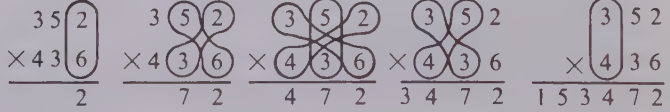
Using the Ideas

Write answers only for each product.

1. 28 ×42	**2.** 46 ×33	**3.** 37 ×42	**4.** 82 ×56	**5.** 53 ×34
6. 64 ×27	**7.** 29 ×63	**8.** 48 ×35	**9.** 53 ×53	**10.** 27 ×64

Extension

1. Study the figures below for finding the product of two 3-digit numbers.



2. Use the method shown in exercise 1 to find each product.

125	757	841	525
×365	×426	×215	×525

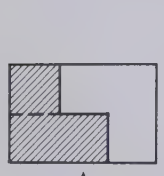
*3. Devise a rule for multiplying two 4-digit numbers.

Lesson 3. Learning a Generalization

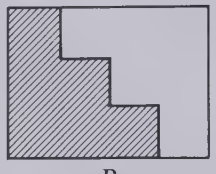
Can you find a pattern?

Investigating the Ideas

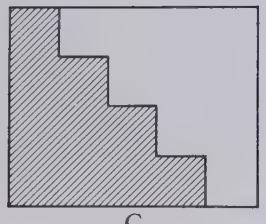
Use the small square in Figure A as the unit. Can you find the area of each shaded part in two different ways? For each part, write an equation to show that the two ways of calculating the area give the same result.



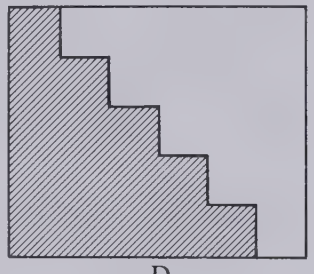
A



B



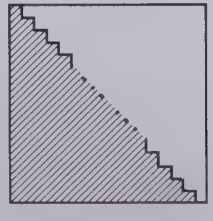
C



D

Discussing the Ideas

1. **A** Describe one way you found for finding area in the figures above.
 B Describe another way you found.
 C Did you find any other way?
2. Can you write an equation to show that these two methods give the same area?
3. **A** Suppose there are 50 vertical segments in the "stairsteps" of Figure E. What is the area of the shaded part?
 B Which of the two methods for finding the area would you use?
 C Can you write an equation about this?



4. Can you find the area of the shaded portion of Figure E if there are 100 vertical segments?
5. Can you use what you have learned so far to explain this generalization?

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{n \cdot (n+1)}{2}$$

Using the Ideas

1. Without adding each number, find the sum of the whole numbers through 25.
2. Find the sum of the first 75 whole numbers.
3. Find the sum of the first 200 whole numbers.
4. What is the sum of the first 1000 whole numbers?

Extension

1. What is this sum? $50 + 51 + 52 + 53 + \ldots + 99 + 100$
2. Can you find a short way to find the sum of
 A these even numbers? $0 + 2 + 4 + 6 + 8 + \ldots + 100$
 B these odd numbers? $1 + 3 + 5 + 7 + 9 + \ldots + 99$
*3. Can you state a rule for what you found in exercise 2 by using a variable?

Lesson 4. Learning Some Facts

Can you learn some "new" facts?

Investigating the Ideas

Many rapid "human Calculators" consider these products to be facts.

×	10	11	12	13	14	15
10						
11						
12						
13						
14						
15						

How many of these "facts" can you give without calculating?

(Record the facts you know and shade that portion of the table with a red pencil. Then fill in the remainder of the table by figuring out the remaining facts.)

Discussing the Ideas

1. Which facts in the table need not be memorized provided you know the others and also know the commutative principle? Shade these facts blue.
2. **A** How many facts altogether are in the table?
 B How many facts remain to be memorized?
3. **A** What is the "largest" fact?
 B Which facts are over 200?
 C Which facts are in the 190's?
 D Do you notice other patterns in the table that might help you remember certain facts?

Using the Ideas

1. Give these products as quickly as possible.

A 15×15	**E** 13×13	**I** 11×13
B 15×14	**F** 14×12	**J** 11×14
C 14×14	**G** 11×11	**K** 11×15
D 13×15	**H** 11×12	**L** 12×13

2. Make flash cards for the "facts" in exercise 1 that you do not know. Practice with a friend.
3. In exercise 1, start with part L and, following reverse order, give each of the products as quickly as possible.

*4. Make a large multiplication table with all numbers up to 20. Mark out the "facts" you know. How many of these "facts" are left to memorize?

*5. A person who knew the distributive principle and the facts in the table referred to in exercise 4 looked at the multiplication 143×15 and wrote 2145. How did he do it so quickly?

Extension

Study the facts for these powers of 2.

$$2^2 = 2 \times 2 = 4$$
$$2^3 = 2 \times 2 \times 2 = 8$$
$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

1. Give the next six powers of 2.

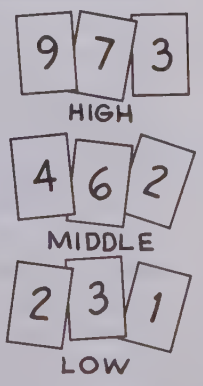
*2. Can you find some mnemonic aids to help you memorize the first ten powers of 2?

Lesson 5. Learning an Attitude

Let's try a place-value game.

Investigating the Ideas

Use 3 sets of 9 cards, each with the digits 1 through 9. Shuffle the 27 cards and deal 3 to each player. Each player then forms a 3-digit numeral, places his cards face down in order, and declares (starting to dealer's left and rotating clockwise) whether his number is high, middle, or low. Play the game in groups of three players.



Discussing the Ideas

1. One player arranged his cards like this and declared that he would try for the low hand. What was wrong with his strategy?



2. What is wrong with this arrangement for a middle hand?



3. If you were dealt these cards, would you try for a high or low? Why?



4. Suppose you are last to declare. Everyone else has declared either low or middle. What would you do with these cards?



Using the Ideas

1. Try playing this game with 2 or more other people.
2. Try the game with the rule that you can declare only high or low.
3. Make up rules for a game in which you turn up the cards one at a time starting with the ones' digit card.

Extension

1. Invent a place-value game in which 4 or 5 cards are dealt to each player.

*2. Find or invent another game or activity that strengthens understanding of the concept of place value.

V. Some Thoughts About Evaluation

The strategy of preparation, investigation, discussion, utilization, and extension is a flexible organizational plan that allows each teacher an opportunity to make a modest beginning toward an activity-oriented mathematics program. The lesson categorization of concept, skill, generalization, fact, and attitude provides a framework that allows each teacher an opportunity to apply the teaching strategy to various types of learning situations. Since there are different types of learning, it is reasonable to assume that there should be different types of evaluation used to measure these learnings.

When considering the facts and skills, for example, emphasis should be placed on child accountability. The teacher should determine the learning outcomes, consider performance objectives for these outcomes, and help the child attain these objectives. The evaluation of this attainment is most easily completed by use of fact and skill tests which determine the child's level of achievement. Since the child needs considerable practice in remembering facts and performing skills, the procedure for helping them is reasonably straightforward.

When evaluating concepts, generalizations, and attitudes, however, the desired performance objectives are often quite difficult to verbalize. We have mentioned earlier that concept learning often takes place

over a relatively long time span, that concepts are extended and broadened, and that concepts mature with each subsequent set of related experiences. Clearly, it is difficult to write a performance objective which specifies the exact level of concept maturity appropriate for a given child at a given time. Whenever possible, objectives for simple concepts should be written, and an attempt should be made to write test items which will show whether children understand these concepts. These items should involve requests for children to give examples of concepts, characteristics of a concept, and even, in some cases, a definition of the concept. For more difficult concepts, the evaluation of children's progress might be made through observation and recorded by means of a check-list which specifies certain levels of development for the given concept. The teacher should be alert for situations in which the child actually uses the concept correctly and should recognize also that understandings which are only partially developed indicate positive achievement. The teacher should also search for instances where the child has shown an ability to form concepts, for this is one of the desired learnings.

When evaluating the child's understanding of generalizations, the teacher should specify the simple generalizations which should be learned by all children. Specific performance objectives and the subsequent test items should be written to evaluate these generalizations. Beyond this, the teacher should again evaluate in greater depth through personal observations or interviews with the children. In the area of generalizations, the teacher should be ever aware that a child who is in the habit of looking for patterns or generalizations has learned a great deal. The teacher should also recognize that a child who can form a generalization from a sequence of specific examples has developed an understanding of a process that is extremely important. We would be remiss if we evaluated only the factual part of the learning of generalizations. As noted earlier, however, although these are important goals of mathematics learning, it is very difficult to write performance objectives for these goals. Whenever possible, objectives should be written which go deeper than facts and skills, but in the absence of objectives, the teacher should feel free to use other means of evaluation, including interviews to evaluate student learning.

While attitudes are not easy to measure in a conventional way, it is suggested that teachers frequently observe children and talk to them about their feelings about mathematics. It is important to realize that one's philosophy toward testing can also have a marked influence on the child's attitude toward mathematics. Testing should be reasonable and realistic, and the child should understand its purpose. The spirit of evaluation should be one of helpful assessment, rather than of critical evaluation. If children participate with teachers in understanding (if not in developing) the goals of instruction, the testing procedure can be a positive influence on the child's attitude and ability to improve.

It is hoped that the teacher will constantly take a broad view toward evaluating mathematics learning among his children. In the long run, evaluation of a child's learning should depend upon the interaction of that child and his teacher. For this interaction to be successful, it may be necessary for the teacher to reexamine his own beliefs about how children learn mathematics. As each teacher makes modest beginnings toward an activity-oriented approach to mathematics learning, he might ask himself the following questions:

1. Do I respect each child as an individual with unique interests, abilities, sensitivities, and significant thoughts?
2. Does the learning environment of my classroom provide a natural, free atmosphere in which children can explore, make decisions, be independent, and encounter exciting new experiences?
3. Does the learning experience also include a supportive, non-judgmental atmosphere in which children have enough routine activities to provide a comfortable threshold of security?
4. Is the child's need for earned success recognized in my classroom?
5. Do I recognize and treat mathematics as a dynamic, ever-growing discipline which offers limitless new vistas to be explored and an inexhaustible variety of new problems to be investigated and solved?
6. Do I view mathematics as a subject of beauty and a source of pleasurable fulfillment of intellectual curiosity?
7. Do I appreciate the significance of my role as a fellow-learner rather than merely a source of information?
8. Is my overall attitude toward mathematics one that encourages a basic freedom to learn through use of manipulative materials in an investigative environment, and through free discussion and exchange of ideas?

As a teacher evaluates the children in his class, he should also reevaluate his approach to mathematical learning. The goal of this short text has been to help in that reevaluation by encouraging the teacher to read, study, observe, experience, experiment, and reconsider. If that goal has been achieved, perhaps his resulting basic beliefs about children, mathematics, and evaluation methods will help him create a new climate of interaction that will spark more effective learning experiences in his classroom.

EXERCISE SET 10

1. Give a set of performance objectives for each lesson completed in Section IV.
2. Create an evaluation tool for each set of behavioral objectives given in exercise 1.

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INTRODUCING THE METRIC SYSTEM

Canada is committed to the metric system of measurement. You may be aware of this but may not have a clear idea of exactly what the metric decision means to you as a *teacher*. It is hoped that this section will serve three purposes—

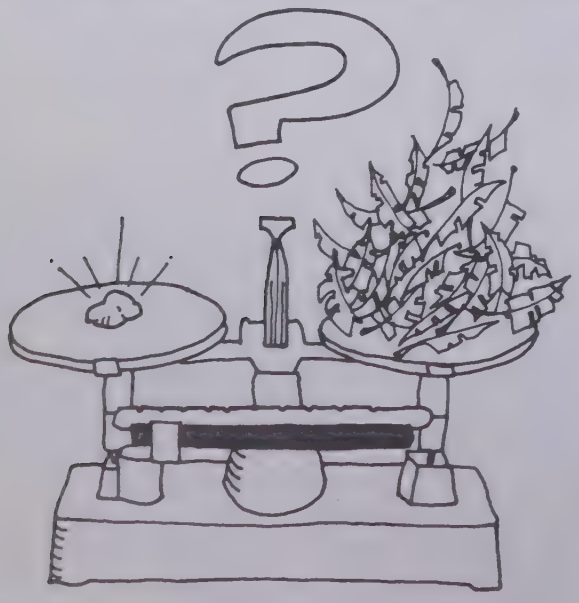
1. give you an idea of how the metric decision will affect you,
2. help you understand the metric system of measurement, and
3. give you some hints for teaching the metric system of measurement to your students.

History and Rationale

The English system of measurement developed from man's need to measure size and distances using units from the most readily available object—himself. He utilized his palm, span, finger, an ell, and a fathom for length; his foot, step, pace, an arrow's flight, and a day's journey for distance; and a handful, shellful, hornful, or gourdful for capacity.

There was little need for standardization until man began to travel and trade with other men. When "standard units" were developed, a new problem arose. Different countries used different definitions for the same unit. The foot was, at first, the length of any man's foot. In some countries, it was the length of the king's foot (since he was the "ruler") and this foot could change as the "rulers" changed. Later an effort was made to standardize some units; for example, England and Scotland decreed the foot to be 12 inches. Unfortunately, England and Scotland didn't use the same definition for the inch.

Today, in the age of technology, one still finds different units in those countries which are not yet metric. Canada and the United States are neighbouring countries, yet they use two different definitions for the gallon. A question at which people in metric countries must laugh is "Which is heavier, a pound of gold or a pound of feathers?" A pound of feathers is heavier since feathers are weighed by the avoirdupois pound (1 avoirdupois pound—7 000 grains) and gold is weighed by the troy pound (1 troy pound—5 760 grains). Which is heavier, an ounce of gold or an ounce of feathers? An ounce of gold is heavier. There are 12 ounces in the troy pound, so one ounce of gold weighs 480 grains; there are 16 ounces in the avoirdupois pound, so an ounce of feathers weighs 437.5 grains.



Out of such confusion there developed a need for a simple, standardized system of measurement. In 1670 Gabriel Mouton, a French abbé, developed a system of measurement organized according to the decimal system of numeration. It took over a hundred years for a system of measurement like the one Mouton put forth to get official sanction. In 1790 the French National Assembly appointed a committee to study the measurement situation and see if a rational system of measurement was possible. In 1795 France adopted a decimal system of measurement, defining the base unit of length to be the *metre* (from the Greek word *metron*, "a measure").

The metric system did not use parts of the human body as units. The metric system did not develop haphazardly adding more and more units as the need arose. The metre was defined as one ten-millionth of the distance from the North Pole to the equator, along the meridian passing near Dunkirk, Paris, and Barcelona. One can see that such a definition would be difficult to replicate in any one country. Also, the length of the metre changes as the position of the North Pole changes; at the time that the metre was defined, scientists were unaware that the position of the North Pole changed.

In 1870, because of the problem of replicating and comparing metric units from country to country, France called a meeting of the metric countries to develop a "unified metric system of measurement". In 1875, the *Treaty of the Metre* was signed to establish the General Conference on Weights and Measures which meets to determine the official definitions for the units used in the metric countries. In 1960 the Conference adopted the Système International des Unités (SI). It is this SI metric system that is most used throughout the world.

A Popular System

The popularity of the metric system stems from two characteristics—the high degree of standardization and its simplicity.

In the entire metric system there are only seven base units! They are **metre** (length), **kilogram** (mass), **second** (time), **ampere** (electric current), **degree kelvin** (thermodynamic temperature), **candela** (luminous intensity), and **mole** (amount of substance).

All units used in the metric system are related to these seven base units. The units you will be most concerned with (because they are the ones used in everyday living) appear in Table 1:

Table 1: Metric Units to be Studied

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Capacity	litre	ℓ*
Temperature	degree Celsius	°C

*As a rule of thumb, the cursive letter (ℓ) is used as a symbol for the litre to avoid confusion with the numeral (1), however, in symbols such as ml (millilitre), kl (kilolitre) the cursive form is not used.

All other units to be discussed can be represented by the product of one of the units and a power of 10. For example, every possible unit of length can be developed by multiplying the number of metres by the appropriate power of 10.

Table 2: Metric Units of Length

Name (Symbol)	Metres
*kilometre (km)	10^3m or 1000 m
hectometre (hm)	10^2m or 100 m
decametre (dam)	10^1m or 10 m
*metre (m)	10^0m or 1 m
decimetre (dm)	10^{-1}m or $\frac{1}{10}$ m
*centimetre (cm)	10^{-2}m or $\frac{1}{100}$ m
*millimetre (mm)	10^{-3}m or $\frac{1}{1000}$ m

*preferred units

To make the system simpler the same prefixes are used with all units. For example, a millimetre (mm) is $\frac{1}{1000}$ of a metre, a millilitre (ml) is $\frac{1}{1000}$ of a litre, a milligram (mg) is $\frac{1}{1000}$ of a gram, etc.

According to the class, you may want to introduce the symbol "m" for metre, "cm" for centimetre, etc. The plurals, metres and centimetres, are also symbolized "m" and "cm", not "ms" or "cms." Remember, these are symbols and not abbreviations and no period is used after a symbol.

Countries which have been completely metric for several years find that some terms such as "decimetre" are not used in everyday living. People will talk of a book being 28 centimetres long rather than 2.8 decimetres long. You may wish to explain the term "decimetre," but it is not necessary.

Most people who feel that the metric system is complex are those who convert back and forth between the metric and English systems of measurement. When teaching the metric system, conversion to the English system is not necessary and should be avoided!

The metre is defined world-wide to be 1 650 763.73 wave lengths in a vacuum of the orange-red line of the spectrum of krypton 86. This is quite a definition! There are two reasons why such a complex definition was adopted –

1. the length never varies and

2. this measurement can be replicated in laboratories throughout the world.

From this brief history of the metric system it is hoped you will take three main thoughts –

1. The metric system resulted from concentrated effort to develop a rational system of measurement. It did not develop haphazardly.

2. The problem of standardization has been solved in the metric system.

3. The metric system is both popular and useful because of its simplicity.

Activities

Experience and activity are key words in the teaching of measurement. Measure things! The success of this material will depend upon the amount of experience each participant has with the activities. The limited number of activities that are presented should stimulate possibilities for many more. Although the content is approached through activities and measuring experiences, there is a need for exercises to further these experiences and to structure metric thinking. Two points should be emphasized –

1. It is *important* that *you* as well as your class do the activities in this section.

2. The activities will be more fun if done in a group situation.

Looking at Table 1 in the *History and Rationale* section, you will notice that you have to be concerned with only four base units. So, let's use the frontal attack, start right in on length, and begin inching our way down the metric road.

Length, Area, and Volume

In the groups where the metric system has been argued for years, there were two camps. One group wanted to use the centimetre, gram, and second for the core of the system and the other the metre, kilogram, and second. The latter group has prevailed.

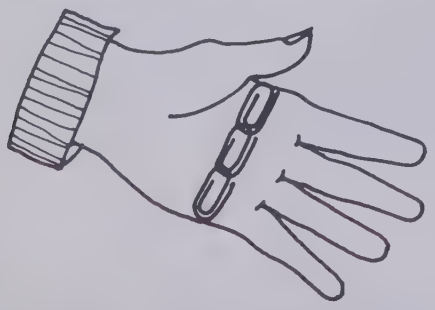
It is strongly urged that first grade teachers **not** start with the metre. It is very difficult for first graders to handle a metre ruler. The same argument may be advanced for the kilogram and litre. Length will be approached as it should be covered with students, i.e., first measure with arbitrary units, then use the centimetre, next use the 10-centimetre (decimetre), and finally the metre. All measurement should be approached as a three step process—

1. Select a unit.
2. Partition the object to be measured into units.
3. Count the number of units used. That number is the measure of the object.

ACTIVITY 1

Measuring objects with an arbitrary unit. Students should do several activities of this type using arbitrary units such as their thumb, a paper clip, pencil, crayon, cutout of their shoe, width of their hand (a unit in the English system used for measuring the height of horses), cubit (another "English" unit, the length of the forearm from the elbow to the tip of the middle finger), or other selected units. For your experience measure the chalk eraser, the width of your hand, the width of this book, and the length of a pencil using a paper clip as the unit.

In the illustration, a "paper clip train" is being used to measure the width of a hand. Follow the three steps mentioned previously in the measurement process.



Record all answers. Then measure the object again using pieces of paper the length of a thumbnail. Repeat the process measuring other objects.

In class emphasize four points—

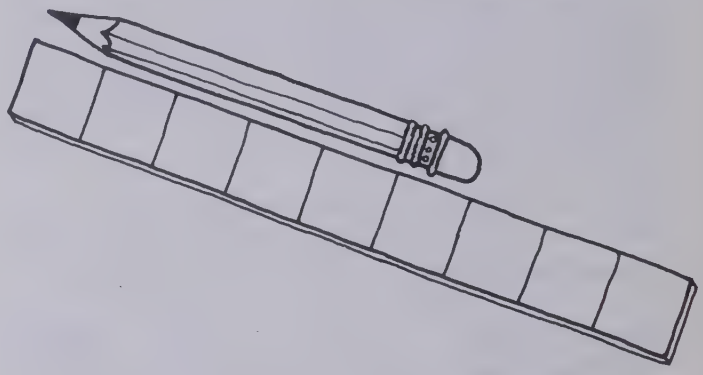
1. The first unit should be lined up with the "starting point" of the object.
2. The units should touch, but not overlap.
3. The "train" should be straight.
4. The units should be "rounded off" to the unit that has its right end nearest to the "finishing point" of the object.

In doing activities where arbitrary units are used, the need for standardized units becomes obvious. Ask several children to measure the same object, each with his own pencil. On the chalkboard, place their statements such as "The table (or whatever object you pick) is 5 pencils wide." "The table is 7 pencils wide." "The table is 8 pencils wide." Children will soon see that when pencils of differing lengths are used, different answers will result.

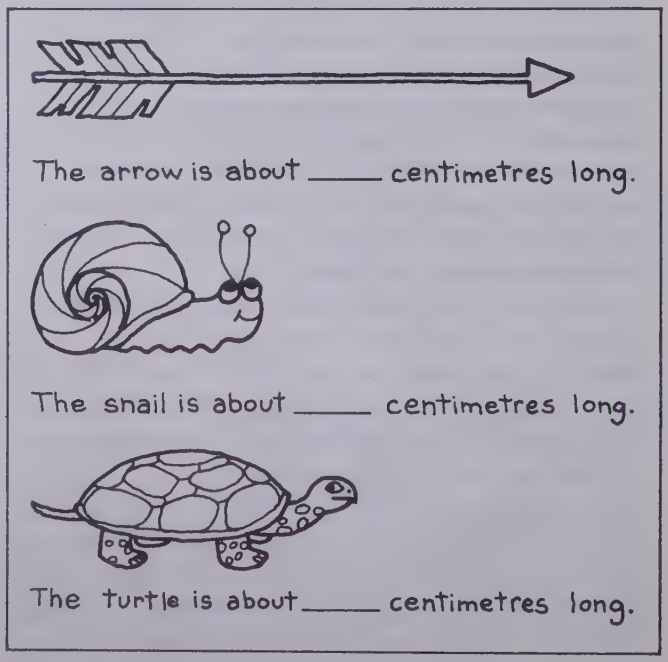
ACTIVITY 2

Developing the concept of a centimetre. Probably the first metric unit the children will make use of is the centimetre. You will need (and each student in the class will need) 9 centimetre strips—9 pieces of paper or cardboard 1 cm by 1 cm square.

The children, especially the younger ones, should have the experience of measuring many objects using centimetre strips. (If at the time you present this activity your students have studied two-digit numbers, have them measure objects longer than 9 cm.)



Using the centimetre strips, measure the length of a paper clip, a piece of chalk, the Cuisenaire 6-rod, the width of a hand, and the width of a thumb to the nearest centimetre. In this initial activity, actually use centimetre strips and not a ruler marked in centimetres. An exercise the children can do at their desks is to measure the pictures of objects drawn on a duplicator master. The pictures can be of predetermined length. Measure the pictures below.



In exercises like these, the length can be controlled. Some answers should require "rounding up," and some "rounding down." The word "about" is important in the sentence since a measurement is an approximation. As the children progress you can have them write not only the number but also the name of the unit.

ACTIVITY 3

Measuring with centimetre rulers. When the children have learned to use the centimetre strips in the measurement process, a ruler marked off in centimetres (not millimetres) should be introduced. It is strongly urged that the child construct his own 10-cm ruler during his first introduction to metric measure. He can do this by constructing a 10-cm train on a 10-cm long piece of paper, pasting the train on the paper, then numbering the cars from 1 to 10. Another approach is to construct a 10-cm ruler in front of the class. Then hand out 10-cm long pieces of paper already marked off in centimetres and have the children number the centimetres from 1 to 10.

The next few activities should involve the measuring of an object with a centimetre train, a 10-cm ruler, and finally with only a 10-cm ruler. When measuring an object with a 10-cm ruler work toward getting your students to "read the ruler" rather than counting the centimetres as they did with the trains.

In the example illustrated the child should learn to round off to the nearest centimetre and then read the ruler, "8 centimetres," instead of counting "1, 2, 3, 4, 5, 6, 7, 8 centimetres."

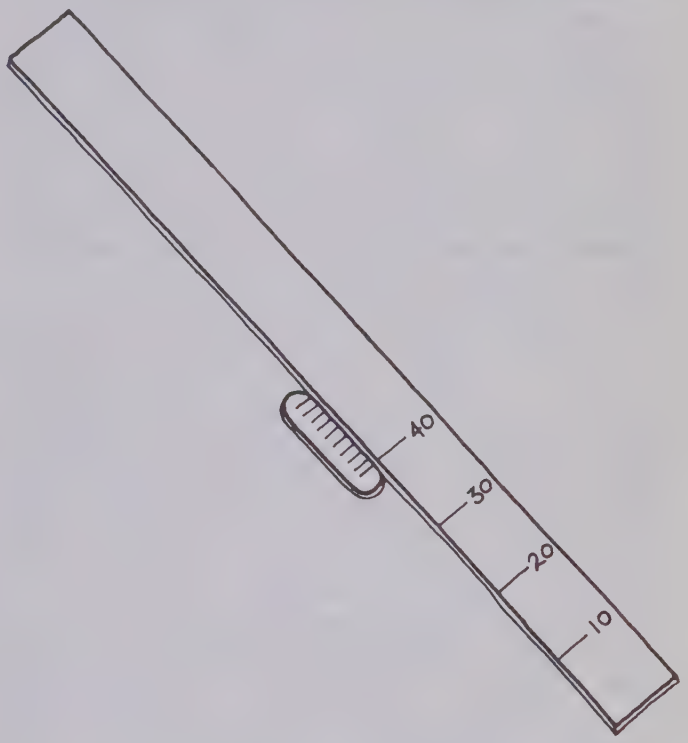


After the children have become skilled in using a 10-cm ruler, they should be given activities requiring them to measure objects which are longer than 10 cm. When working with 5-and-6-year olds, be careful that the measure of the object is not a number the children haven't studied. In the activities concerning measurement it is the process that should be emphasized; the numbers themselves should never be a source of difficulty.

Now, using your 10-cm ruler, measure the length and width of this book and length of your forearm, the length of your foot, and length of your span (what is your span?).

ACTIVITY 4

The metre and notation. Initially, you may want to have your students measure objects with metre-long strips of unmarked cardboard. Then ask them to number the centimetres on the metre strip in groups of 10 using their 10-cm strips. Before proceeding



further, have the class subdivide these cardboard metre rulers into centimetres. It is important that you do the activities with the same type of ruler your students will use. If you have a classroom set of wooden metre rulers, use one of them. Ideally, the rulers used should be marked off in centimetres, but if the ruler is marked off in centimetres (cm) and millimetres (mm) no harm is done. Measure the length, width, and height of your desk rounding off to the nearest metre.

The measurements for a desk, accurate to the nearest metre, might be 2 m long, 1 m wide, and 1 m high. Such measurements would not be helpful. The metre is used for much longer measurements, such as the length and width of the classroom, the playground, the school, the block, etc. To measure the dimensions of objects such as desks, tables, bookshelves, and people, a metre ruler may be used and the results recorded in centimetres. For example, a desk may be 152 cm long, 76 cm wide, and 74 cm high.

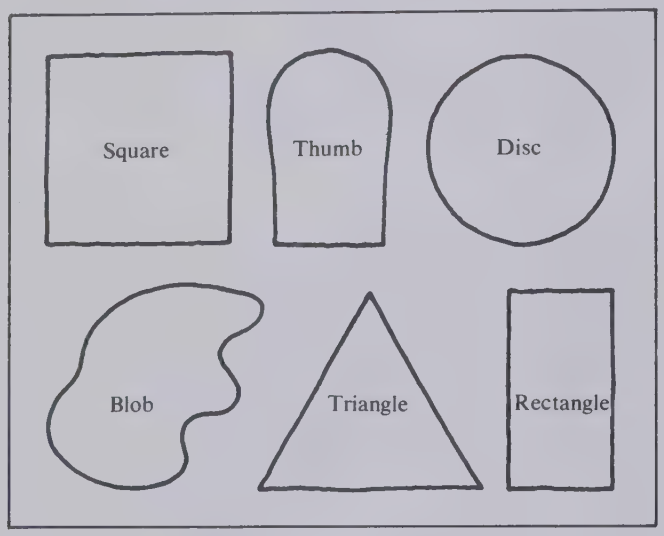
You might say: I am 178 cm tall; what is your height (in centimetres)?

Just as 153 cents is written as \$1.53, 153 centimetres is written as 1.53 metres. This can be interpreted as 1 metre and 53 centimetres which is read as "one point five three" metres. Do not dwell on the mathematical use of the notation—it is not necessary!

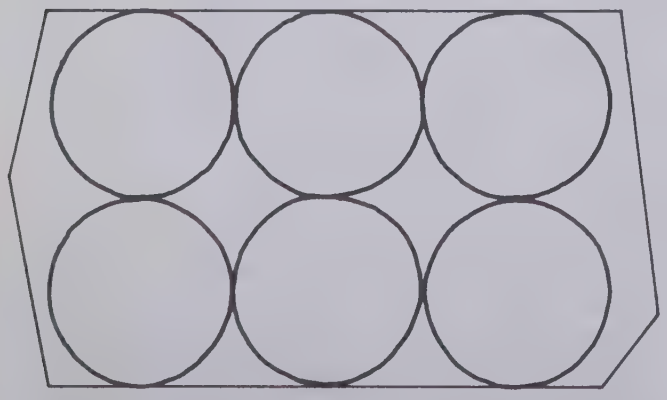
With your class,record the dimensions of your classroom, your desk, their desks, your height, and their heights in terms of centimetres, then in terms of metres using the decimal notation.

ACTIVITY 5

Area using arbitrary units. Here are some examples of area units:



Let the children give names to the units. Then follow the measurement process: select one of these units, match it against the area of some object, and count the number of units used. For example, the irregular figure below has an area of about 6 discs



(if disc is the name given to the unit used). Emphasize that you are trying to "cover" the object. The units should be "even with the edge" of the object, the units should touch, but not overlap, each other. Direct the children's attention to the parts of the object that are not "covered."

Make a cutout of some irregular area such as your thumb and make copies of it out of paper. Use your "thumb" to find the area of the top of a chalk eraser, of the irregular figure measured with the discs, of a cutout of your shoe, and of figure X.

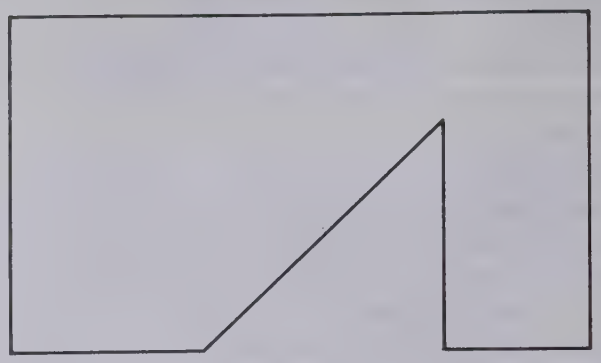


Figure X

Record the answers on the chalkboard in sentence form—

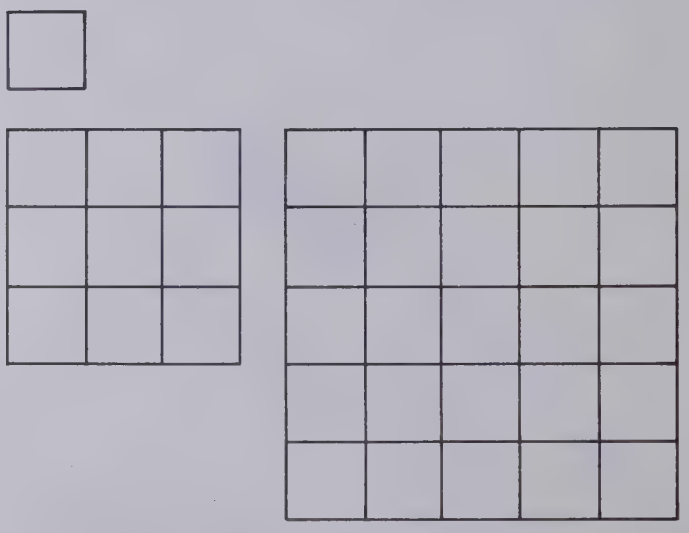
"The figure has an area of about _____ thumbs."

Have your class perform similar activities.

ACTIVITY 6

Area using the centimetre square (cm^2). Have the children make centimetre squares (or have them available for use). The children should have the experience of finding the area of many objects.

Make duplicator masters for some areas that the class can measure with their centimetre squares. The figures below are 1 cm^2, 9 cm^2, 25 cm^2, respectively.



You might point out that the square containing the 9 cm^2 has a side of 3 cm and the square containing the 25 cm^2 has a side of 5 cm.

Have the children use their centimetre squares to find the *area* of a stamp, a 10-cm ruler, the cutout of their thumb, the irregular figure which had an area of 6 discs, and figure X.

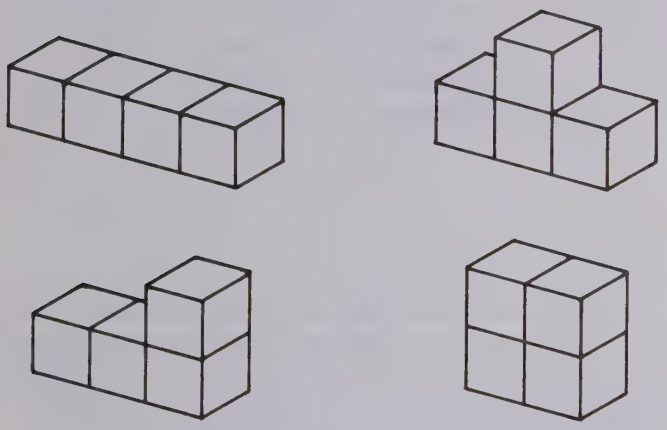
ACTIVITY 7

Volume, using the centimetre cube. In the initial development of the concept of volume, it is important

that children have the opportunity to construct several differently shaped objects each having the same number of volume units.

As with length and area, the study of volume should be introduced with activities making use of arbitrary units of volume, such as blocks, Cuisenaire rods, pencils, erasers, or even marbles.

Use 10 or 12 centimetre cubes in this activity. At first, let the children work on their own, constructing any objects they like. Encourage them to see that an object built of a specific number of cubes has a volume of the same number of cubes regardless of its shape. For example, the illustration shows 4 different constructions, each having a volume of 4 centimetre cubes (4 cm^3).



How many differently shaped objects can be constructed with a volume of 8 centimetre cubes? When those possibilities have been exhausted, try the activity with 10 cubes.

REVIEW: LENGTH, AREA, AND VOLUME

1. Have your class compare the length of their feet, spans, and cubits. Why are these units useless as standard units?
2. Complete these statements.

a.	128 cm = ____ m	**e.**	1.06 m = ____ cm
b.	108 cm = ____ m	**f.**	10.01 m = ____ cm
c.	15 cm = ____ m	**g.**	23.86 m = ____ cm
d.	1010 cm = ____ m	**h.**	0.09 m = ____ cm

3. What would be the length of the sides in a square containing:
 a. 36 cm^2 → ____ cm?
 b. 25 cm^2 → ____ cm?
 c. 4 cm^2 → ____ cm?
 d. 16 cm^2 → ____ cm?
4. How many different-shaped objects can you form with 6 centimetre cubes?

Capacity

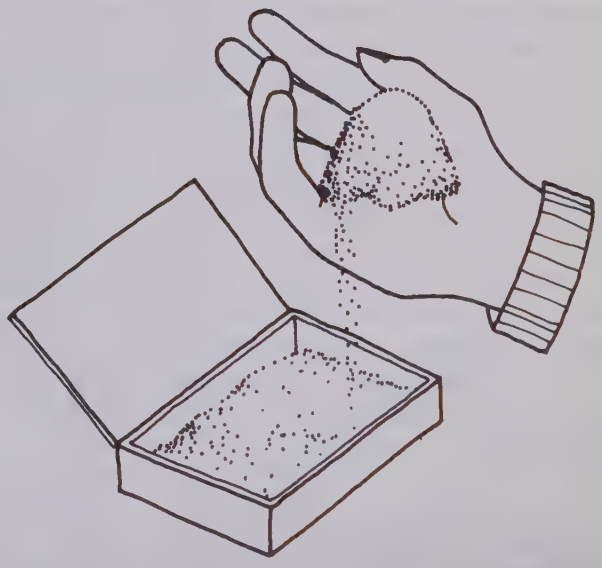
Capacity can be thought of as the amount of material a container will hold. Capacity is usually linked to liquid measure though you may have already had your classes measure capacity by using sand to avoid using liquids.

In the metric system of measurement, volume and capacity are directly related. A container with a volume of 1 cubic centimetre (1 cm^3) will hold 1 millilitre of water. One millilitre (1 ml) is one thousandth of a litre (0.001 ℓ).

The need for fractional names such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ etc. will diminish. The parts of the whole which need emphasis are—0.1, 0.2, 0.3, . . . , 0.9. Of course, in measurement, fractions could disappear completely, since $\frac{3}{8}$ of a meter is 0.375 m or 375 mm. However, when working with the litre (the unit of capacity in the metric system) don't worry now about using $\frac{3}{4}\ \ell$, $\frac{2}{3}\ \ell$, etc. if it is the amount you want the children to see or work with. Since the metric system is based on 10 and since 1, 2, 5 and 10 are the only divisors of 10, we will probably talk about halves, fifths, and tenths of metric units. The decimal notation ($\frac{1}{2}$ is 0.5) will prevail eventually, even at the primary level.

ACTIVITY 8

Capacity and arbitrary units. The most obvious capacity units are handfuls. Give each child a container to fill with water or sand or other material you prefer to use. Have the children fill the container



(milk carton, ice cream carton, cigar box, etc.) with "handfuls" of material. Have them record their results on a piece of paper: "My carton holds _____ handfuls of _____." Compare the wide range of results. Re-emphasize the need for a standard unit to measure capacity. If further experience is necessary, you may want to repeat the project with cups brought from home (since there are so many different sized and shaped cups). Try the activity yourself or get several containers such as an ice cream carton, a milk carton, a wastebasket, a big cooking pan, and a litre container.

On a piece of paper write a pair of sentences for each container:

"The (name of container) holds about (guess) litres.

The (name of the container) actually holds (result) litres.

In the first blank "guestimate" the number of litres the container will hold. In the second, write in the results of measuring the object.

Don't forget the three step measuring process—

1. Select the unit—the litre.
2. Match the unit against the object—fill the object using the litre.
3. Count the number of units (litres) used.

When the container is full (it is best to have a "fill line" just below the top of the container) round off to the nearest whole litre according to whether more or less than half of the last litre was used.

ACTIVITY 9

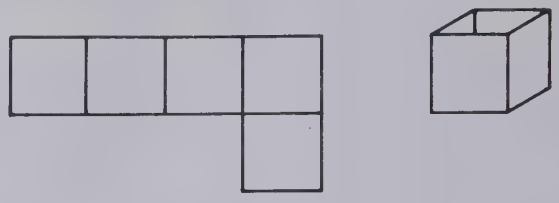
Working with the litre. Get a container that holds a litre of water(and, ideally, has submarkings for each 100 ml). When you are collecting containers for your classroom, try to get as many different shapes as you can. It is important, especially in early experiences, that the children see that litre containers can come in many different shapes. It is the quantity the container will hold, not its shape that determines a capacity of 1 litre.

Once you get a litre container you can make many more. Pour a litre of water into a container and mark a "fill line" for 1 litre on the outside with tape, or, if possible, cut the container so that it holds just 1 litre. Suggested existing containers which can be cut are quart, half-gallon, and gallon milk cartons, round quart, half-gallon, and gallon ice cream cartons. Containers that can be marked might be various shaped pans, cooking bowls, large tin cans, and bottles or jugs. Most activities for introducing the metric units should be accompanied by some estimation exercises. Have the students estimate and record how many litres a container will hold, then measure the container to see about how many litres it does hold. Compare records.

ACTIVITY 10

Introducing the millilitre. The litre is a unit for capacity that is used for milk, gasoline, paint, and other quantities of considerable size. The litre is not used to measure small quantities, such as toothpaste, soda pop, medicines, frozen orange juice, etc. The unit used for the smaller measures is the millilitre (ml). If your school is going to get a set of metric capacity containers, try to get them in these sizes—1 ℓ, 500 ml, 200 ml, 100 ml, 50 ml, 20 ml, and 10 ml. With such a set (whether bought, given, or constructed) one can do all the activities that are necessary.

Construct a container with a volume of 1 cubic centimetre (1 cm^3) to demonstrate the size of the millilitre (ml). Trace the figure below, then cut it out and tape it together along the edges. If you avoid spillage your cube will hold 1 ml of water.



The children need several activities measuring the capacity of objects and recording the results in millilitres. Have them first guess and then measure the capacity of a thimble, a match box, a tablespoon, and a teaspoon. Record the results in sentences like—

"I estimate that the thimble holds about _____ ml.
It actually holds about _____ ml."

Mass

As the metric system becomes the predominant system of measurement you may hear talk about the difference between mass and weight. A lunar example may be the best way to show the difference. Now that we are in the space age, practically everyone knows that a man weighs less on the moon than he does on the earth. For example, a 300-kg man on earth would weigh about 50 kg on the moon, but he would have the same mass on the moon as he does on earth. Weight is dependent upon gravity, mass is not. Begin to stress the use of the correct metric term, mass.

The base unit of mass in the metric system is the kilogram (kg). For example, we say "I have a mass of 78 kg.

ACTIVITY 11

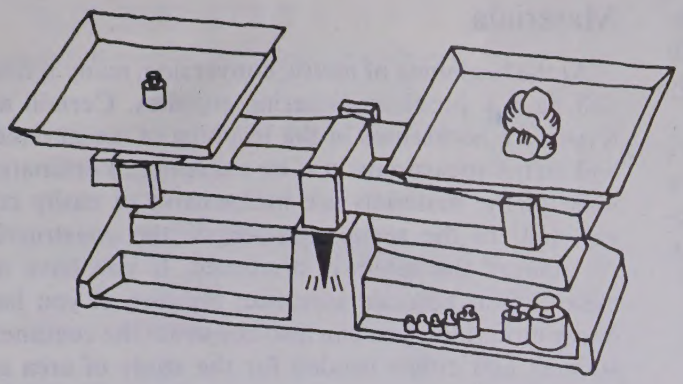
Arbitrary units of mass. To find the mass of an object you will need a balance and some arbitrary units such as paper clips, pencils, Cuisenaire rods, pennies, or other objects. Put a pencil on one side of the beam and then "balance the pencil" with pennies (or multiples of any other small unit). Record the results on paper in a sentence like:

"The pencil has a mass of about _____ pennies."

Repeat the activity with at least three other objects.

ACTIVITY 12

The unit used for small masses is the gram (g). This activity is very similar to the last. You will need gram masses. If you have a classroom set, that's great! If you don't, you can make one.



Put a gram mass on one side of the balance and balance it with a lump of clay or plasticine. Label your clay "1 g." In a similar manner make a set of clay or plasticine "masses" in multiples such as: 5 g, 10 g, 20 g, and 50 g. Use several small objects as test objects (a paper clip, a nickel, a penny, and a pencil). However, before you have the children put one of the test objects on the balance, ask them to estimate its mass in grams. Then find the mass of the object. Record both the guess and the result.

The quarter has a mass of about (guess) grams.

It actually has a mass of (result) grams.

Repeat the activity using other objects. Do you and the class get better at estimating mass?

ACTIVITY 13

Measuring mass using the kilogram. Hopefully, all schools will have metric scales available for finding the mass of children and other large objects using kilograms. For this activity, have each child find his own mass and then make and label a cutout of himself (perhaps using his projected shadow). Have him record his height and mass in metric units on the cutout.

Then you and your class might measure the mass of other objects, such as your own chairs, the textbooks used in the course of one day, litre of water (don't count the container—first find its mass when empty), a dictionary, and even the principal of the school (if he agrees). As mentioned earlier, there is a direct relationship between volume and capacity in the metric system of measurement. In fact, there is a direct relationship between volume, capacity, and mass. A container whose volume is 1 cubic cm (cm^3) holds 1 ml of water and the 1 ml of water has a mass of 1 g. A container whose volume is 1000 cubic cm (or 1 cubic decimetre) holds 1000 ml of water (or 1 litre), and the water has a mass of 1000 g (or 1 kilogram). What did you get for the mass of one litre of water?

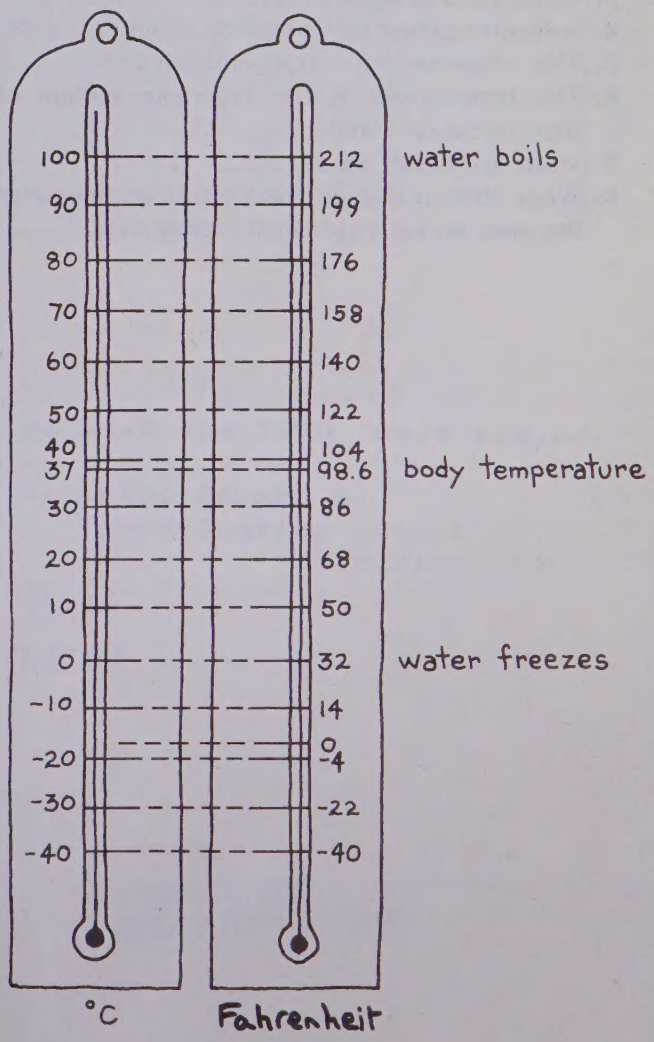
N.B. It is a good idea to label some of the objects in the room as you introduce each metric unit. For example, the aquarium may be 70 cm long, 40 cm wide, 35 cm high; have a water surface area of 2 800 cm^2, volume of 98 000 cm^3; a capacity of 98 ℓ of water and a mass of 12 kg. If the children label the objects as they study particular units, they will begin to think metric.

REVIEW: CAPACITY AND MASS

1. When finding the mass of something using a balance beam, how do you decide which unit to round off to?
2. Fill in the answers:
 - **a.** 28 ml of water has a mass of about _____ grams.
 - **b.** 170 ℓ is _____ ml.
 - **c.** 3.12 kg is _____ g and 438 g or _____ kg.
 - **d.** It would take _____ ml of water to balance 1 kg.
3. **a.** Will a car get a higher or a lower number of miles per litre than miles per gallon? (Is the litre larger or smaller than the gallon?)
 - **b.** Will a car get a higher or a lower number of kilometres per gallon than miles per gallon? (Is the kilometre longer or shorter than the mile?)
 - ★ **c.** Gasoline consumption rates will be given in kilometres per litre. Will a car get a higher or a lower number of kilometres per litre than miles per gallon?

Temperature

This last section covers the introduction of a metric unit, the degree Celsius (°C), for which there is no physical model. On the Celsius scale for temperature, water boils at 100° C and freezes at 0° C. The unit is named after the Swedish scientist, Anders Celsius, who created the centigrade temperature scale. The



Celsius and centigrade scales are the same, but centigrade is no longer the proper term since the centigrade is a unit used to measure angles in the metric system.

The best way to get used to the Celsius temperature scale is to use it! It is almost a necessity that you have a Celsius thermometer. However, if you have a demonstration model of the Fahrenheit thermometer, you can rescale it using the nomograph shown here.

ACTIVITY 14

Graphing temperatures. Be sure to give the children lots of opportunities to read the temperature and record it in degrees Celsius (° C). Perhaps you could institute a morning weather report given by a different child each day to get the class to use Celsius thermometers and to give them a feeling for what the temperature is when expressed in degrees Celsius (° C). The previous day's high and low temperatures (taken from a newspaper account) could be recorded on a wall graph.

REVIEW: TEMPERATURE

1. My body temperature is about _____ °C.
2. Normal room temperature is about _____ °C.
3. Water boils at about _____ °C.
4. A warm summer day would be about _____ °C.
5. The temperature in a refrigerator is about ____ °C.
6. The temperature in the vegetable section of a supermarket is about _____ °C.
7. Water freezes at about _____ °C.
8. What temperature on the Celsius thermometer is the same on the Fahrenheit thermometer? ____ °C

Material

At the beginning of metric conversion, many schools will have a problem gathering supplies. Certain materials are necessities in the teaching of measurement and metric measurement is no exception. Fortunately, most of the materials are inexpensive or easily constructed. In the section on length, the construction of some of the rulers is discussed. If you have one metric ruler, you can construct the rest. If you have one metric ruler, you can also construct the centimetre squares and cubes needed for the study of area and volume.

The construction of units of capacity and mass have also been discussed. When it comes to temperature you should have a thermometer available for classroom use. If it is a Fahrenheit thermometer, then you should rescale it to degree Celsius using the nomograph given earlier.

Following is a list of companies and government agencies that are currently producing materials or can give some assistance with this problem of teaching the metric system of measurement.

Addison-Wesley (Canada) Ltd.—Don Mills, Ontario
Buntin Gillies & Co. Ltd.—Ottawa, Ontario
Cameron Products—Bramalea, Ontario
Canadian Metric Association—(P.O. Box 35)—Fonthill, Ontario
Contrasts 20—Calgary, Edmonton, Vancouver, Winnipeg, Regina (Nearest Barber-Ellis Office)
Kruger Pulp and Paper Ltd.—Moncton, Toronto, Hull, Montreal (Nearest Office)
Information Canada (Under Government of Canada) (Nearest Office)
Jack Hood School Supplies Co. Ltd.—Stratford, Ontario
Lufkin Rule Co. of Canada Ltd.—Don Mills, Ontario
Lily Cups Ltd.—Scarborough, Ontario
MacLean-Hunter Learning Materials Co.—Toronto 101, Ontario
Metric-Aids Ltd.—Toronto, Ontario
Moyer-Vico Ltd.—Moncton Weston, Winnipeg, Saskatoon, Edmonton, Vancouver and the Longueuil Co. in Chambly (Nearest Office)
The National Council of Teachers of Mathematics—1906 Association Drive, Reston, Virginia 22091
Sargent-Welch Scientific Co. of Canada Ltd.—Weston, Ontario
Spectrum Education Ltd.—Toronto, Ontario
Spicars International Ltd.—Scarborough, Ontario
Toronto Dominion Bank (Nearest Office)

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